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# Economic efficiency of methods of imperfect hedging financial options

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# EXECUTIVE SUMMARY


Based on the positive trends of the futures market, it can be concluded that in addition to active speculators in the market, there are hedgers as well. Position protection against the price risk becomes more relevant in the unstable individual segments of the global economy and the crisis in the international markets.

This study deals with the theory of hedging in derivatives market, as well as, the practice of using these operations of financial derivatives in developed countries. The study focuses on the main hedging strategy, which is the basis of complex strategies creation. In this study less attention is paid to more complex strategies, as they include the basic strategy. With knowledge of the technique of basic strategies, the investor will be able to build one that will satisfy all the exact needs.

The aim of this paper is to study the latest techniques of pricing and hedging of financial options, comparing new techniques with traditional techniques , in the financial industry, as well as an analysis of the possibility of imperfect hedging in the Russian derivatives market.

The first chapter is defined relevance of the chosen topic and described the history and development of the derivatives market in Russia.

The second chapter consists of theoretical aspects, it describes the concept of derivatives market. Description of the theory of hedging of financial options and the strategies of hedging using options contracts, The Black-Scholes model and volatility.

The third chapter describes Methods of imperfect hedging of financial options, Algorithm of successful hedging, and successful hedging variety, described the major foreign futures and options markets. Moreover, the third chapter indentifies examples of hedging various risks in companies, the use of hedging instruments at the foreign trade operations by Forex market participants.

The fourth chapter (practical) chapter shows the practical aspects of hedging on Sberbank shares and a currency pair USD - RUR example. In this study various sources of information, both Russian and foreign authors, were used. The foundation of this study can be found in John Hull "Options, futures and other derivatives" research and Burenin A. N. "Forwards, futures, options, exotic, and weather derivatives" research, in addition, was used the information of interviews with investment banks and assistance of the project manager (Prof. Kostas Giannopoulos).

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# CHAPTER 1: INTRODUCTION

An important result of ongoing structural reforms in the country was a significant increase in interest to the derivatives market. Economic agents begun to realize the importance of hedging mechanisms as a tool of risk management, and some of them began to use derivative financial instruments to protect their investments against possible losses. In many ways, it has become possible due to legislative changes in the taxation of corporate profits, including the profit from transactions with financial instruments of futures contracts.

The problem of hedging of derivative instruments is more actual in the domestic financial science. Real economic needs require a construction of new financial instruments from the organizers and the leading operators of the derivatives market, that allows to hedge the majority of operations performed on the cash market. At the time, when the Russian market of futures is sufficiently formed, the market for financial options, which gives higher flexibility to manage the risk, is still in the beginning of its developing. Need to increase the amount of the risk management tools and implementation in practice methods of scientific risk management of the professional participants of the Russian derivatives market, based on a strict formalization of investment decisions, determines **relevance of the topic of this research.**

The classic solution of the problem of hedging of financial options is given in the concept of a perfect hedge. Its foundations were laid by well-known works of Black and Scholes, Merton, Cox, Ross and Rubinstein. The concept of a perfect hedging is focused on constructing a portfolio of market assets that generates payments equal to payments of financial instruments of derivatives. In the absence of arbitrage, value of this portfolio determines the value of the derivative.

At the moment, the strategy of perfect hedging is established as the standard approach of financial engineering, which have found wide acceptance in the theory and practice of risk management. The main feature is that the value of a financial option is determined regardless of the preferences and characteristics of its owner. Methods of perfect hedge does not take into account the expectations of the holder of urgent position, his attitude to risk, as well as the characteristics of investment strategy.

A new approach to risk management of urgent positions offer methods imperfect hedge. A main feature of the imperfect methods of hedging is that the construction of a hedging strategy allows the possibility of loss. Due to this, at a manageable level of risk, releases a new capital of financial transactions, which opens up additional opportunities of risk management for comprehensive investment strategy.

Undoubted advantage of imperfect hedging techniques is the ability to take into account the expectations of market participants and its attitude to risk. In practical terms, the investor, who is active in the futures market and ready to take a controlled risk, methods of imperfect hedging are new tools to support investment decisions.

Despite these advantages of an imperfect hedge, the first steps of theoretical interpretation have been made only in the 1990s in the study of Duffy and Leykerta (1999,2000). Moreover, research has focused on finding a mathematical solution to the issue of the construction of imperfect hedge.

It identified the need for theoretical developments that are based on mathematical models of imperfect hedging techniques, but at the same time taking into account economic content, as well as empirical studies of the practical application of these methods. Implementation of this research is a prerequisite for the establishment of the methods of risk management in the practice of professional participants of the Russian derivatives market.

At the present stage, imperfect hedging methods have not been sufficiently studied and require active attention. The need to understand methods of imperfect hedging of financial options, use of mentioned methods in the Russian derivatives market and led to the choice of the topic of this study, as well as its purpose, objectives, structure and content.

The objectives are to know the latest pricing techniques and financial options hedging, comparison of new techniques with traditionally used in the financial industry, as well as an analysis of the possible use of imperfect hedging on the Russian derivatives market.

To achieve the above mentioned objectives, the following tasks have been made:

1. To study the features of the theory and practice imperfect hedging methods of financial options;

2. Set the economic advantages and disadvantages of the various optimality indicators of hedging strategy;

3. Determine the optimal conditions for the use of imperfect hedging methods of financial options.

4. Develop a step-by-step algorithm implementing these methods in practice for various types of options.

5. Carry out an empirical study of the possible usage of imperfect hedging of financial options in the Russian derivatives market.

6. Create a spreadsheet in Microsoft Excel application to select the necessary method of hedging by entering data of the dynamics of the asset and the expectations of the hedger.

The object of study is the Russian derivatives market, and the subject - price risks of urgent position holder.

The information base for the study is the data on the derivatives market FORTS from Moscow Stock Exchange RTS and the Central Bank of Russia.

As a methodological basis of the study were selected system analysis, methods of synthesis and comparison, methods of mathematical and statistical analysis, probability theory.

The practical significance - results can be used by business entities for financial risk management.

## 1.1 The value of the derivatives market for the country's economy.

The market economy is a set of different markets. One of them is the financial market, in which distribution of funds between the participants of economic relations is made. If you look at the financial market in the time section, it can be divided into two segments: the cash and derivatives market.

The cash market - a market in which a deal is delivered immediately. The time of execution is not intentionally separated in time from the moment of the transaction. In fact sometimes there is a certain delay between them due to technical reasons, but usually it does not exceed two days.

In the derivatives market there are futures contracts. Futures deal is a contract between two or more parties, the execution of which is deliberately separated in time from the moment of its signing that is directly settled. Usually the date of execution lags behind the date of the transaction for more than two days. An important feature of the futures deal is that the conditions of performance are specified at the time of its conclusion. In the derivatives market derivatives are traded. Derivatives are financial instruments whose value depends on the price of some underlying asset (the goods, currency, stocks, bonds), interest rate, stock index, the temperature or other value of a quantitative indicator, which is generally called the foundation.

As a rule, the implementation of the rights of the derivative is separated in time from the date of the transaction, so in the professional literature derivatives are often called fixed-term instruments. By the type of implement of the rights there is distinguished delivery and cash settlement. In the first case it is a deliverable derivative instruments, in the second - the settlement.

Futures market is high-yield though risky field for investment. However, derivative transactions bring not only great opportunities for speculative game, but also bring real benefits to society.

It is necessary to distinguish the following benefits for society related to the use of the derivatives.

Firstly, the financial derivatives market open new opportunities to participants to manage the risk of economic activity. They allow decomposing it into components. By buying or selling these components, the company can focus on managing certain kind of risk, and pass on other risks to the market participants that are better in dealing with them.

Second, market participants can use derivative instruments for speculative purposes. Speculator assumes the risk in an effort to obtain a certain profit based on one’s expectations.

Third, the derivatives market provides new opportunities for arbitrageurs. These market participants are seeking to make a profit due to the simultaneous opening of the long and short positions that have the same value, but one of the functions of payments is clearly preferable to the other.

Fourth, derivatives open investment opportunities that cannot be done in any other way. Thanks to financial engineering, market participants can develop customized solutions for specific problems of economic activity. In particular, the derivatives allow greater flexibility in terms of regulatory restrictions.

Fifth, derivatives allow using the effect of the leverage quite easily and at relatively low cost. That gives the opportunity to reach an entirely new level in relations between return and risk, which would not be implemented by the position in cash market instruments.

Sixth, the futures market instruments reduce transaction costs, in particular, for small and medium market participants to achieve a certain payment structure.

Only large companies can effectively implement dynamic hedging derivative instruments, as they usually have a well-established infrastructure and the high volume of transactions reduces the cost of fees. For example, the purchase of a derivative with multiple underlying assets leads to lower costs for the diversification of the investment portfolio.

Seventh, the futures market performs the informational function. Many institutions involved in the analysis or the conduct of monetary policy, are constantly analyzing information embedded in the prices of financial assets.

For example, implied volatility is the volatility of the market valuation of the underlying asset during the term of the fixed-term contract.

The price of such contracts is an indicator of investors' economic expectations. Empirical studies have confirmed that the application of risk management in practice promotes success to commercial activities of companies. For example, Gechi, Minton and Shranda (1997) studied foreign currency hedging of 500 largest firms in the annual ranking of the magazine "Fortune". The authors concluded that the use of currency derivatives has a positive impact on the growth of the company's potential, measured by the cost of research and development activities.

Alleyannisa and Weston (1998) research confirms direct dependence of the use of hedging techniques and the value of the company. The authors examined the use of currency derivatives on a sample of 720 large non-financial companies in the USA from 1990 to 1995 and its impact on the value of the company, expressed by Tobin's Q ratio. This ratio represents the ratio of the market value of debt and equity to the current replacement value of the assets the company. Under the replacement value means the value of all assets in which they can be replaced at this time. Empirical research has shown that the use of derivatives for hedging increases the value of company. Statistically significant that it was found that the coefficient of Tobin's Q is in average for 5.7% higher for companies hedging currency risks, than companies that do not hedge.

The owner of derivative position is also exposed to credit risk, which is in danger because of the other party or parties default the obligations for a futures contract. For futures contracts traded on the Exchange, the risk is quite low due to the clearing system. More attention deserves credit risks in the OTC market, where the parties negotiate their own characteristics of a derivative meeting their needs, and monitor its implementation.

Derivatives also are characterized by liquidity risk, which reflects the ability of a financial instrument being convertible into cash. Liquidity risk plays an important role not only in the futures market, but also in other markets, especially during periods of high volatility and severe economic shocks.

Prudent risk management strategy, consistent with the objectives of the economic activity of the company and confirmed by the reliable empirical studies is essential for the use of derivatives. Only in this case, the benefits of using instruments of the derivatives market are the highest.

## 1.2. Structure and dynamics of the global derivatives market.

The development of exchange and OTC segments of the global derivatives market began in the early 1970s., derivatives on financial assets. Growth continued rapidly until 2009, during which the market could only maintain current trading volumes. Nevertheless, in 2010 the global market of exchange derivatives returned to pre-crisis growth rates, with an annual increase in turnover of 26%, which is 2% less than the increase in 2007. The volume of world trade with standard contracts has increased almost 10 times from 2.4 billion contracts in 1999 to 22.3 billion contracts in 2010, also the geographical structure of the derivatives market has changed. If the beginning of the 2000s. absolute urgent trade leaders were the United States and Western Europe, today the total leader is the Korean market. The second leader after Korean market could be the United States Futures Exchange. Almost 1 billion contracts less has US-European Futures Exchange alliances EUREX and NYSE Euronext. The top ten of world's leading exchanges, trading futures and options, consists of India, Brazil, China etc. RTS shows consistently high growth rates and trading volumes, moreover, at the end of 2011 it is in 11th place in the list of the largest futures exchanges in the world.

The main components of a permanent high-speed growth of the world futures and options market are as follows:

- Intensive growth in trading volumes on the futures market in the Asian-Pacific region and Latin America (42.8% and 49.6% in 2011. compared to 2010), Exchange Zhen-zhou and Shanghai (China) Exchanges showed maximum growth in trading commodity futures. On two of the largest trading platforms in India, turnover of currency futures tripled. In general, the highest increase in trading volume showed Indian Exchange Market - 142% in 2011 compared with the previous period;
- The gradual recovery of the segment of interest rate derivatives, which for many years held a leading position of trading in derivatives markets (information on the distribution of trading volumes are presented in Table 1.1) lost 23% in 2009 compared to 2008, when the underlying assets market was in a deep crisis;
- Keeping strong growth of commodity futures trade, among which the main interest of developing markets participants were on agricultural products and non-ferrous metals.

**Table 1.1. Global futures and options market structures dynamics.**

| Category of tool | 1999 | | 2009 | | 2013 | |
|---|---|---|---|---|---|---|
| | billion. contracts | % Of total | billion. contracts | % Of total | billion. contracts | % Of total |
| Stock indices | 0,51 | 21,25 | 6,38 | 35,96 | 7,74 | 34,71 |
| Individual shares | 0,7 | 29,12 | 5,59 | 31,51 | 6,29 | 28,21 |
| Interest rates | 0,79 | 32,92 | 2,47 | 13,92 | 3,21 | 14,39 |
| Currencies | 0,05 | 2,13 | 0,99 | 5,59 | 2,4 | 10,76 |
| Commodities | 0,35 | 14,58 | 2,31 | 13,02 | 2,66 | 11,93 |
| Total | 2,4 | 100 | 17,74 | 100 | 22,3 | 100 |

## 1.3. Russian derivatives market.

Over the past 10 years, development of this segment of the domestic economy was not slower, in terms of quantitative and qualitative indicators, than the global markets, moreover, Stock Exchange RTS is among the 20 largest futures exchanges, offering fixed-term contracts for market participants, to meet the current needs of economic entities. Table 1.2 shows the structure and dynamics of the Russian derivatives market.

**Table 1.2 Structure and dynamics of the derivatives market of Stock Exchange RTS (FORTS)**

| Indicator | 2008 | | 2009 | | 2010 | | 2011 | |
|---|---|---|---|---|---|---|---|---|
| | futures | options | futures | options | futures | options | futures | options |
| Volume, bln.rub | 6207,3 | 1305,9 | 9394,9 | 1762,7 | 13660 | 509,1 | 27986 | 1364, 3 |
| Turnover, mln. Contracts | 119,7 | 25,2 | 193,6 | 46,2 | 454,5 | 20,0 | 593,7 | 23,8 |
| The number of transactions, million. | 11,4 | 0,3 | 28,0 | 0,7 | 73,4 | 0,9 | 114,9 | 2,0 |
| Total open interest at the end of the year, bln. Rubles. | 73,04 | 68,7 | 28,08 | 10,0 | 67,1 | 17,4 | 100,8 | 40,8 |
| The amount of open positions end of year, contracts | 1,7 | 1,5 | 1,3 | 0,5 | 2,2 | 0,9 | 3,2 | 0,9 |

RTS Stock Exchange derivatives market, which is the only developed segment of the derivatives market in Russia, has both - similarities and differences with the development of world derivatives market. Options market still develops much slower than futures market, and the crisis of 2008-2009. was an additional deterrent, reducing the volume of trading options for almost 3 times. By the end of 2011, the options market trading volume achieved the level of development of Year 2007. In addition to the crisis, replacement of options to futures options had the impact on the decline of volume of trading options.

FORTS offers necessary tools for all major groups of the underlying assets. The main share of trading, in Russia and global market, has contracts issued on the stocks and shares. However, the weakest segment of the domestic futures market is the market of interest rate derivatives (opposed of global market), on which there are no options on interest rates, as well as during 2008-2011. there were no contracts for bonds.

This research focuses on financial options, so it is necessary to propose a set of given options contracts on the Russian derivatives market FORTS on the Stock Exchange RTS. Below is a set of provided option contracts on FORTS market.

