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Don't look back

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Historical simulation may be a natural setting for scenario analysis, but it must take account of current market conditions, caution Giovanni Barone-Adesi, Frederick Bourgoin and Kostas Giannopoulos

Value-at-risk is becoming increasingly popular as a management and regulatory tool. But before this acceptance goes much further, we need to assess its reliability under financial market conditions. Most VAR models deal either with the non-normality of security returns or with their conditional heteroscedasticity, but not with both. We are developing a modified historical simulation approach that allows for both effects.

Historical simulation relies on a specific distribution (usually uniform or normal) to select returns from the past. These returns are applied to current asset prices to simulate their future returns. Once enough different paths have been explored, it is possible to determine a portfolio VAR without making arbitrary assumptions about the distribution of portfolio returns. This is especially useful where there are abnormally large portfolio returns.

It is well known that large returns cluster in time (see, for example, Mandelbrot, 1963, and Black, 1976). The resulting fluctuations in daily volatility make the confidence levels of some VAR calculations unreliable (Boudoukh *et al*, 1995). This is the case with those that ignore clustering, such as VAR measurements based on the standard variance-covariance matrix and Monte Carlo methods, which typically ignore current market conditions to produce flat volatility forecasts for future days. Moreover, the use of the covariance matrix of security returns or the choice of an arbitrary distribution in the Monte Carlo method usually destroys valuable information about the distribution of portfolio returns.

To make our historical simulation consistent with the clustering of large returns, we model the volatility of our portfolio as an asymmetric Garch (Generalised autoregressive conditional heteroscedasticity) process (Engle & Ng, 1993) that generalises the Garch model. This model allows positive and negative returns to have different impacts on volatility (known as the leverage effect, see Black, 1976). Past daily portfolio returns are divided by the Garch volatility estimated for the same date to obtain standardised residuals. These are independent and identically distributed (IID) and are therefore suitable for historical simulation.

To adjust them to current market conditions, we multiply a randomly selected standardised residual by the Garch forecast of tomorrow's volatility. In this way, a simulated portfolio return for tomorrow is obtained. This simulated return is used to update the Garch forecast for the following day, which is then multiplied by a newly selected, standardised residual to simulate the return for the second day. Our recursive procedure is repeated until the VAR horizon (ie, 10 days) is reached, generating a sample path of portfolio volatilities and returns. We repeat our procedure to obtain a batch of sample paths of portfolio returns. A confidence band for the corresponding portfolio values is built by taking the kernel (empirical) frequency distribution of values at each time. The lower 1% area identifies the worst case over the next 10 days.

To illustrate our procedure, we constructed a hypothetical portfolio, diversified across all 13 national equity markets in our data sample. To form our portfolio, each equity market is weighted in proportion to its capitalisation in the world index (MSCI) as at December 1995. The portfolio weights are reported in table A.

These weights are held constant for the entire 10-year period and multiplied by the 13 local index returns. So the portfolio returns are calculated again backwards to reflect the current weightings. Since the aim of market risk is to quantify eventual portfolio losses in a single currency, all local portfolio returns are measured in dollars. The descriptive statistics, together with the Jarque-Bera (1980) test for normality, are shown in table B, where the p-value indicates the probability that our portfolio returns are generated from a normal distribution.

Figure 1 shows the empirical distribution of the portfolio's returns. The rejection of normality in table A and the pattern of clustering visible in figure 1 leads us to model our portfolio returns, r_t, as a Garch process with asymmetries, with daily volatility, h_t, given by:

$$r_t = R_t = \mu + \varepsilon_t \tag{1a}$$

$$h_t = \omega + \alpha(\varepsilon_{t-1} + \gamma)^2 + \beta h_{t-1} \tag{1b}$$

The variance for small increments on the other end can be written as:

$$h_t^2 = c_t^2 \Delta t = O(\Delta t)$$

The daily return in equation (1a) is the sum of each expected value, μ, plus a random residual, ε_t. Because of the small, statistically insignificant value of μ^2, this term will be neglected in the calculation of daily volatilities.[1] Equation (1b) defines the volatility of ε_t, h_t, as an asymmetric Garch process. h_t is the sum of a constant, ω, plus two terms reflecting the contributions of the most recent "surprise", ε_{t-1}, and the last period's volatility, h_{t-1}. Finally, γ allows for the asymmetric response of the innovation on the volatility and is statistically significant.

Therefore, our portfolio volatility is modelled to depend on the most recently observed portfolio returns. The combination of asymmetric Garch volatility and portfolio historical returns offers us a fast and accurate

A. Portfolio weights

Country	Our portfolio	World index (Dec 1995)
Denmark	0.004854	0.004528
France	0.038444	0.035857
Germany	0.041905	0.039086
Hong Kong	0.018918	0.017645
Italy	0.013626	0.012709
Japan	0.250371	0.233527
Netherlands	0.024552	0.022900
Singapore	0.007147	0.006667
Spain	0.010993	0.010254
Sweden	0.012406	0.011571
Switzerland	0.036343	0.033898
UK	0.103207	0.096264
US	0.437233	0.407818

B. Descriptive statistics of the equally weighted portfolio

Mean (pa)	10.92%	Std dev (annual)	12.34%
Skewness	−2.82	Kurtosis	62.36
Normality	3,474.39	p-value	0

[1] *In fact, for stock prices, μ^2 is in the order of $\mu^2\Delta_t$:*

$$\mu^2 = c^2 \Delta t^2 = O(\Delta t^2)$$