**1. Indices**

**MX** Futures-style call option on a futures contract on the MICEX Index

**MX** Futures-style put option on a futures contract on the MICEX Index

**RI** Futures-style call option on a futures contract on RTS

**RI** Futures-style put option on a futures contract on RTS

**2. Shares**

**GM** Futures-style call option on a futures contract on ordinary shares of MMC "Norilsk Nickel"

**GM** Futures-style put option on a futures contract on ordinary shares of MMC "Norilsk Nickel"

**GZ** Futures-style call option on a futures contract on ordinary shares of OAO "Gazprom"

**GZ** Futures-style put option on a futures contract on ordinary shares of OAO "Gazprom"

**LK** Futures-style call option on a futures contract on ordinary shares of OAO "NK" LUKOIL "

**LK** Futures-style put option on a futures contract on ordinary shares of OAO "NK" LUKOIL "

**RN** Futures-style call option on a futures contract on ordinary shares of OAO "NK" Rosneft

**RN** Futures-style put option on a futures contract on ordinary shares of OAO "NK" Rosneft

**SN** Futures-style call option on a futures contract on ordinary "Surgutneftegaz"

**SN** Futures-style put option on a futures contract on ordinary "Surgutneftegaz"

**SR** Futures-style call option on a futures contract on OJSC "Sberbank of Russia"

**SR** Futures-style put option on a futures contract on OJSC "Sberbank of Russia"

**TN** Futures-style call option on a futures contract on the preferred shares of "Transneft"

**TN** Futures-style put option on a futures contract on the preferred shares of "Transneft"

**VB** Futures-style call option on a futures contract on ordinary shares of JSC VTB Bank

**VB** Futures-style put option on a futures contract on ordinary shares of JSC VTB Bank

**3. Currencies**

**ED** Futures-style call option on a futures contract on the euro-dollar

**ED** Futures-style put option on a futures contract on the euro-dollar

**Eu** Futures-style call option on a futures contract on the euro - the Russian ruble

**Eu** Futures-style put option on a futures contract on the euro - the Russian ruble

**Si** Futures-style call option on a futures contract on the exchange rate USD - Russian Rouble

**Si** Futures-style put option on a futures contract on the exchange rate USD - Russian Rouble

**4. Commodity contracts**

**BR** Futures-style call option on a futures contract on Brent oil

**BR** Futures-style put option on a futures contract on Brent oil

**GD** Futures-style call option on a futures contract on refined gold bullion

**GD** Futures-style put option on a futures contract on refined gold bullion

**PT** Futures-style call option on a futures contract on platinum bullion

**PT** Futures-style put option on a futures contract on platinum bullion

**SV** Futures-style call option on a futures contract on refined silver

**SV** Futures-style put option on a futures contract on refined silver

### 1.3.1. Russian derivatives market formation and problems in the current stage.

The first stage of the derivatives instrument market development was in the period since 1992. to 1994. This stage is characterizing significance of foreign exchange contracts. Date of birth of the derivatives market in the country is considered to be October 21, 1992. On this day, at the Moscow Commodity Exchange (Московская товарная биржа) was held its first trading in futures on the $ 10 with delivery at the expiration of 2 months. December 8, 1992 on the Tyumen-Moscow Stock Exchange "Hermes" (later renamed as Options and Futures Exchange) was introduced first option on US dollar - Russian Ruble exchange rate. The rapid depreciation of the Russian ruble in "Black Tuesday" October 11, 1994 and the events that followed after, led to a reduction in the volatility of exchange rates, and the derivatives market has lost its investment attractiveness.

The second stage took place from the end of 1995. and the first half of 1997. During this period, the formation of the derivatives market is different, main significance has of contracts in the government bonds. On November 9, 1995. on the Options and Futures Exchange market appeared futures option to OGSZ. On April 10, 1996 SPSE launches trade options on futures based on the profitability index coupon OFZ, and in May 1996, these exchanges are offering options on futures on T-bills. Since the beginning of 1997, MCSE offers futures on primary auctions and secondary trading, as well as options on these futures.

The third stage took place in the second half of 1996 and the beginning of 1998 and differs with a replacement of different settlement contracts for short-term bonds on deliverable contracts for corporate securities, which now is monopolist on the futures market. First trades on futures contracts of "Lukoil" and JSC "Mosenergo" shares were held in September 1996 on the Russian Stock Exchange (Российская биржа). The Russian derivatives market growth rate in this segment were unbelievable. In February 1997, the volume of trading futures on the shares of "Mosenergo" on Russian Stock Exchange exceeded 1500% compared to January. In this period was changing a range of tools offered by different trading platforms.

The fourth stage (second half of 1998 - the first half of 2000) - the period of analysis, awareness of mistakes and the beginning of efficient derivative securities market. After the crisis of August 17, 1998, the volume of securities transactions sharply reduced, and, as a consequence, the derivatives market had virtually disappeared. At the same time plenty of stock and futures exchanges closed. The only trading platform, do not interrupt trading derivatives, becomes Exchange "Saint-Petersburg". Business entities were realizing that the financial system can survive without hedging only for a while. In addition, large financial companies appreciate the advantages of using options.

The fifth stage of the formation of the derivatives market began in the end of Year 2000**.** Market is experiencing not only a quantitative but also a qualitative revival. A leader of the futures market initially becomes NP "Saint-Petersburg Stock Exchange". In late 2001, the derivatives market of the Exchange "Saint-Petersburg" had been transferred in the Derivatives Trading Exchange RTS (FORTS). In addition, in April 2002, trading in the derivatives section of the MICEX resumed.

2011 can be considered as the beginning of the sixth stage of development of the Russian derivatives market, which is associated with the already combined RTS and MICEX market platforms. The first joint project of the two exchanges was - futures contract settlement on the MICEX index. The specification was approved in late August on both exchanges.

# CHAPTER 2: THE THEORY OF HEDGING OPTIONS AND HEDGING WITH OPTIONS.

## 2.1 Derivatives market.

For over half a century of economic science is actively exploring the origin and background stages of the derivatives market, its role and functions and the characteristics of transactions made in this market. However still there is no strict definition of what derivatives market is.

In the specialized literature, the term "derivatives market" is used with other terms such as "financial instruments", "futures market", "financial market for derivatives", while often they are used as synonyms. Some authors refer to these terms the market of futures contracts with future asset delivery at the price of the transaction, and the others - a market in which transactions are concluded with a maturity of greater than two working days.

A lot of scientists agree that there are essential differences in terms of "futures market" and "derivatives market". Thus, Galanov V. A. believes that the futures market is a market with fixed-term contracts. Though, derivatives market, according to writer, is a market where participants exchange their obligations arising under contracts on a temporary basis, or to repay them earlier. Popular writer Kuznetsova L. G agrees with above mentioned, moreover, she notes that "the futures market does not have a mechanism, releasing from the commitments, and therefore other markets were done - derivatives markets, that are focused on other contractual relations and provided the institutional structure for market participants that is able to eliminate the risk of non-term liabilities. "[1].

In the economic writings dominates the view that the derivatives market was made due to the need for society to eliminate the negative effects of increased uncertainty of economic conditions. Derivatives market allows business entities regulate the financial risk effectively, create the desired risk profile, conduct operations to diversify portfolio risk, and thus ensure the stability of their financial situation and the achievement of planned financial results.

Function of the derivatives market for the prevention of financial risk and reducing potential losses caused by its implementation, expressed as a hedging - protecting one's capital against effects of inflation through investing in high-yield financial instruments (bonds, notes, shares), real estate, or precious metals.

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[1] *Kuznetsova L.G. Derivatives in the economic space of Russia: problems of terminology // Securities Market.*2006. № 7. P. 38.

American researcher J. R. Hicks, first emphasizing the objective due to the appearance of the market, wrote: ...entrepreneurs will want to hedge their sales for the reason; supplies in the near future are largely governed by decisions taken in the past, so that if these planned supplies can be covered by forward sales, risk is reduced. But although the same thing sometimes happens with planned purchases as well, it is almost inevitably rarer; technical conditions give the entrepreneur a much freer hand about the acquisition of inputs (which are largely needed to start new processes) than about the completion of outputs (whose process of production- in the ordinary business sense-may be already begun). Thus, while there is likely to be some desire to hedge planned purchases, it tends to be less insistent than the desire to hedge planned sales. "[2].

Hedging mechanism works as follows: hedger (one who is hedged), is, for example, the buyer (seller) of an asset A in cash market and at the same time the seller (buyer) of the same asset A in the derivatives market. As a rule, the movement of prices in both markets is agreed in direction and is predictable, and therefore that the hedger lost (won) on the cash market, offset by a gain (loss) on derivatives markets.

The possibility of such financial security is based on the fact that the derivatives market provides a balance of cash flows regardless of the size and the number of market participants, because the amount of money that has lost a certain number of market participants, the same amount won those who correctly assessed the dynamics of the market .

In addition, the function of the derivatives market is - ability to display an overview of the future state of the economy.

According to the type of instruments, trading market is divided into **forward, futures, options market** and **swap market**.

Depending on the forms of organization, derivatives market is divided into the exchange and OTC. Mechanism to use or trade derivatives requires the presence of the organizer of the market of these instruments. With the exchange form of trade organizer is the stock exchange by itself. With OTC forms of institutional trading its organizers are major dealers, for example, banks swaps dealers or global financial companies.

## 2.2. History of research hedging of financial options.

The traditional solution to the problem of hedging of financial options is described in the theory of perfect hedge. The basics of this theory were laid by well-known research of **Black and Scholes, Merton, Kokca, Rocsa and Rubinstein.** The theory of perfect hedge is used in the

[2] *Hicks J. R.* Capital and Value. M.: Progress, 1988. Page. 245.

creation of some portfolio market assets that generates payments corresponding derivative payments.

Imperfect methods of hedge are the newest way of risk management. The start of this theoretical understanding was laid only in the 1990s in research by **Daffy and Richardson (1991), Nechaev and Melnikov (1998), Fellmer and Leykert (1999, 2000).** In addition, this work was aimed to identify the mathematical solution for the problem of constructing imperfect strategies of hedge, and issues, such as the rationale of economic efficiency and feasibility of using these methods in the research, were not solved.

Crucial role in the creation of the theory of imperfect hedge have had studies of Black F., M. Scholes, R. Merton, A.N. Shiryaeva, A.V.Melnikova, D.O. Kramkova, N.I. Berzona, V.A. Galanova, Y.M. Mirkina, M. Harrisona, D. Krepsa, S. Pliska, G. Fellmera, S.N. Volkova.

## 2.3. Option and option models.

There are four possible types of option transactions:

• Purchase of call option

• Sale of call option

• Purchase of put option

• Sale of put option

The most commonly used types of options - American and European. They have some significant differences.

• American options can be repaid at any pre-specified period of the option. This means that at the signing of the transaction of the option, there is time of maturity, during which the parties may strike this option.

• However, European option can be repaid only on the maturity date of the option (pre-agreed by the parties).

In this dissertation the European type of options will be used.

**The option premium** - amount that is paid by the buyer of the option to seller at the signing of fixed-term contract. The economic meaning of the option premium - e.g., the buyer of the option pays money for the possibility to enter into the transaction in the future.

Often with the term "price of the option" understands "option premium". The premium of stock options is its financial quote. The premium is usually set in accordance with the rule of supply and demand by equalizing supply and demand. In addition, there are some mathematical models that allows to calculate the premium based on the current value of the underlying asset and its stochastic parameters such as volatility, profitability, etc. With this calculation - option premium theoretically refers to the price of option. Usually it is calculated by the organizer or broker and is provided with information of financial quote during the term of trading.

The idea of an efficient market is the foundation of all mathematical models of calculating the option premium. There is an assumption that "fair" option premium is equal to the value at which neither the buyer nor the seller, on average, will not have profit.

In order to calculate the option premium, a stochastic process features, that models the price change of risky assets held by the option database, are used. One of the most important statistical parameter that affect the value of the premium is the price volatility of the option of underlying asset. The higher the price, the higher risk of incorrect predictions of the asset price in future period, and therefore the premium value increases and the option seller receives more money. The second most important parameter - the time of option maturity. The farther to maturity date, the higher the premium (at the same price the asset was agreed in the options contract). The parameters of such a option pricing model is calculated on the basis of historical dynamics.

The most well-known and widely-used option models:

- Black - Scholes option pricing model;
- Binomial model;
- Heston model;
- Monte Carlo model;
- Bjerksund-Stensland model;
- Cox-Rubenstein model;
- Yates model.

In this study, the classical Black-Scholes model will be used, to illustrate the methods of imperfect hedging of financial options.

## 2.4. Strategies of hedging with using options contracts.

Option - it is a contract, in terms of which a buyer or seller of a risky asset, such as a product or a security, receives the right, but not the obligation as in the case with futures, make a purchase or sale the risky asset at a price agreed in advance at a certain moment in the future or agreed in advance and called the maturity date or during the all period of the contract, defined in advance. When concluding an option contract, the seller of the option has the obligation to implement reciprocal sale or purchase of risky asset on conditions of the sold option.

Options transactions are made in over-the-counter and exchange-traded markets.

Types and series of options distinguish depending on the "origins of the option." Options can be classified following:

1) The form of implementation.

• With a physical delivery of the underlying asset.

• With a cash settlement on the "spot" conditions.

2) According to the terms of execution.

• Standard Terms and Conditions.

• Identical conditions.

3) According to the terms of reference.

• Common options (options of one series, traded on one options market).

• Multiple traded options (one series options, traded on several markets at the same time).

• Internationally traded options (options that are traded in the markets of other countries).

4) By the time of execution.

• American option (executed at any time until it expires).

• European option (performed during a specific period of up to its expiration).

• Bermudan option (performed on certain dates during the validity period of the option).

• Interest rate option (an option, which is based on the credit financial instruments by which is paid interest: bills, bonds, deposits, etc.). Executed automatically before the expiration date, when the market will indicate the financial viability of the option).

5) According to the nature of the underlying asset.

• Securities options.

• Debt options.

• Flexible options.

• Index options.

• Currency options.

• Future options.

Since the exercise price of the option is set, and the market price of an asset changes constantly, at the time the option is exercised price can be higher or lower than the exercise price. If the exercise price of the option is above the market price of the asset then the buyer of the option "call", if he was going to purchase the asset, gives up the right demand fulfillment of the option, or he can purchase the asset in the market at a lower price. Amount paid by the seller premiums offset by the cheapness of asset purchases on the spot market. If the price of the option (plus a premium) will be lower than the current market price of the asset, the buyer of the option will gain benefit if buyer uses the right to demand its execution, that is purchase asset from option seller at the strike price and sell on the spot market at a higher price, having thus, speculative profit.

Figure 2.1 shows a graphical interpretation of financial result for the buyer and seller call and put options dependence.

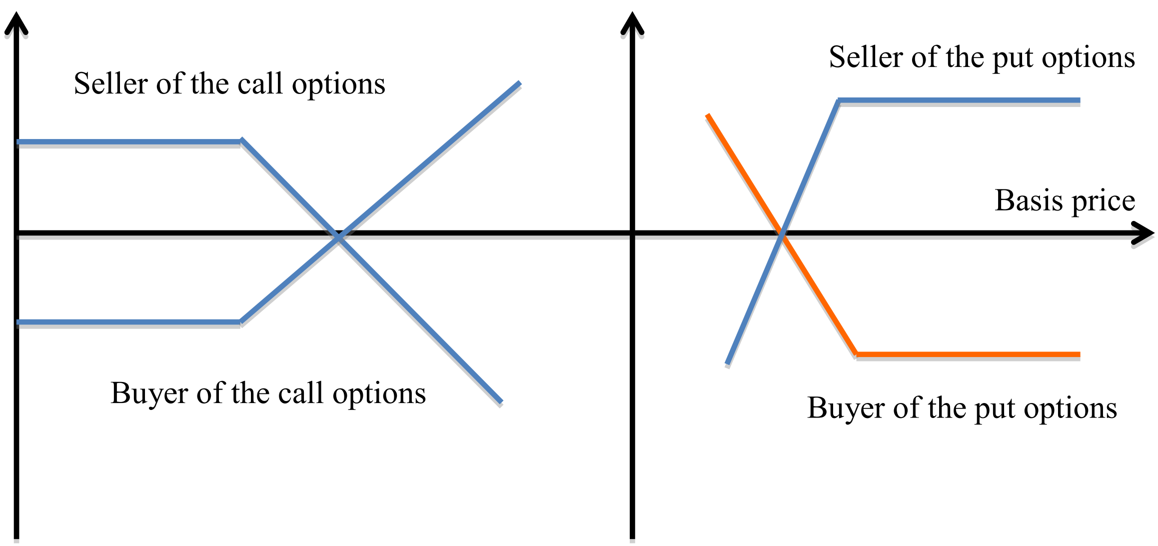


**Figure 2.1 Financial result for the buyer and seller call and put options dependence**

There are roughly separated 5 situations in which the purchase of "put" option is feasible:
1) In a situation where we want to use the effect of the financial leverage (but assume that the market will go down), and our funds are limited. For example: there are $ 100, which can be used to purchase action 1 or 5 put options. Put option will limit our losses to the value of the award into 20 $, because of 5 options our maximum loss will be 5 * 20 = $ 100. But our income is not limited, and income of 5 options under certain circumstances may be ten times higher than the income from operations with the action.
2) As an alternative to a stop order by limiting possible losses is the purchase of the option with a long strike, i.e. distant time of execution, which helps to hedge a long (long position) of the underlying asset. I.e. there are 100 rubles, which a year later are spent on purchase of 5 securities, there may be an alternative buying 5 put options. This will help to avoid the undesirable effects of the price increase and a year later, depending on the situation, the losses paid by the seller as a premium will either limit and the paper in the spot market will be bought, or exercise the option. The situation is similar to the use of a peculiar stop order.
3) To create a long position in the underlying asset at a favorable level in the case of a market correction. To do this, cheap put option “out of money” (OTM) about the technical level and far below are bought.  If the market falls to this level, the investor can create a long position in the underlying asset, using an option as a prepaid stop. (This position creates a significant psychological comfort, helping to overcome investor the panic that reigns in the market at the

time of correction). This version differs from the preceding one by implying the lack of position in the underlying asset.

4) As a way to play of volatility for investment in long-term options. Strategy for those who mastered the expected volatility in the investment through the purchase of delta hedged long OTM put options. It is a unique way of earning money. For example, very often in an election year expected volatility increases until the last terms of the event; however the spot often stays in one place. Since the amortization of premium of long-term options is small, there is a high return on investment due to high volatility.

5) Every trader operating in the market has limits on sales, based on the risk limits. In the situation where the underlying asset cannot be sold because of the risk limits, traders begin to work with options, using them to avoid the restrictions, and the fact that the sale of "puts" limits the risk of loss to the seller must not be forgotten.

Expediency of investment in call options can be determined from analogous logic.

The use of options gives parties of the agreement multivariate tactical choices during market strategies. Conventionally strategy can be divided into 4 groups.

1) In situations when price growth in the underlying asset is expected it is reasonable to use a long call, short put, bull call spread, bull put spread strategy.
2) In situations when the devaluation of underlying asset is expected following strategies should be used: long put, short call, bear call spread, bear put spread.
3) In anticipation of increased price volatility basis long straddle, short call butterfly spread strategies can be used.
4) In situations when reducing fluctuations in the basis are expected it is reasonable to use short straddle, long call butterfly spread strategies.

In fact, strategies used for the hedging namely are long call, short call, long put, short put. Other types of strategies are used mainly to conduct arbitration operations either for speculative operations.

Consider key strategies in detail:

1. **Long call strategy. (Purchase of a call option).**

The decision to use strategy of long call may be adopted in the following cases:

- The buyer intends to reduce the risk on short positions over the basis.

- It is supposed to fix the price for the upcoming purchase basis. The application of this strategy can be described in more detail in the examples:

Assume player A in the securities market sold short (i.e., not having it in stock) to player B a certain amount of shares at a certain price, expecting that by the time he will have to deliver these shares, the price will go down, and he can, purchase required number of shares at a good price and make a profit. However, probable and quite an opposite turn of events, i.e., the price may not fall but rise. To limit their losses at bad increasing prices for these shares in this case, player A can buy call options for the required number of shares with a strike price equal to the price of delivery of these shares to player B. Thus, player A has hedged its position from an unwanted change in prices by limiting the size of potential losses by premium paid.

Let's assume a different situation when an investor considers prices for some assets attractive on the given market level, but there are no funds to purchase the asset at the moment - the inflow of funds is planned in some period of time (the duration of it knows investor). And, at the same time, investor assumes that in this period of time the cost of asset will increase and purchase will be unprofitable. In this case, the investor can fix the value of the asset at this level through the purchase of the option "call" on the asset with the price equal to the spot price of the asset at the moment. Similarly to the first example the investor hedged its position on the growth of the asset market value by limiting the size of losses by paid premium.

**2. Long put strategy (the purchase of a put option).**

This strategy involves buying an option to sell a specified quantity of the underlying asset during the date of maturity.

The decision to use this strategy may be adopted in the following cases:

- The buyer intends to reduce the risk on a long position over the underlying asset.
- The buyer wants to benefit from an expected decline in the price of the underlying asset.
- It is required to fix the price of the underlying asset for the expected price reduction.
- As an example, consider the following scenario:

Assume that an investor A has a certain amount of the underlying asset in his possession, that investor wants to hedge against the exchange value falls. To reduce the risk of losses from lower prices of assets in this case, investor might purchase a put option on that asset. Hedger can set the lowest possible price for a sale of the underlying asset, thereby limiting the risk of undesirable changes in the price level of the premium paid. Indeed, when price underlying asset will fall too low the investor A, may sell his existing asset at the strike price and the reduce losses with the size of seller option premium (if the option performance price will correspond to the current price level on the basic asset) or profit (if the price of the underlying asset is lower than the strike price). If the market value of the underlying asset increases, for the investor A it will not be necessary to strike the option, because it is possible to implement assets in the current market at a price higher than the strike price of the option. In this case, the maximum amount of losses incurred by the investor A will be equal to the amount of the premium paid.

The investor A expects to reduce the prices of certain assets and the investor B, on the contrary, expects to increase the market value of the asset. Investor A, buying a put option on the asset, secures to receive virtually unlimited amount of profit, in case if market will behave exactly as investor A expects during the term of the option. The reason for this is the right to sell the underlying asset at the strike price, which the buyer has with buying an option contract. Otherwise, investor B, the option seller, has a profit of premiums, paid by the investor A, and the investor A will incur a loss - value of the premium paid. If the current price of the underlying asset is higher than the strike price, mortgaged in the option, the investor A can refuse to strike the option and limit the losses with amount of option premium of seller.

A situation, in which it is appropriate to use the long put strategy long, is when the investor A has a certain amount of some assets. However, investor does not want to sell the asset at a given time. At the same time, the investor A has fear that by the time when he will be able or willing to sell assets, the market value will fall and the investor will have loss by selling the asset at a lower price, or investor will wait for the next asset price increase, what in this situation is not appropriate. In order to protect from potential losses, investor A may purchase a put option. If the price of the underlying asset is lower than the strike price, the investor A may strike the option by selling the asset at the strike price. With an increase of the underlying asset market value, to implement the right of the option is not necessary, and the investor A may sell assets in the current market at a price higher than the put price of the option.

**3. Short Call Strategy (sale of a call option).**

The use of this strategy comes with selling the right to buy the underlying asset with an obligation to deliver the asset at the strike price during the term of the option. In this case, the maximum income is limited with the size of premium paid by the buyer, while the amount potential losses is almost unlimited. Undoubtedly, this strategy carries a much higher risk than the long call strategy. The seller of the call option takes unlimited risk if the underlying price increases compared to the strike price. At the same time the seller expects to receive an income of premiums, if the underlying price of will not change, or will fall lower than the strike price.

The use of this strategy as a hedging instrument requires careful and detailed approach to the analysis of the market situation.

A very important factor in the usage of this strategy is to determine the strike price of the option. Are available the following options:

The strike price for the current market price of the underlying asset ("at the money" ATM option).

The strike price is lower than the current market price of the underlying asset ("in the money" ITM option).

The strike price is higher than the current market price of the underlying asset ("out the money" OTM option).

By selling a "at the money" call option, the investor limits maximum premium income, that is paid by the buyer of the option. Selling a call option in the money, the investor can expect an income equal to the paid premium, which is reduced by the difference between the purchase price and the strike price of the underlying asset. Selling an option "out the money", the investor may receive the maximum income, that is equal to the amount of the premium, which is increased by the difference between the strike price and the purchase price of the asset.

As an example of short call strategy - investor can sale covered call option. In this case, the investor's purpose may be to obtain additional income of the underlying asset by option premium or protection from the underlying asset market value reduction within the premium. An attention should be paid to the fact that with the sale of covered call option (ie, if investor has the underlying asset), the investor has to be sure that the market price of the underlying asset will not be higher than the strike price.

**4. Short put strategy (sale of a put option).**

This strategy involves the provision of the right to sell a specified amount of the underlying asset during the term of the option at a specified strike price, for the premium. The investor, using the short put strategy (seller), assumes to take a delivery transaction of the underlying asset from the counterparty, with the specified strike price of the option contract during the term of the option. At the same time, the option buyer acquires all the rights of this transaction.

Using this strategy, the seller of a put option expects that the market value of the underlying asset will rise over the duration of the option.

As in the short call strategy, in this strategy the investor voluntarily limits the amount of the income of the transaction with amount of option premium paid by the buyer of the contract. Possible amount of losses incurred by the seller of the contract, is almost unlimited. In a situation where the current price of the underlying asset will fall to the level of significance of the difference between the strike price and the premium, the seller will not incur losses, and will not receive income, but if the current price of the underlying asset falls below the set difference, the seller of the option contract will incur a loss.

As an example, consider the following scenario:

Suppose an investor A sells a certain amount of a put option of the underlying asset at a specified strike price, that is lower than the current market price of the underlying asset. If the underlying price goes up, for the buyer of a put option it will not be necessary to strike the option, and the investor A will gain a profit within the amount of the option premium. If the market value of the asset goes down, the further development depends on the underlying asset price fall. If the price of the underlying asset will fall to the level of the strike price or lower, but not lower than the value of the difference between the strike price and the premium paid by the seller, investor A will cover losses within the premium received. If the price of the underlying asset will be lower than the difference between the strike price and the premium paid by the seller, the investor A risks to have unlimited loss, as the counterparty will require to strike the option on a strike price specified in the contract.

## 2.5. The Black–Scholes model.

The Black–Scholes or Black–Scholes–Merton model is a mathematical model of a financial market containing certain derivative investment instruments. From the model, one can deduce the Black–Scholes formula, which gives a theoretical estimate of the price of European-style options. The formula led to a boom in options trading and legitimised scientifically the activities of the Chicago Board Options Exchange and other options markets around the world. lt is widely used, althoughoften with adjustments and corrections, by options market participants. Many empirical tests have shown that the Black–Scholes price is "fairly close" to the observed prices, although there are well-known discrepancies such as the "option smile".

In this model, the main element in determining the price of an option is the volatility of the asset underlying the derivative instruments. Depending on the value of volatility, the price of the asset is decreased or increases, and it is directly proportional effect on the price of the option. Thus, for a known value of the option it is possible to determine the amount of market volatility.

The Black–Scholes model assumes that the market consists of at least one risky asset, usually called the stock, and one riskless asset, usually called the money market, cash, or bond.

Now we make assumptions on the assets (which explain their names):

- The stock does not pay a dividend.
- There are no transaction costs associated with the buying or saling an asset or the option.
- Short-term risk-free bank interest rate is known and does not change until the date of execution.

- It is possible to borrow and lend any amount, even fractional, of cash at the riskless rate.
- It is possible to buy and sell any amount, even fractional, of the stock (this includes short selling).
- The above transactions do not incur any fees or costs.
- There is no arbitrage opportunity (i.e., there is no way to make a riskless profit).
- The trading underlying asset on the option is being continuously and asset price dynamics obeys on a geometric Brownian motion with some known parameters.

Creating a Black-Scholes model is based on a model of riskless hedge. That is when you buy the asset and simultaneously sale call options on this asset, the investor can create a risk-free position, where the profit of the asset will cover losses on options, and vice versa.

Risk free hedged position must be repaid at a rate equal to the risk-free interest rate, otherwise there would be an opportunity to extract arbitrage profits and investors trying to take advantage of this opportunity, the option price would lead to an equilibrium level, which is determined by this model.

Price of European call option is determined by the formula:

$C(S,t) = SN(d_1) - Ke^{-r(T-t)}N(d_2)$, where

$$d_1 = \frac{\ln(S/K) + (r + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}},$$

$$d_2 = d_1 - \sigma\sqrt{T-t}.$$

Price (European) put option is determined by the formula:

$$P(S,t) = Ke^{-r(T-t)}N(-d_2) - SN(-d_1).$$

Designations:

$C(S,t)$ - the present value of the call option at time t before the expiration of the option;

$S$ - is the spot price of the underlying asset;

$N(x)$ - is the cumulative distribution function of the standard normal distribution

(To determine $N(x)$ can be used the standard normal curve or Excel-function NORMSDIST(x). It returns the standard normal cumulative distribution, which has a mean of zero and a standard deviation equal to one);

$K$ - is the strike price;

$r$ - is the risk free rate (annual rate, expressed in terms of continuous compounding);

$T-t$ - is the time to maturity;

$\sigma$ - is the volatility of returns of the underlying asset.

**Alternative formulation**

Introducing some auxiliary variables allows the formula to be simplified and reformulated in a form that is often more convenient (this is a special case of the:

$$C(F,\tau) = D\left(N(d_+)F - N(d_-)K\right)$$

$$d_\pm = \frac{1}{\sigma\sqrt{\tau}}\left[\ln\left(\frac{F}{K}\right) \pm \frac{1}{2}\sigma^2\tau\right]$$

$$d_\pm = d_\mp \pm \sigma\sqrt{\tau}$$

The auxiliary variables are:

. $\tau = T - t$ is the time to expiry (remaining time, backwards time)

. $D = e^{-r\tau}$ is the discount factor

. $F = e^{r\tau}S = \frac{S}{D}$ is the forward price of the underlying asset, and

. $S = DF$

with $d_+ = d_1$ and $d_- = d_2$ to clarify notation.

Given put-call parity, which is expressed in these terms as:

$$C - P = D(F - K) = S - DK$$

the price of a put option is:

$$P(F,\tau) = D\left[N(-d_-)K - N(-d_+)F\right]$$

"The Greeks" measure the sensitivity of the value of a derivative or a portfolio to changes in parameter value(s) while holding the other parameters fixed. They are partial derivatives of the price with respect to the parameter values. One Greek, "gamma" (as well as others not listed here) is a partial derivative of another Greek, "delta" in this case.

The Greeks are important not only in the mathematical theory of finance, but also for those actively trading. Financial institutions will typically set (risk) limit values for each of the Greeks that their traders must not exceed. Delta is the most important Greek since this usually confers the largest risk. Many traders will zero their delta at the end of the day if they are not speculating and following a delta-neutral hedging approach as defined by Black–Scholes. Table of coefficients and methods of their calculation is as follows:

**Table 2.1 Coefficients «The Greeks» in the Black-Scholes model.**

| The Greeks | | **Calls options** | **Puts options** |
|---|---|---|---|
| Delta | $\frac{\partial c}{\partial S}$ | $N(d_1)$ | $-N(-d_1) = N(d_1) - 1$ |
| Gamma | $\frac{\partial^2 c}{\partial S^2}$ | $\frac{N'(d_1)}{S\sigma\sqrt{T-t}}$ | |
| Vega | $\frac{\partial c}{\partial \sigma}$ | $SN'(d_1)\sqrt{T-t}$ | |
| Theta | $\frac{\partial c}{\partial t}$ | $-\frac{SN'(d_1)\sigma}{2\sqrt{T-t}} - rKe^{-r(T-t)}N(d_2)$ | $-\frac{SN'(d_1)\sigma}{2\sqrt{T-t}} + rKe^{-r(T-t)}N(-d_2)$ |
| Rho | $\frac{\partial c}{\partial r}$ | $K(T-t)e^{-r(T-t)}N(d_2)$ | $-K(T-t)e^{-r(T-t)}N(-d_2)$ |

In an empirical study in the third chapter of our work is used perfect delta hedge within the Black-Scholes model.

### 2.5.1 Volatility

**A) LOGNORMAL PROPERTY OF STOCK PRICES**

The model of stock price behaviour used by Black-Scholes, and Merton. It assumes that percentage changes in the stock price in a short period of time are normally distributed. Define

$\mu$: Expected return on stock per year

$\sigma$: Volatility of the stock price per year

The mean of the return in time $\Delta t$ is $\mu\Delta t$ and the standard deviation of the return is $\sigma\sqrt{}\Delta t$ ,so that

(0) $\Delta S/S \sim \phi(\mu\Delta t, \sigma^2\Delta t)$

where $\Delta S$ is the change in the stock price $S$ in time $\Delta t$ and $\phi(m, v)$ denotes a normal distribution with mean $m$ and variance $v$.

The model implies that

(1) $\ln S_T - \ln S_0 \sim \phi\ (\left(\mu - \frac{\sigma^2}{2}\right)T, \sigma^2 T)$

from this, it follows that

(2) $\ln S_T/S_0 \sim \phi\ ((\mu - \frac{\sigma^2}{2})\, T, \sigma^2 T)$

and

(3) $\ln S_T \sim \phi\ (\ln S_0 + (\mu - \frac{\sigma^2}{2})\, T, \sigma^2 T)$

where $S_T$ is the stock price at a future time T and $S_0$ is the stock price at time 0. The last formula shows that ln $S_T$ is normally distributed, so that $S_T$ has a lognormal distribution. The mean of ln $S_T$ is ( $\ln S_0 + (\mu - \frac{\sigma^2}{2})\, T, \sigma^2 T$) and the standard deviation is $\sigma\sqrt{T}$.

**Example**

Consider a stock with an initial price of 40$, an expected return of 16% per annum, and a volatility of 20% per annum. From formula(3) , the probability distribution of the stock price $S_T$ in 6 months time is given by

$\ln S_T \sim \phi\ (\ln 40 + (0.16 - \frac{0.2^2}{2}) * 0.5, 0.2^2 * 0.5)$

$\ln S_T \sim \phi\ (3.759, 0.02)$

there is a 95% probability that a normally distributed variable has a value within 1.96 standard deviations of its mean. In this case, the standard deviation is $\sqrt{0.02}$ = 0.141. Hence, with 95% confidence,

$3.759 - 1.96*0.141 < \ln S_T < 3.759 + 1.96*0.141$

this can be written

$$e^{3.759-1.96*0.141} < S_T < e^{3.759+1.96*0.141}$$

or

$32.55 < S_T < 56.56$

thus, there is a 95% probability that the stock price in 6 months will lie between 32.55 and 56.56.

A variable that has a lognormal distribution can take any value between zero and infinity. This figure illustrates the shape of a lognormal distribution. Unlike the normal distribution, it is skewed so that the mean, median, and mode are all different. From formula (3) and the properties of the lognormal distribution, it can be shown that the expected value E($S_T$) of $S_T$ is given by

$E(S_T) = S_0 * e^{\mu T}$

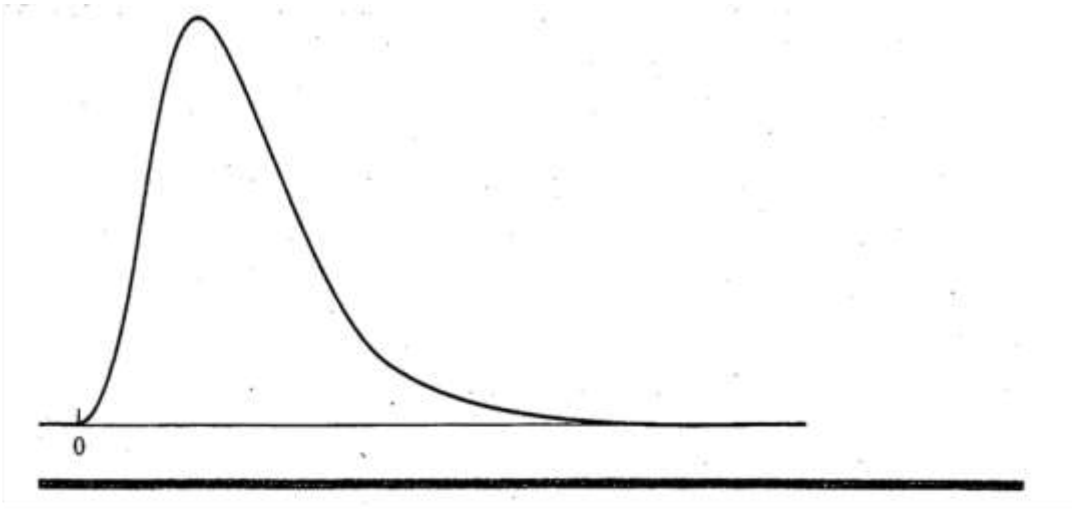


**Figure 2.2 Lognormal distribution**

this fits in with the definition of $\mu$ as the expected rate of return. The variance var($S_T$) of $S_T$, can be shown to be given by

var($S_T$) = $S_0^2$* $e^{2\mu T}$*($e^{\sigma^2 T} - 1$)

**The volatility** $\sigma$ of a stock is a measure of our uncertainty about the returns provided by the stock. Stocks typically have a volatility between 15% and 60%.

When $\Delta t$ is small, formula (0) shows that $\sigma^2 \Delta t$ is aproximately equal to the variance of the percentage change in the stock price in time $\Delta t$. This means that $\sigma\sqrt{\Delta t}$ is approximately equal to the standard deviation of the percentage change in the stock price in time $\Delta t$. Suppose that $\sigma$ = 0.3 or 30%, per annum and the current stock price is 50$. The standard deviation of the percentage change in the stock price in 1 week is approximately

30*$\sqrt{(1/52)}$= 4.16%

A standard-deviation move in the stock price in 1 week is therefore 50*0.0416, or 2.08$

Uncertainty about a future stock price, as measured by its standard deviation, increases-at least approximately-with the square root of how far ahead we are looking. For example, the standard deviation of the stock price in 4 weeks is approximately twice the standard deviation in 1 week.

**B) ESTIMATING VOLATILITY FROM HISTORICAL DATA**

To estimate the volatility of a stock price empirically, the stock price is usually observed at fixed intervals of time (e.g., every day, week or month). Define:

n+1: Number of observations

$S_i$: Stock price at end of 1th interval, with $i$ = 0,1,….,n

$\tau$: Length of time interval in years

and let

$u_i = \ln(\frac{S_i}{S_{i-1}})$ for $i$ = 1,2,…..,n

The usual estimate, $S$, of the standart deviation of the $u_i$ is given by

$$S = \sqrt{\frac{1}{n-1}\sum_{i-1}^{n}(u_i - \bar{u})^2}$$

$$S = \sqrt{\frac{1}{n-1}\sum_{i-1}^{n}u_i^2 - \frac{1}{n(n-1)} * (\sum_{i=1}^{n} u_i)^2}$$

Where $\bar{u}$ is the mean of the $u_i$.

The standard deviation of the $u_i$ is $\sigma\sqrt{\tau}$. The variable $s$ is therefore an estimate of $\sigma\sqrt{\tau}$. It follows that $\sigma$ itself can be estimated as $\hat{\sigma}$, where

$$\hat{\sigma} = \frac{s}{\sqrt{\tau}}$$

The standard error of this estimate can be shown to be approximately $\hat{\sigma}/\sqrt{2n}$.

Choosing an appropriate value for n is not easy. More data generally lead to more accuracy, but $\sigma$ does change over time and data that are too old may not be relevant for predicting the future volatility. A compromise that seems to work reasonably well is to use closing prices from daily over the most recent 90 to 180 days. Alternatively, as a rule of thumb, n can be set equal to the number of days to which the volatility is to be applied. Thus, if the volatility estimate is to be used to value a 2-year option, daily data for the last 2 years are used.

**Example**

Table shows a possible sequence of stock prices during 21 consecutive trading days. In this case,

$\sum u_i = 0.09531$ and $\sum u_i^2 = 0.00326$

**Table 2.2 Possible sequence of stock prices**

| Day | Closing stock price($) | Price relative $S_i/S_{i-1}$ | Daily return $u_i = \ln(S_i/S_{i-1})$ |
|---|---|---|---|
| 0 | 20.00 | | |
| 1 | 20.10 | 1.00500 | 0.00499 |
| 2 | 19.90 | 0.99005 | -0.01000 |
| 3 | 20.00 | 1.00503 | 0.00501 |
| 4 | 20.50 | 1.02500 | 0.02469 |
| 5 | 20.25 | 0.98780 | -0.01227 |

| | | | |
|---|---|---|---|
| 6 | 20.90 | 1.03210 | 0.03159 |
| 7 | 20.90 | 1.00000 | 0.00000 |
| 8 | 20.90 | 1.00000 | 0.00000 |
| 9 | 20.75 | 0.99282 | -0.00720 |
| 10 | 20.75 | 1.00000 | 0.00000 |
| 11 | 21.00 | 1.01205 | 0.01198 |
| 12 | 21.10 | 1.00476 | 0.00475 |
| 13 | 20.90 | 0.99052 | -0.00952 |
| 14 | 20.90 | 1.00000 | 0.00000 |
| 15 | 21.25 | 1.01675 | 0.01661 |
| 16 | 21.40 | 1.00706 | 0.00703 |
| 17 | 21.40 | 1.00000 | 0.00000 |
| 18 | 21.25 | 0.99299 | -0.00703 |
| 19 | 21.75 | 1.02353 | 0.02326 |
| 20 | 22.00 | 1.01149 | 0.01143 |

and the estimate of the standard deviation of the daily return is

$\sqrt{\frac{0.00326}{19}} - \sqrt{\frac{0.09531^2}{380}}$= 0.01216 or 1.216%.

Assuming that there are 252 trading days per year, $\tau$=1/252 and data give an estimate for the volatility per annum of 0.01216$*\sqrt{252}$ =0.193, or 19.3%. The standart error of this estimate is

$\frac{0.193}{\sqrt{2*20}}$ = 0.031 or 3.1$ per annum.

The foregoing analysis assumes that the stock pays no dividends, but it can be adapted to accommodate dividend-paying stocks. The return, $u_i$, during a time interval that includes an ex-divident day is given by

$u_i$ = ln $\frac{S_i+D}{S_{i-1}}$

where D is the amount of the dividend. The return in other time intervals is still

$u_i$ = ln $\frac{S_i}{S_{i-1}}$

However, as tax factors play a pert in determining returns around an ex-dividend data, it is probably best to discard altogether data intervals that include an ex-dividend date.

**C) TRADING DAYS VS. CALENDAR DAYS**

An important issue is whether time should be measured in calendar days or trading days when volatility parameters are being estimated and used. Research shows that volatility is much higher when the exchange is open foe trading than when it is closed. As a result, practitioners tend to ignore days when the exchange is closed when estimating volatility from historical data and when calculating the live of an option. The volatility per annum is calculated from the volatility per trading day using the formula

$$\begin{aligned} &Volatility\ per\ annum \\ &\quad = Volatility\ per\ trading\ day\ * \sqrt{Number\ of\ trading\ days\ per\ annum} \end{aligned}$$

This is what we did in example when calculating volatility from the data in table. The number of trading days in a year is usually assumed to be 252 for stocks. The life of an option is also usually measured using trading days rather than calendar days. It is calculated as $T$ years, where

$$T = \frac{Number\ of\ trading\ days\ until\ option\ maturity}{252}$$

# CHAPTER 3. METHODS OF IMPERFECT HEDGING.

Honours from methods of imperfect hedging of financial options within the Black-Scholes model is the use of a combination of a number of options of different types and different strike price. Thus the combination depends on the model parameters such as volatility, the average growth rate and bank interest rate. In the application of imperfect hedging under Black-Scholes model there are used two option exercise prices:

- K – strike price, equal to the price of the underlying asset in the initial period of the hedging
- h – boundary, using the formula:

$h = e^{\sigma \cdot b + \ln(S_0 + (r + \frac{1}{2} \cdot \sigma^2) \cdot T}$, where

b = $\sqrt{T} \cdot \Phi^{-1}(1 - \varepsilon) + \frac{\mu - r}{\sigma} \cdot T$,

μ – the average growth rate of the underlying asset,

r – bank interest rate,

σ – volatility,

T – term of execution,

S – the price of the stock, which sometimes can be a random variable or a constant,

ε – the probability of losses.

## 3.1. Methods of imperfect hedging of financial options.

Methods of imperfect hedging provide an opportunity to reduce the cost of protection against the risk by accepting a controlled risk of losses. The traditional model of decision making suggests that, a rational investor will always prefer a greater profit, if other things are equal. Rational market participant avoids excessive risks and are ready for it only if compensation is offered, especially higher yields. That is why risk management of option contract includes expected return maximizing at a controlled level of risk, or equivalently, the risk minimizing at the more likely level of profitability. It is the main idea and the main feature of the methods of imperfect hedging of financial options.

Methods of imperfect hedging of financial options are divided into three groups depending on the methods of managing a risk - method of the mean hedging , quantile hedging and expected shortfall hedging.

Method of mean hedging is one of the most studied methods of imperfect hedging of financial options. In this method "quality measurement" of hedging strategy is carried out with the difference of the square of the terminal capital and the payment obligation. This method was proposed and described in Daffi Richardson studies in 1991, a further development of the method comes from Melnikova and Nechaeva studies in 1998 and of Schweitzer work in 1999. A more detailed list of development of the method of the mean square hedging research is given in the Melnikova, Volkova and Nechaeva (1998) book .

The mean square hedging enables controlled reducing of costs of hedging the option, although, this method has the disadvantages, specific to all the quadratic risk measures, risk of strategy increases after the both, positive and negative, deviations of payments. As this method is described in details in various studies and is widely known, in this study, its review is not carried out and the focus is on looking at other methods of imperfect hedging of financial options.

The next group of methods of imperfect hedging of financial options is the ***quantile hedging***. Value of the option in the case of quantile hedging is calculated as a result of interaction of a combination of factors; unexpected change of factors causes the risk of hedger. This risk of hedger consists of the unpredictability of market dynamics or uncontrolled changes in the economic environment and refers to a group of market risks.

Often, as a measure of market risk, the concept of Value-at- Risk is applied. The essence of this concept is to determine the amount of damages at a given interval of confidence. More formally, the method consists of finding the quantile of profit and loss distribution, expected in some time interval in the future. With determination of confidence interval through e, Value-at-Risk corresponds to the bottom quintile of the order 1-e. This method gives a detailed view of the risk of urgent position and allows to measure the economic value of an urgent position.

With the Value-at-Risk concept use as a risk measure, the issue of finding the optimal distribution of risk-return is limited with the general theory of confident statistical estimation. The basic concept of this theory is the quantile or assessment border at a predetermined interval. That is why this method of imperfect hedging of financial options, in which quantile distribution of profit or loss of hedging strategy is used as a criterion of success, is called ***quantile***.

In this study, the method of quantile hedging strategy that maximizes the probability of successful hedge with restrictions on the cost of implementing this strategy, is described. The idea of quantile hedging is fairly new and not described in the appropriate literature. This concept was first presented by G. Fellmerom in March 1995 in the framework of the traditional Black-Scholes model, and the main features of this method are described in the Fellmera Leykerta study in 1999. In this study, the method of quantile hedging problem is considered from the mathematical side. The problem is solved by a generalization of the method Kulldorfa (1993) to maximize the probability of achieving a certain level of Brownian movement at a given moment of time. Applying a generalized method, the researchers were able to summarize the problem. However, the economic content of this method did not have evidence, that was in their theories.

A little more detailed concept of economic features of quantile hedging are in Melnikov A.V., Volkov S.N. and Nechaev M.L. study in 2001. With a brief description of the mathematical algorithm for determining the hedging conditions, researchers are paying enough attention to the analysis of the structure of these conditions, and on the example of a standard European call option describing the main features of hedging conditions.

The method of quantile hedging considers only the probability of loss, but does not consider its amount. Condition with very high and very low amount of loss is assumed to be equally risky. From a practical point of view, this approach is not perfect and has a serious criticism.

The third group of methods of imperfect hedging of financial options is expected shortfall hedging. This concept takes into account the magnitude of the expected loss. This method defines risk as the mathematical expectation with physical possibility and minimizes this risk by limiting the cost of hedging. The main idea of the method of expected shortfall hedging is described in the study of Follmera Leykerta in 2000. Basically, the authors focused on the mathematical formulation of the problem.

The above-described methods of imperfect hedging (quantile hedging and expected shortfall hedging) are often described as similar methods of imperfect hedging. A comparison of these methods in this study shows that the hedging strategy may be optimal for both criteria only under certain conditions. In general, the structure of the hedging conditions depends on the used method. Therefore, the hedging portfolio which is optimal for one of the criteria often is not optimal for another criterion.

## 3. 2. Algorithm of successful hedging.

Depending on the model of the set of successful hedging will be different, and the boundary of $h$ will be used in different ways, as described below. Therefore, the correct choice of hedging strategy needs to adhere the following algorithm:

Step 1: Finding the structure of the set of successful hedging, based on the type of the option and the parameters such as volatility, the average growth rate and bank interest rate.

Step 2: Determination of payments hedging strategy on the set of hedging strategies.

Step 3: Selecting the method of replication payments hedging strategy (for example, in the third chapter of the research method chosen was delta hedging within the Black-Scholes model).

## 3. 3. Successful hedging variety.

In the study, there are examined two methods for imperfect hedging of financial options - quantile hedging method and the expected-shortfall hedging method. Both of these methods decrease the cost of hedging by assuming the risk of loss.

The main difference between methods include the determination of the risk. In quantile hedging risk is defined as the probability of loss. However in the expected-shortfall hedging method - risk is defined as the expected loss.

It has been described that the methods assume the use of an imperfect hedge combination of several options. The structure of this combination is determined depending on the method of hedge and model parameters. The definition of the structure is influenced by such indicators as the volatility $\sigma$, the growth rate $\mu$ and the interest rate $r$ . Figure 3.1 illustrates different combinations of these parameters, which determine the form of the set of successful hedging.

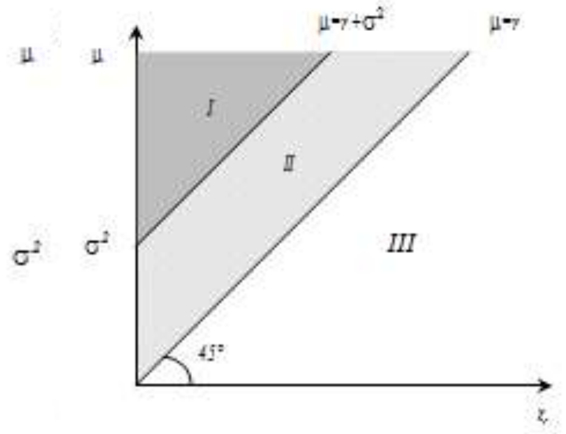


**Figure 3.1 The set of succesfull hedging.**

The figure shows that there are three possible forms of the set of successful hedging. Consider these cases as an example of the call option. When quantile hedging form the set varies depending on the expressions $(\mu-r)/\sigma^2$ , more then 1 or not. In a first variant the set consists of two regions. The first region consists of the states with low quotation of asset close to the strike price. The boundary of h1 restricts this area from above. The second region are included the states in which the course of the underlying asset is higher than the boundary h1.

If the expression *$(\mu - r)/\sigma^2 < 1$*, then the set are included with the course below the $h_3$.

In the method of expected shortfall hedging form the set determines like the difference between $\mu$ and $r$. If $\mu > r$ payments are made from the right side of oт $h_4$, otherwise there is upper limit $h_5$.

Table 3.1 shows all the possible structure of the set of successful hedging depending on various parameters of the model by the example of a call option.

**Table 3.1 The set of successful hedging of a call option depending on the model parameters.**

| Area | Dependence | Quantile hedging | Expected shortfall hedging |
|---|---|---|---|
| I | $(\mu\text{-r})/\sigma^2 > 1$ | $(0;h_1]\cap[h2;\infty)$ | $[h_4;\infty)$ |
| II | $(\mu\text{-r})/\sigma^2 \leq 1 \cap \mu > r$ | $(0;h_3]$ | $[h_4;\infty)$ |
| III | $\mu \leq r$ | $(0;h_3]$ | $(0;h_5]$ |

Based on the choice of a method of hedging, the structure of the set is represented in different ways in the first and second region. In these areas, the average growth rate higher than the bank interest rate. In that case hedging strategy is optimal only by one of the risk parameters - quantile or expected-shortfall. But if the growth rate is not higher than the interest rate of the bank, then the set of hedging strategies coincide. In this case if the strategy is optimal for one of the criteria, then the strategy is also optimal for other criteria of risk. Typically, the market is experiencing a situation where interest rate is lower than the rate of the underlying asset.

### 3.3.1. A numerical example.

Consider the above described interconnection via an example. For example, consider a call option, the exercise price is equal to 100, and the period of performance - 0.1. Assume that volatility is 0.2, and the process begins with the hedging level equal to 100. Bank interest rate is 6% per year (0.06), and the average rate of growth of the price of the underlying asset is equal to 0.15. With these parameters (μ-r) / σ2> 1, and thus corresponds to the first area of the figure 100500. Dependence of the probability of success and the expected success of the value of the hedging portfolio by using method of quantile hedging is described by the graph shown in Figure 3.2.

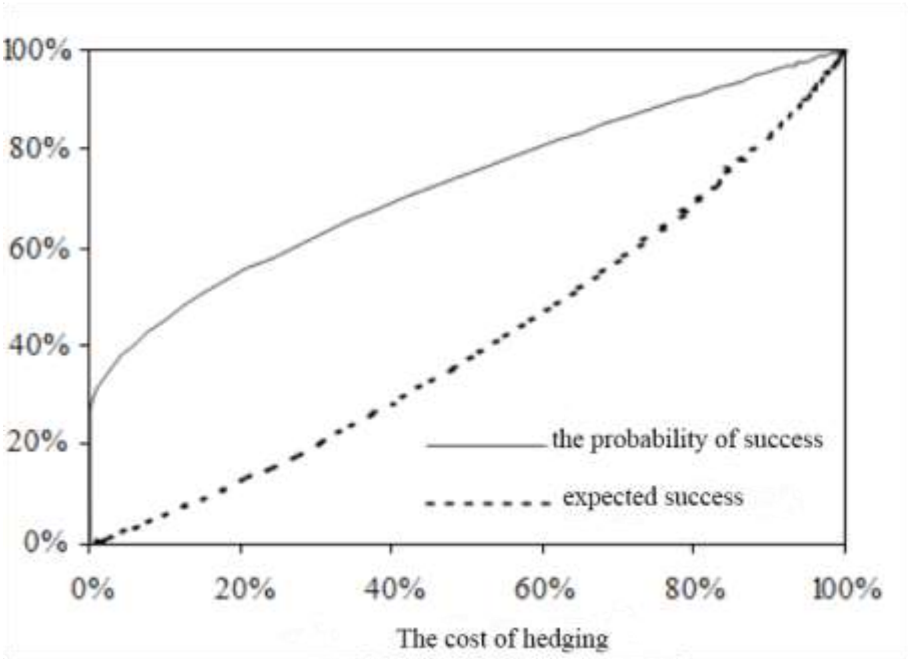


**Figure 3.2, The probability of success and expected success in quantile hedging in condition 1.**

Graph of the probability of success is convex, and the schedule of expected success - concave. Therefore, the optimal hedging strategy will be the criterion of probability of success, but this strategy cannot minimize the expected loss for a given constraint on the capital.

The expected-shortfall method of hedging in dependence of the expected success of the cost of hedging is a convex function. Also, the probability of success increases slower than the cost of hedging. Therefore, this strategy will be optimal by the criterion of the expected success, but will not be optimal for the quantile criterion. Figure 3.3 shows the success of the hedge in this case.

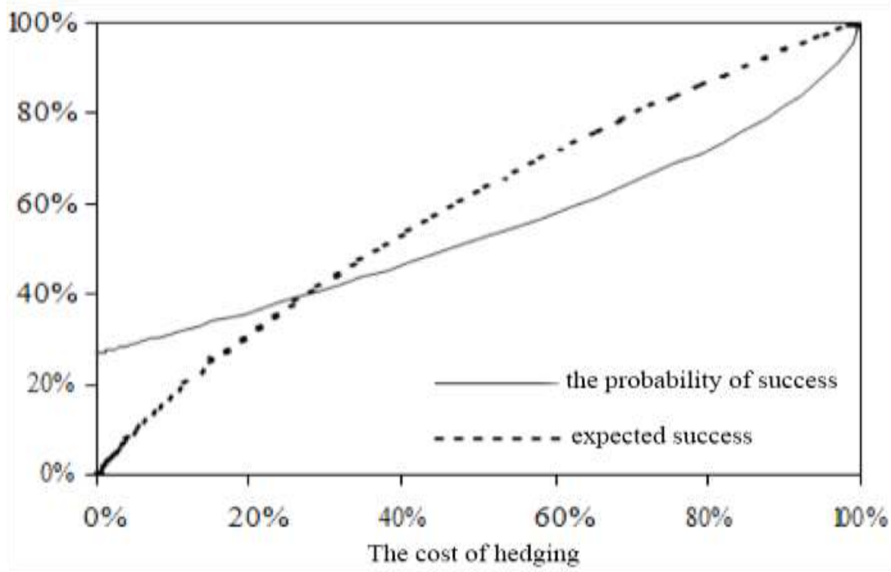


**Figure 3.3 The probability of success and expected success in expected-shortfall hedging in condition 1.**

By lowering the average growth rate to the level of 0.07 it is proceed to the second area. Apllication of quantile hedging strategy will be optimal by the criterion of the probability of success, but there is some imperfection in the strategy on the criterion of minimizing expected losses (a graph slightly concave). Probability of success and expected success are represented in Figure 3.4.

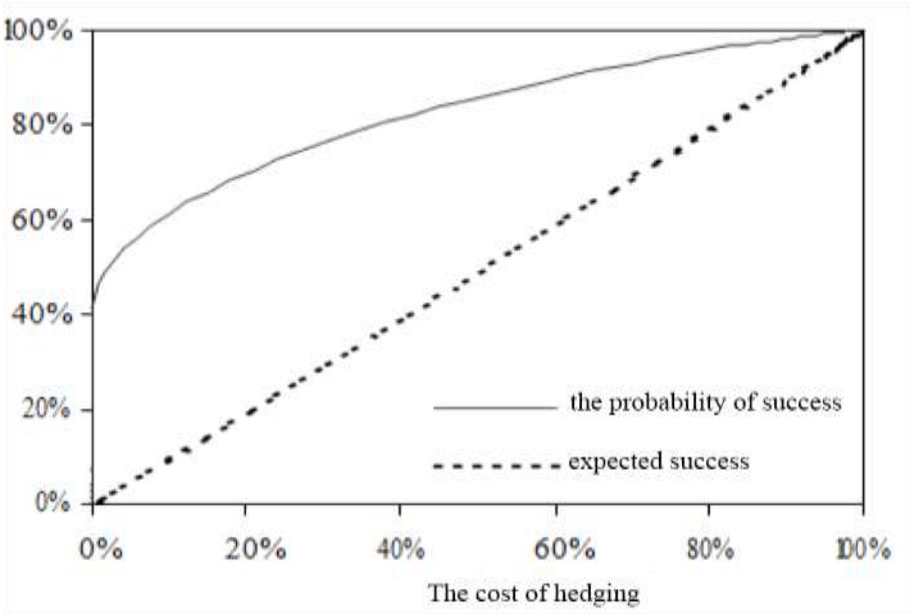


**Figure 3.4 The probability of success and expected success in quantile hedging in the condition 2.**

If the mothod of expected-shortfall is used the function depending on the expected success of the cost of hedging is convex. The probability of success increases with an increase initial capital. In this case this strategy is optimal only for a few risk criterion, which formed the basis of a method of hedging. Figure 3.5 shows the dependence.

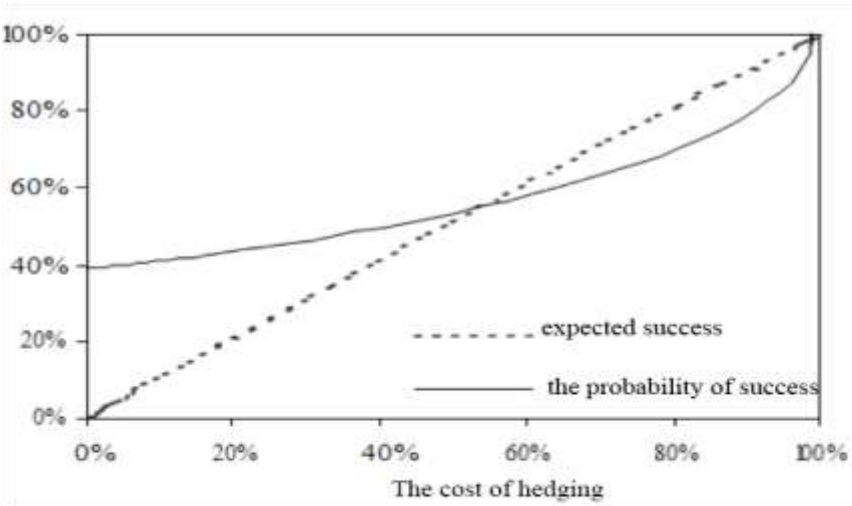


**Figure 3.5 The probability of success and expected success in the expected-shortfall hedging in the condition 2.**

In the situation when the bank interest rate is higher than the average rate of growth of the price of the underlying asset the set of successful hedging coincide. Therefore, they are optimal for both criteria. Figure 3.6 shows that the function of the probability of success and expected success function are convex. Since the sets are the same, the same graph for both hedging techniques will be used.

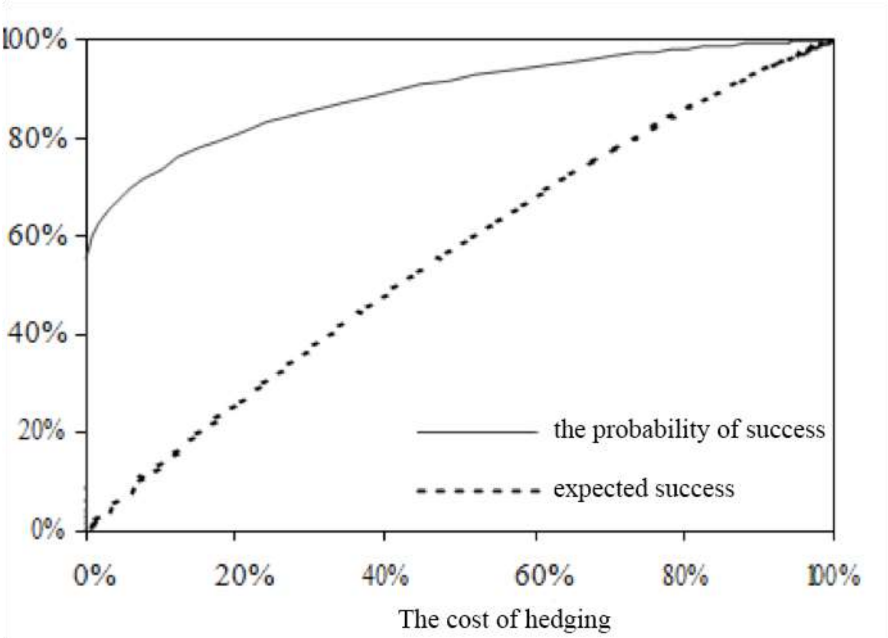


**Figure 3.6 The probability of success and expected success in quantile hedging and the expected-shortfall hedging in the condition 3.**

Type and characteristics of the option play an important role in forming dependence between the success and cost of hedging. Table of set of successful hedge changes if a put option instead of the call option. Such structure of the set of successful hedging is presented in Table 3.2.

**Table 3.2 Set of the successful hedge of a call option, depending on the parameters of the model by the example of a put option.**

| Area | Dependence | Quantile hedging | Expected shortfall hedging |
|---|---|---|---|
| I | $(\mu-r)/\sigma^2 > 1$ | $[h_1;\infty)$ | $[h_4;\infty)$ |
| II | $(\mu-r)/\sigma^2 \leq 1 \cap \mu > r$ | $[h_1;\infty)$ | $[h_4;\infty)$ |
| III | $\mu \leq r$ | $(0;h_2]\cap[h_3;\infty)$ | $(0;h_5]$ |

Set of the successful hedge coincide in methods of quantile hedging and expected-shortfall hedging. Therefore hedging portfolio is optimal not only by the criterion of the probability of success, but also by the criterion of minimizing losses. In the case of condition 3, the hedge will be optimal only from the viewpoint of one of the criteria.

### 3.3.2. Conclusion

Examples of hedging of call and put options show that the hedging strategy is optimal from the perspective of the criteria of maximizing the probability of success and minimize the expected loss under certain conditions, the relationship between the model parameters such as volatility, the average growth rate and the interest rate. The precise form of such relationship varies depending on the type of option and its parameters.

### 3.3.3. International practice of hedging.

The modern economy is characterized by fluctuations in the prices of goods and services. Manufacturers and consumers are interested in establishing effective mechanisms to protect them from unexpected changes in prices and to minimize the adverse of economic consequences. In the activities of any company, whether it is an investment fund, or agricultural producer, there is always a financial risk. It may be associated with any actions of economic entities: the sale of manufactured products, long term ownership of investment instruments, vulnerablity to depreciation, the purchase of assets.

It means that in the course of business, companies and entrepreneurs are facing the possibility that in the result of their operations, they will have a loss or a profit that was not expected due to unexpected changes in the price of one asset with which operation is conducted. The risk implies the possibility of loss, and the possibility of winning, but there is a significant category of persons that are not clined to take risks, willing to give up a greater profit for the sake of reducing the risk of losses.

Hedging has more than a century of history of existence but the bulk of publications and scientific researches in this area accounted for 80’s and 90’s of the twentieth century. However, even in 1848, draws on the CBOT (Chicago Board of Trade) futures contracts on grains to a great extent were used by traders in order to neutralize market risk.

### 3.3.4. The largest foreign markets of futures

Table 3.3 provides information about the instruments, which are traded on a stock exchanges in countries.

**Table 3.3 Trading instruments on a stock exchange**

| | Traded instruments | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Exchange | Shares | Bonds | Funds | Exchange-traded funds, ETF | Structured Products | Futures | Options | Commodity spot market |
| **NYSE** | + | + | + | + | + | | | |
| Nasdaq | + | | + | + | + | | | |
| Amex | + | + | + | + | + | | shares, ADR, stock index, ETF, HOLDR, including options LEAP and FLEX | |
| Nymex | | | | | | for crude oil, natural gas, petroleum, gasoline, coal, propane, gold, silver, copper, aluminum, platinum, palladium. | Crude oil, natural gas, oil, gasoline, gold, silver, copper, aluminum, platinum | |
| CME | | | | | | for agricultural products and ethanol, stock indexes,currencie,interest rates, market indices USA real estate, weather and futures TRAKRS, on indices TRAKRS, calculated taking into account dividends and other | Crude oil, natural gas, oil, gasoline, gold, silver, copper, aluminum, platinum | butter, cheese, skim milk powder |
| CBOT | | | | | | for agricultural commodities, metals,interest rates, the Dow Jones | for agricultural commodities, metals, interest rates, including FLEX options on | |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | interest rates, the Dow Jones | |
| CBOE | | | | | | | shares, stocks, ETF, HOLDR, interest rates, including options LEAP and FLEX | |

The existence of three major stock exchanges (Chicago Board of Trade, Chicago Mercantile Exchange, Chicago Board Options Exchange) gives Chicago status of financial center. In 2007 the volume of derivatives traded on the Chicago Stock Exchange reached about two thirds of trades of in the USA. Chicago Stock Exchange dominated in trading derivative financial instruments for long time in the world, but during the 1990s, lost some market share. European and Asian markets offered electronic trading system earlier than Chicago. Until a certain moment on the Chicago Stock Exchange existed voice trading, but when it stopped, electronic trading on competitive stock exchanges did not stop.

Shareholders of Chicago Mercantile Exchange Holdings Inc. (CME) and CBOT Holdings Inc. (SBOT) approved the merger of the two largest stock exchanges in the United States. The result of the vote in each of the companies identified overwhelmingly approvement of the absorption of CBOT by CME. The transaction is estimated in 11.9 billion USD. During the discussion to the conditions for the shareholders were improved several times in the deal. The result: in latest proposal of CME, CBOT shareholders received 0.375 common shares of the combined company, instead of 0.35. The combined company was named CME Group. According to some media reports, CBOT shareholders approved the merger with CME, and formed a market that has become the world's largest platform specializing in derivatives instruments.

However, hedging remains to some extent "destiny of elite" and not only in Russia. This activity accepts only professionals of the highest caliber, and there's a simple explanation: the careless use of financial instruments may apply to "play with fire", and the achieved effect is directly opposite the expected one.

As an example there can be brought practices of various companies:

**Experience of NATIONAL INSTRUMENTS CORPORATION (Germany).**

NATIONAL INSTRUMENTS Corporation is one of the market leaders in Europe’s instruments. The list of products includes various kinds of tools for commercial, scientific, industrial and other equipment, the total number of items are hundreds.

NATIONAL INSTRUMENTS Corporation uses a full-scale hedging strategy for the whole volume of foreign currency earnings. The basis of the strategy are the forward and futures contracts on the euro, the Japanese yen and the pound sterling. Contracts are not bought by the company to achieve speculative purposes. By the end of 2005, hedging strategy has reduced the loss from exchange rate fluctuations by 1.4 million dollars, increasing additional revenue by 11.5 million dollars.

**Experience of INTERNATIONAL BUSINESS MACHINES Corporation (IBM).**

Corporation INTERNATIONAL BUSINESS MACHINES (IBM) is a long-time leader of the global computer hardware market. One of the main requirements of IBM is an effective system of internal controls.

The issue about hedging occurs at IBM in connection with the implementation of large amounts of products in markets worldwide. In 2008, due to the weakening of the dollar, IBM received additional revenue from exchange transactions, part of which was leveled by hedging transactions. The share of revenue received directly in dollars, is not more than 56 per cent of total income, while the number of currencies used in the turnover reaches 35.

On 31 December 2009 in the possession of IBM, there were derivatives in the value of $ 450 million. In 2009, the time value of derivatives in the management of IBM increased by 52%.

Nevertheless, the hedging strategy applied by the company, can not cover all the IBM risks, related to the exchange of currencies.

Due to the volatility of the current market situation hedging becomes a particular need for many market participants. Further in the research there will be discussed examples of hedging using currency and oil futures held in overseas markets.

Example:

Company «XXX» is planning a conversion operation to exchange dollars for euros. Volume of transactions – 1 000 000 USD. In this case, there is a a risk - that in the time of transaction the euro will increase significantly to dollar, which will cause additional costs.

Eliminate the risk associated with a possible increase the course, there can be used transactions with futures on the euro to the dollar.

Currently, on the futures market the company «XXX» is interested in instrument - estimated futures on the euro to the dollar.

Date of execution - 15 September 2014.

Final settlement price - the euro against the dollar, set by the European Central Bank on the day of the futures. Volume of one contract - 30 000 rubles. At the moment futures quoted on company «XXX» arrange level 1.7000.

Then in the described conversion operation is necessary to conclude a deal to buy a certain number of futures at the price of 1.5000 for hedging risk.

Let's calculate how many futures contracts are needed to be bought. On August 15, 2014. rates were:

USD/RUR = 36,2521

EUR/USD = 1,3400

EUR/RUR = 46,8730

Based on the trends prevailing in the market, it is assumed that on September 15, 2014 exchange rate will be:

USD / RUR = 35,3529

EUR/USD = 1,5000

EUR/RUR = 48,0000

In the case of the sale of 1 000 000 USD company will receive:

1,000,000 / 1.5000 = 666 666.667 euros

Calculate the losses in case if the euro against the dollar on 15September 2014 will be higher for example by 0.1. Then the rates would be:

USD/RUR = 34,6667

EUR/USD = 1,6000

EUR/RUR = 49,0000

In this case, the sale of one million USD company will receive: 1 000 000 / 1.6000 = 625 000 euros

The losses of appreciation of the euro to the dollar at 0.1000 will be 666 666.667 - 625,000 = 41 666.667 euros or 41 666.667 * 49.0000 = 2 041 666.68 rubles.

Now consider the profits of one futures contract if the price on euros to dollar changes to 0.1000:

The cost of purchase of the futures contract at the rate of EUR / USD = 1,5000 is 30 000 rubles. * 1.5000 = 45 000.00 rubles. The cost of purchase of the futures contract at the rate of EUR / USD = 1,6000 is 30 000 rubles. * 1.6000 = 48 000.00 rubles.

So the profit is 48 000.00 - 45 000.00 = 3 000,00 rubles.

Thus, the losses caused by the growth of rate in the amount of 0.1000 will be 2041 666.68 rubles., And at the same time a profit from one of the futures contract is 3 000,00 rubles.

Then to hedge the operations it is needed to buy 2041 666.68 rubles / 3000 = 680,55 ≈ 681 futures contracts. It means that in situation of the appreciation of the euro against the dollar profit from 681futures contracts will fully compensate the losses of buying euros for USD at the exchange rate overvaluation.

To hedge described conversion operation company «XXX» bought 681 futures contract at a price of 1.5000. Presume that at the time of contract execution (September 15 2014.) Euro rate against the dollar will be 1.7000.

Courses are:

USD/RUR = 34,002

EUR/USD = 1,7000

EUR/RUR = 50,0000

Converting one million dollars by the rate of 1 000 000/666 = 1.5000 666.667 euro company acquired only 1 000 000 / 1.7000 = 588 235.294 euros.

Losses amounted to 666.667 666 - 588 = 78 235.29 431.377 euros or 78 431.377 * 50.0000 = 3 921 568.85 rubles. At the same time profit from the purchase of futures contracts were as follows: (30 000 rubles. * 681) * (1.7000 - 1.5000) = 4.086 million rubles (amount gained is a little higher because of rounding the number of contracts in a big way to the nearest whole number).

As a result, the profit on the futures position is fully compensated the losses from the change rates. Thus, the futures help to minimize the risks of transactions in the foreign exchange markets. So the investor hedge this transaction on adverse changes rates.

As a rule, all foreign exchange transactions are carried out through the Forex market, where investors can also make transactions to hedge positions.

### 3.3.5. The use of hedging instruments at the foreign trade operations by Forex market participants

In Russia, about Forex learned in the early 90s due to the beginning of market relations. First who entered in the market were large banks, which quickly oriented that the operations in this market can provide high returns. In many banks were created specialized departments, which conducted operations on FOREX. Appeared the first professional currency traders, this specialty immediately became very popular.

Exchange rates in the international Forex market are constantly changing. As a result, the real cost of buying or selling goods for the currency of goods may change significantly, and the contract that seemed initially beneficial may ultimately appear unprofitable. Of course, the opposite situation is possible when the change in the currency exchange rate makes a profit, but the task of a trading company is not to gain profit from changes in exchange rates. For a trading company, it is important to be able to plan the real cost either for bought or sold goods.

Trades in the Forex market are made on the basis of margin trading. This kind of trading has several features which have made it very popular.

Small start-up capital allows implement transactions in amounts that is many times (in the tens or hundreds of times) exceeding it. This excess is called leverage.

Trade is carried out without real money supply, which reduces overhead costs and gives the opportunity to open positions as buying and selling currencies (including different from the currency of the deposit).

Hedging of currency risk with the help of deals without the actual movement of funds (using leverage) provides additional opportunities:

- Allows not distract significant amounts of money from the turnover of the company;
- Allows to sell the currency that will be received in the future.

In order to use the advantages provided by the hedges it is necessary to open a trading account in company that provides trading services in the Forex market. The general principle of hedging in the foreign trade operations is to open foreign currency position on the trading account in the direction of the future operations for conversion of funds. Importer need to buy foreign currency, so it opens a position by buying currency on the trading account, and at the moment of real currency purchases in bank, the position closes. Exporter is required to sell foreign currency, so it opens a position by selling currencies in the trading account, and at the moment of real currency selling in bank, the position closes.

Consider the example.

Importing company waits delivery consignments from Europe in the amount of EUR 100 000 during the month. The company has dollars on the account and it will have to convert them at their bank to the euro. Based on the calculation of costs and future profits, the current euro exchange is suitable for business. But the manager does not want to buy euros now for the full amount of the contract and thereby preserve their funds. So he decides to hedge the risk of euro rate growth by making deals on the trading account without actual delivery of funds. To do this, he transfers $ 10 000 into his trading account and opens a long position on the EUR / USD (buy euro, sell dollars) in the amount of EUR 100 000. An amount of $ 10 000 on the trading account allows you to "withstand" unfavorable movement of the rate of more than 800 points (100 000 * 0.0800 = 8000).

**Table 3.4 The sequence of actions of hedging**

| Date and Event | EUR/USD | Transaction in the bank with physical delivery | Transactions on trading account without actual delivery |
|---|---|---|---|
| Conclude the Contract | 1.54560 | | Buy EUR 100 000, sale dollars |
| A month later, the receipt of goods and | 1.60000 | Buy EUR 100 000, sale dollars | Sale of 100 000 euros, buy dollars |

| | | | |
|---|---|---|---|
| the transfer 100 000 EUR to seller | | | |
| Result | | Loss of $ 5,440 on a course 1.54560 | Income $ 5 440 |

As seen in the table, loss received in the transaction with the actual delivery, offset by a profit on the trading account. Thus, the result turned to zero.

Thus, the management of the company rid themselves from concerns about the possible increase in euro exchange rate and retained funds for other operations.

Consider another example.

Within six months, are expected several shipments of raw materials abroad with payment in euros for the sum of 1 000 000 EUR. However, the exporting company prefers to keep their money in dollars, so it will have to convert received euros into dollars. If the current rate of the euro to the dollar the is favorable for the company, it is necessary to open a short position in the euro / dollar (sell euros and buy dollars) to the whole amount of the contract, and then close it partly depending on the amount of each consignment, (Table 3.5).

**Table 3.5 The sequence of actions of hedging (Example 2)**

| Date and Event | EUR/USD | Transaction in the bank with physical delivery | Transactions on trading account without actual delivery |
|---|---|---|---|
| Conclude the Contract | 1.54560 | | Sale of 1000 000 euros, buy dollars |
| After month: Delivery of the consignment and receive € 500 000 from the buyer | 1.6000 | Sale 500 000 euros, buy dollars<br>Income 27 200 | Buy 500 000 euros, sale dollars |
| After 2 month: Delivery of the consignment and receive € 300 000 from the buyer | 1.5800 | Sale 300 000 euros, buy dollars<br>Income 10 320 | Buy 300 000 euros, sale dollars |

| | | | |
|---|---|---|---|
| After month: Receipt of goods and the transfer of EUR 200 000 to seller | 1.5000 | Sale 200 000 euros, buy dollars<br>Loss 9 120 | Buy 200 000 euros, sale dollars |
| Result | | Income 28,400 dollars relative to the course 1.54560 | Income 28 4000 |

Thus, the resulting loss on the trading account, is compensated by the profits from the euro conversion in the bank at a better rate.

It should be noted that at the time of conclusion of the contract the company-exporter does not have the euro in the presence, but the use of schemes of work with no real supply means it has the ability to sell the full amount of the euro, that would be received in the future.

To open a position in the amount of 1 000 000 euros, on the trading account there has to be about 15 500 (1 000 000 * 1.54560 / 100 = 15 456 100 - this is the maximum lever 1.54560 - euro against the dollar). However, to "withstand" the fluctuations in the amount of up to 500 points, on the trading account must be additionally about $ 75,000 (every 100 points "are" 15,456 dollars: 1000000 * 0.0100 = 10, 000). Even when moving to 1,000 points per month (which happens quite rare) from turnover is diverted only $ 100 000 (1 000 000 * 0.1000 = 100 000), ie, 10% of the contract amount.

Position has to be kept open for a long time, and every day the position is migrating on the following date (rollover) taking into account differences in interest rates in the currencies involved in the transaction. In the current market practice, this amounts to approximately 0.01% per day, per month it is 0.3% of the transaction amount. However, the client will either pay this sum for the migration of the position, or receive this amount depending on the direction of the transaction (buy or sell).

To open a position is required to make a security deposit. The value of this deposit is generally from 1% to 5% of the bargain. After closing the position deposit can be removed from the trading account (including profit or loss).

But the practice of hedging is present not only in the foreign exchange market. Taking into account rising oil prices an example of buyer protection from unwanted oil price increases is quite relevant.

Consider an example where the company XXX have to buy 1 million barrels of crude oil in 3 months, and like any buyer it is hedging against adverse price increases. It was decided to hedge deliverable futures contract for Brent «Urals». The contract price is determined by the price of 1 barrel of oil, and the contract includes 10 units of the underlying asset.

At the beginning of August 2014 the price for a barrel of oil was 103,38 USD (with a fixed price of 106 USD per barrel), so each contract will cost the same amount. To hedge 1 million barrels, yhere are needed 1,000,000 / 10 = 100 000 contracts.

**Table 3.6 The sequence of actions of hedging (Example 3)**

| Date | Derivatives Market | Physical delivery |
|---|---|---|
| August 5, 2014 | Buy 100 000 contracts with price 103,38. Expenses 10 338 000 USD | Does not occur. |
| September 5, 2014 | Execute contracts for a fixed price. Pay 106 USD * 1 000 000 = 106 000 000 USD | Price on the spot market increased to 120,57 USD per barrel. |
| Total | Our income was (120.57 - 106) * 1 000 000 = 14 570 000 USD minus cost of hedging<br>Total 4 232 000 | |

The table shows that by hedging our position from the price increase we have not lost more than 4 000 000 USD, but we had relevant costs of hedge, by strengthening our position in front of the adverse effects of the market.

Thus, the cost of hedging are very small compared to the amount of the hedged contracts. The aim of hedging is not the extraction of additional revenue, but to reduce potential losses. Therefore the effectiveness of the hedge can be assessed only taking into account the basic activities of the trading company. Well-built hedging program reduces not only risk but also costs due to the release of the company's resources, it also helps the manager to focus on the main aspects of the business.

Modern derivatives market allows its participants manage risk through the use of the following basic hedging instruments: futures contracts, options and operations "swap".

Inclusion in a portfolio hedge of an asset in all aspects can improve the quality of existing investments. Among foreign investors this statement for a long time is an indisputable fact.

In the recent years Russian investors are moving towards a gradual awareness of the opportunities and the need of risk hedging to their business.

# CHAPTER 4: EFFECTIVNESS OF METHODS OF IMPERFECT HEDGING OF FINANCIAL OPTIONS ON RUSSIAN DERIVATIVES MARKET

In this chapter the empirical study of the concepts of hedging will be carried out. The study will done on the basis of one of the standard methods of hedging - delta hedging. This method is quite simple, which explains its prevalence. The essence of this method is as follows: By using some of the risky underlying assets portfolio and bank accounts (credit), payoff function of fixed-term contract is dynamically reproduced. At the same time, the part of shares or other risky asset in the portfolio corresponds to the delta option (one of the indicators of "Greeks" in the Black-Scholes model).

**Delta**

Delta is the first derivative of option value, indicating how much the value of the option will move at a modification in underlying price. This is why delta is called **percent change**. Delta is also called **percent chance**, its value indicating what is the chance for that option to expire in the money. Delta is at the same time **hedge ratio**, indicating how much of the underlying contract must be bought/sold in order to equal the movement in option price. Deltas are additive. The delta of an option portfolio made up by several options on the same underlying asset, is the sum of individual options delta times position sizes. This kind of delta is called *position delta*. If we are to include the underlying in a portfolio, we can say the underlying has a delta of 1.

**Option Delta**

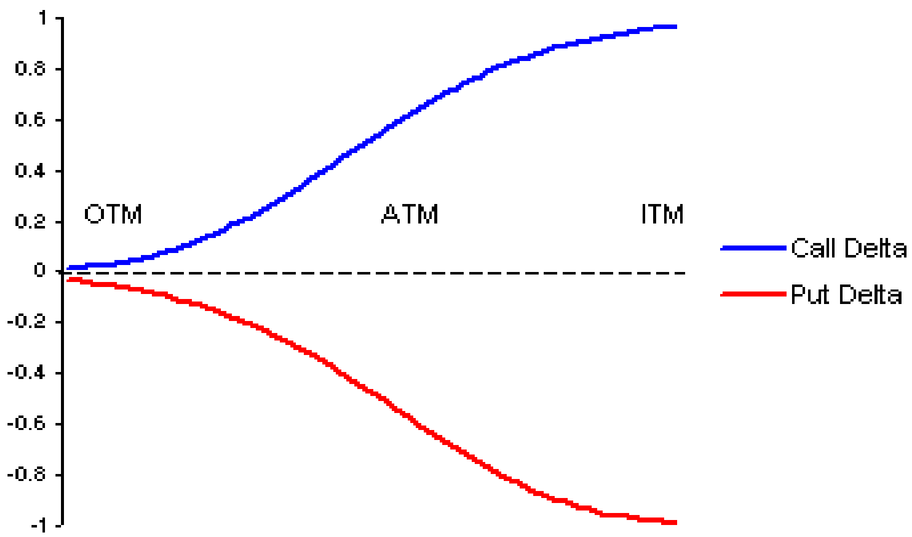


**Figure 4.1 Option delta**

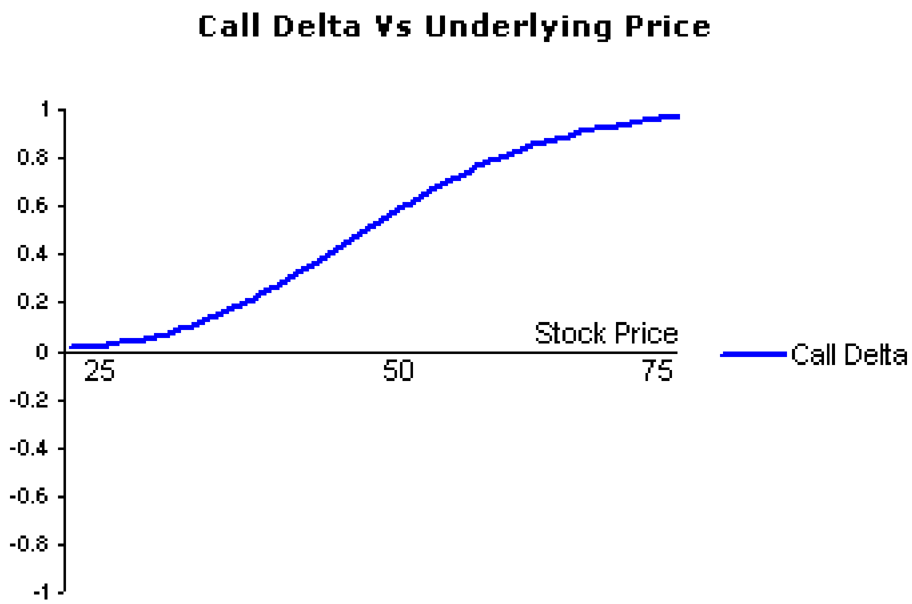


**Figure 4.2 Call delta Vs Underlying delta**

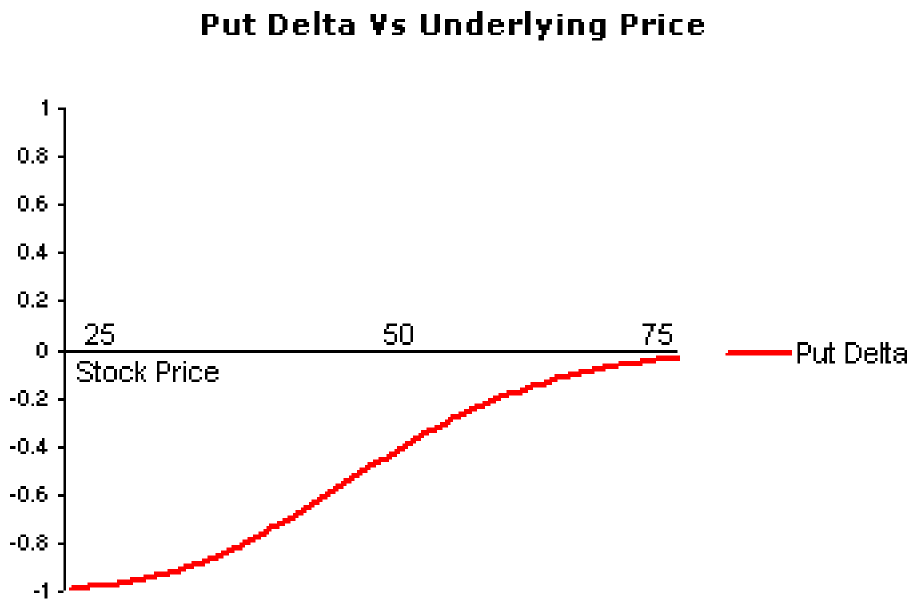


**Figure 4.3 Put Delta Vs Underlying Price**

Look carefully at these figures. Figure two and three might seem to contradict the first, but in the first figure the horizontal axis is the on money status of the options, whether in two and free the horizontal axis is the underlying price. Delta ranges from 0 to 1 for Call options, and from 0 to -1 for Put options. Please note that there is also a percentual notation for option, where options might have deltas of 100 or -100. The more out of the money options are, the closer to 0 delta is, i.e. options are almost insensitive to underlying shifts; the more in the money options are, the more extreme values it takes, i.e. options value almost copies underlying movement. Also, the sum of the call delta and the absolute value of it's correspondent put must yield 1 (or 100, if perceptually).

**Table 4.1 Call Deltas and expiries**

| **Strike** | **Jun** | **Jul** | **Oct** | **Jan** |
|---|---|---|---|---|
| 25 | 1.00 | 0.99 | 0.94 | 0.90 |
| 35 | 0.80 | 0.80 | 0.78 | 0.77 |
| 45 | 0.60 | 0.61 | 0.63 | 0.64 |
| 55 | 0.40 | 0.42 | 0.48 | 0.51 |
| 65 | 0.20 | 0.23 | 0.33 | 0.38 |

**Table 4.2 Put Deltas and expiries**

| **Strike** | **Jun** | **Jul** | **Oct** | **Jan** |
|---|---|---|---|---|
| 25 | 0.00 | -0.01 | -0.06 | -0.10 |
| 35 | -0.20 | -0.20 | -0.21 | -0.23 |
| 45 | -0.40 | -0.39 | -0.37 | -0.36 |
| 55 | -0.60 | -0.58 | -0.52 | -0.48 |
| 65 | -0.80 | -0.77 | -0.67 | -0.61 |

As we look at farther expiries, we find decreasing deltas for ITM Calls, and increasing deltas for OTM Calls. For Put options, the situation is reversed: we find increasing deltas for ITM Puts, and decreasing deltas for OTM Puts (we may consider the same situation, but applied to the **absolute delta value**). Delta has a tendency to remain quite constant for ATM options. For farther expiries, the delta range (maximum and minimum delta per all strikes) is shrinking. As days go by, the ITM calls delta will be increasing (as chance to remain ITM goes higher) and the OTM calls delta will be decreasing (as their chance to become ITM decreases). Same behavior applies to the absolute value of Put options delta. This effect is known as **trumpification**, which is **a delta effect caused by time and volatility, that increases the deltas of OTM options, decreases the delta of ITM options, pushing them all towards 0.5 (absolute)**.

Option delta is the ratio of changing price of the option, which is caused by a change in prices of an risky underlying asset, with a changing price of risky assets. Mathematically, the option delta can be represented as the first derivative of the option value of the asset, underlying the option. Like any first derivative, this figure reflects the rate of option price change relatively to the dynamics of the price of the underlying asset.

In the beginning lets look closer at the algorithm of delta hedging. The understanding of the simple perfect hedge model is necessary for further study of the methods of imperfect hedging. To study delta hedging mechanism, the call option in the Black-Scholes model will be used. In addition to this model, there are other, more advanced methods of dynamic hedging, which operate in discrete time, but in this study we will use is the Black-Scholes model will be used, as it sufficiently illustrates the mechanisms of methods of imperfect hedging, and remains simple.

In this model, the delta index is a value of the standard normal distribution of the coefficient d1, the formula of the coefficient d1 is shown below, together with the formula of finding the coefficient of the delta of the call option:

$$\Delta_i = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(d_1-\mu)^2}{2\sigma^2}}, \text{ where}$$

$$d_1 = \frac{\ln\left(\frac{S_i}{x}\right)+\left(r+\frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}, \text{ where}$$

$S_i$ – The current price of an asset, T – time of maturity in years, r – Interest Rate, σ – volatility, and μ – the average growth rate, x – the option strike price.

Algorithm of delta hedging is quite simple and can be described by the following stages of its implementation:

1. With the release, a call option has a price, which can be found by using the Black-Scholes formula shown below:

   $$C_0(x)=S_0*\Phi(d_1(x)) - x*e^{-rT}*\Phi(d_2(x)), \text{ where}$$

   $$\Phi(d_1)=\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(d_1-\mu)^2}{2\sigma^2}},$$

   $$\Phi(d_2)=\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(d_2-\mu)^2}{2\sigma^2}},$$

   $$d_1(x)=\frac{\ln\left(\frac{S_i}{x}\right)+\left(r+\frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}},$$

   $$d_2(x) = d_1(x)-\sigma*\sqrt{T}$$

2. In order to hedge, in the portfolio will be bought the risky assets, in the equal amount to the index - delta of the call.
3. Funding, of the purchase of an asset in the portfolio, is made of two sources: the premium of the option and the bank loan at an interest rate $r$, expressed in percent per annum.

4. By the end of the first day of the hedge, there are delta units of shares (or other risky assets) in portfolio and loan $B$, the amount of which is determined as the difference between the value of the purchased shares and the option premium. The formula for calculating the amount of bank loan is as follows::

   $B_0 = \Delta_0 * S_0 - C_0^{BS}$, where

   $C_0^{BS}$ – price of the call option, calculated according to the formula of the preceding paragraph.
5. In this case, the value of all hedging portfolio is the sum of the value of shares and the bank loan. Note that, the first day of the hedging, value of the option is equal to the cost of the hedging portfolio, that is easy to understand, by comparing the formula of the fourth paragraph with the formula of the fifth paragraph.

From the second day of hedging the action takes place in the different way: suppose that the rate of risky asset has changed. Also on the same day the term of fixed-term contract is reduced by one day. Both figures lead to a change in the delta of the call option, and therefore it is necessary to adjust the hedging portfolio. Correction of the hedging portfolio is done as follows:

1. With a positive change in price of the underlying asset, delta of the call option is increased as well, so the investor should buy additional shares in portfolio in an amount equal to the increase value of the delta index of the call option. Purchase price in such a case is the result of multiplication of incremental delta of the call option at the current price of the underlying asset. This purchase is also financed by the additional loan. So the amount of the loan increases. Hedger also need to pay interest to the bank for the use of the loan in the first day of the hedge. The amount of interest is calculated by the formula:

   $r * B_0 * dt.$

   Therefore, the amount of bank loan increases, it can be calculated by the following formula:

   $B_1 = B_0 + r * B_0 * dt + (\Delta_1 - \Delta_0) * S_1$
2. With the negative dynamics of the underlying asset, correction of hedging portfolio can be made in a similar way, the only difference is that as a result of negative changes in the call option coefficient of delta, investor has to sell a number of shares of the hedging portfolio, moreover, the amount of the bank loan is reduced.

   As a result of these corrections, the cost of hedging portfolio will be calculated as follows:

   $\pi_1 = \Delta_1 * S_1 + B_1$

The cost of hedging portfolio may not be equal the theoretical value of a call option in the Black-Scholes model due to such phenomena as the hedging error. Hedging error may occur due to the fact that the dependence of the option price to changes in the price of risky asset, underlying in the foundation of the derivative, is not a linear function, whereas the delta hedge insures only the linear component of the change in the price of the option. This discrepancy between theory and practice of financial risk management is also due to the fact that in the theory, portfolio parameters are in the continuous model, whereas in practice, hedging occurs at discrete moments in time.

Below are presented formulas and algorithm of correction of hedging portfolio for the second day of the hedge. Due to changes in the price of the underlying asset and changes in the maturity of the option, in each of the following days it is necessary to make corrections the portfolio and review the size of all components of the hedging portfolio, then for the next few days the size of bank loan and total cost of hedging portfolio can be represented by the following formulas:

$$B_t = B_{t-dt} + r * B_{t-dt} * dt + (\Delta_t - \Delta_{t-dt}) * S_t$$

$$\pi_t = \Delta_t * S_t + B_t$$

Position of risky assets is liquidated on the date of maturity of fixed-term contract. The profit from the asset sale is used to repay the debt of a bank loans.

The illustration of the mechanism of delta hedging is shown in Figure 4.4.

| | Hedging portfolio | |
|---|---|---|
| | The underlying asset | Bank loan |
| $(\Delta_t - \Delta_{t-dt})>0$ | Buy | Increase |
| $(\Delta_t - \Delta_{t-dt})<0$ | Sell | Decrease |
| $(\Delta_t - \Delta_{t-dt})=0$ | - | - |

**Figure 4.4 Scheme of delta hedging.**

## 4.1. Empirical testing of methods of imperfect hedging.

Empirical testing of various methods of hedging is based on stock price and other risky assets that are involved in the FORTS (futures market) on the Moscow Stock Exchange RTS.

First, indicators such as market volatility and average growth rate of the asset price should be found, it is necessary to calculate the price of the call option.

**Stock volatility -** a statistical financial measure, which characterizes the average annual change (fluctuation) of asset price or price volatility. Volatility is an important indicator of risk management as it shows a measure of the risk of the use of certain financial instruments in a given period of time. Volatility is the mean square root (standard) deviation of the stock price divided by the square root of the time period, expressed in years. The formula for calculating the average annual volatility is presented below:

$$\sigma = \frac{\sqrt{\frac{1}{n}\sum_{i=1}^{n}(S_i - \bar{S})^2}}{\sqrt{P}}, \text{ where}$$

$$\bar{S} = \frac{\sum_{i=1}^{n} S_i}{n},$$

n - the number of trading days considered in the sample, S - quotation of shares in the period i, P - the time period in years.

Calculation of these indicators requires data of the dynamics of the asset price. In this study, the data of the Moscow Stock Exchange RTS will be used.

Below are graphs of the dynamics of the price change of the asset (shares of Sberbank and the currency pair USD RUR) for the period from January 2013 to May 2013 for illustrating the various methods of perfect and imperfect hedging.

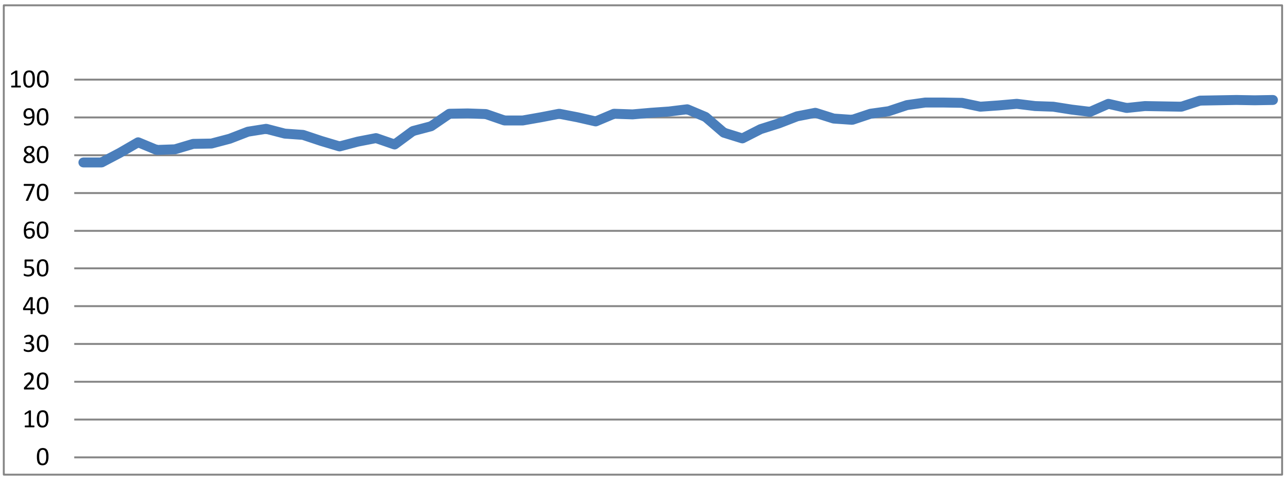


**Figure 4.5. Stock price dynamic of shares of Sberbank**

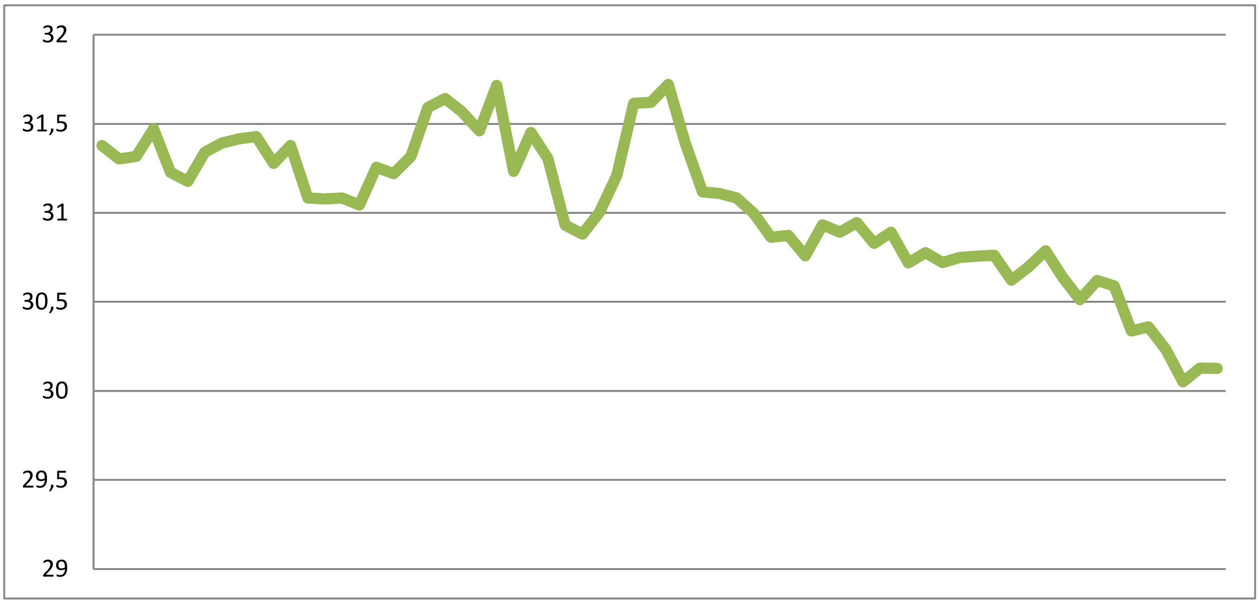


**Figure 4.6 Dynamics of the dollar against the Russian ruble.**

For the constant bank interest rate is taken the refinancing rate of the Central Bank of Russia, from September 14, 2012 until the present time (May 2013) and it is equal to 8.25% per annum. In reality, the interest rate is slightly different from the refinancing rate, but to illustrate the examples, assume the refinancing rate as a constant bank rate.

### 4.1.1. Perfect delta hedging.

Tables 4.3 and 4.4 shows the process of a call option hedging in the classical Black-Scholes model for various assets (shares of Sberbank and the currency pair USD RUR). Calculation will be done for 22 trading days. For each period (each date of the trade) are given the following indicators, that reflects the order and structure of the forward contract hedging portfolio:

1. Date - day of trade.
2. The quotation - The current value of the asset at the moment of closing the day of trade. In the formulas, ths figure is shown as the variability Si, where i - number of days of the hedge.
3. № - number of the hedging day
4. T – period of the option, expressed in years. Since the calculation period is 22 days, the initial day of the hedging parameter T is equal to 0.0880, with the subsequent increments of 0.004, in every day of the hedge.
5. ΔC(K) – the delta of the call option, the strike price is equal to K. K in this case is the quote of the asset at the beginning of the hedging.
6. Purchase - the number of shares or other assets purchased or sold at the time of correction of hedging portfolio.

7. Cost of purchase - the cost of the purchase or sale of an asset for the correction of the hedging portfolio.
8. Amount of the loan - the amount owed for the loan taken for the purchase of an asset in the hedging portfolio for each day of the hedging.
9. Value of an asset in the portfolio - the value of risky asset, which is the foundation of derivative instruments in hedging portfolio.
10. Value of the portfolio - the value of the hedging portfolio, which is equal to the value of shares, net of payments on a bank loan.

**Table 4.3 An example of delta of perfect hedging of shares of Sberbank.**

| The Quotation | № | T | ΔC(K) | Purchase | Cost of purchase | Amount of the loan | Value of an asset in the portfolio | Value of the portfolio |
|---|---|---|---|---|---|---|---|---|
| 92,60 | 0 | 0,088 | 0,5035 | 0,5035 | 46,6219 | 45,0743 | 46,6219 | 1,5477 |
| 92,70 | 1 | 0,084 | 0,5036 | 0,0001 | 0,0139 | 45,1043 | 46,6709 | 1,5667 |
| 92,75 | 2 | 0,08 | 0,5036 | 0,0000 | 0,0019 | 45,1223 | 46,6928 | 1,5706 |
| 92,50 | 3 | 0,076 | 0,5025 | -0,0011 | -0,1019 | 45,0367 | 46,4802 | 1,4436 |
| 93,00 | 4 | 0,072 | 0,5045 | 0,0020 | 0,1859 | 45,2387 | 46,9172 | 1,6786 |
| 93,90 | 5 | 0,068 | 0,5080 | 0,0035 | 0,3304 | 45,5852 | 47,6813 | 2,0961 |
| 92,85 | 6 | 0,064 | 0,5035 | -0,0045 | -0,4190 | 45,1828 | 46,7443 | 1,5616 |
| 93,19 | 7 | 0,06 | 0,5048 | 0,0013 | 0,1177 | 45,3167 | 47,0280 | 1,7115 |
| 93,60 | 8 | 0,056 | 0,5064 | 0,0016 | 0,1457 | 45,4786 | 47,3755 | 1,8970 |
| 92,98 | 9 | 0,052 | 0,5036 | -0,0027 | -0,2529 | 45,2421 | 46,8189 | 1,5769 |
| 92,80 | 10 | 0,048 | 0,5028 | -0,0008 | -0,0781 | 45,1804 | 46,6604 | 1,4801 |
| 92,07 | 11 | 0,044 | 0,4993 | -0,0035 | -0,3201 | 44,8767 | 45,9533 | 1,0767 |
| 91,55 | 12 | 0,04 | 0,4969 | -0,0025 | -0,2249 | 44,6680 | 45,4639 | 0,7960 |
| 93,59 | 13 | 0,036 | 0,5055 | 0,0087 | 0,8127 | 45,4967 | 47,2949 | 1,7983 |
| 92,56 | 14 | 0,032 | 0,5010 | -0,0045 | -0,4189 | 45,0942 | 46,3605 | 1,2663 |
| 93,07 | 15 | 0,028 | 0,5030 | 0,0020 | 0,1819 | 45,2922 | 46,7877 | 1,4955 |
| 92,90 | 16 | 0,024 | 0,5023 | -0,0007 | -0,0661 | 45,2425 | 46,6563 | 1,4139 |
| 92,80 | 17 | 0,02 | 0,5017 | -0,0006 | -0,0542 | 45,2047 | 46,5520 | 1,3474 |
| 94,47 | 18 | 0,016 | 0,5086 | 0,0070 | 0,6578 | 45,8787 | 48,0473 | 2,1687 |
| 94,55 | 19 | 0,012 | 0,5087 | 0,0001 | 0,0096 | 45,9045 | 48,0873 | 2,1827 |
| 94,65 | 20 | 0,008 | 0,5090 | 0,0002 | 0,0215 | 45,9426 | 48,1545 | 2,2120 |
| 94,59 | 21 | 0,004 | 0,5086 | -0,0004 | -0,0385 | 45,9208 | 48,0856 | 2,1649 |
| 94,65 | 22 | 0,000 | 0,5087 | 0,0001 | 0,0136 | 45,9508 | 48,1346 | 2,1840 |

**Table 4.4 Example of delta of perfect hedging of USD RUB pair (USD - Russian ruble).**

| The Quotation | № | T | ΔC(K) | Purchase | Cost of purchase | Amount of the loan | Value of an asset in the portfolio | Value of the portfolio |
|---|---|---|---|---|---|---|---|---|
| 30,1140 | 0 | 0,088 | 0,5033 | 0,5032 | 15,1522 | 14,9976 | 15,1522 | 0,1548 |
| 30,0775 | 1 | 0,084 | 0,5026 | -0,0006 | -0,0190 | 14,9840 | 15,1149 | 0,1310 |
| 30,0695 | 2 | 0,08 | 0,5024 | -0,0003 | -0,0077 | 14,9819 | 15,1033 | 0,1215 |
| 30,1710 | 3 | 0,076 | 0,5036 | 0,0012 | 0,0366 | 15,0237 | 15,1910 | 0,1674 |
| 30,1593 | 4 | 0,072 | 0,5033 | -0,0003 | -0,0093 | 15,0199 | 15,1756 | 0,1558 |
| 30,1580 | 5 | 0,068 | 0,5031 | -0,0002 | -0,0050 | 15,0204 | 15,1699 | 0,1496 |
| 30,0499 | 6 | 0,064 | 0,5015 | -0,0016 | -0,0474 | 14,9785 | 15,0684 | 0,0900 |
| 29,9599 | 7 | 0,06 | 0,5002 | -0,0013 | -0,0402 | 14,9438 | 14,9833 | 0,0395 |
| 30,1234 | 8 | 0,056 | 0,5022 | 0,0020 | 0,0611 | 15,0102 | 15,1259 | 0,1158 |
| 29,9255 | 9 | 0,052 | 0,4995 | -0,0028 | -0,0831 | 14,9326 | 14,9435 | 0,0110 |
| 29,9968 | 10 | 0,048 | 0,5003 | 0,0008 | 0,0243 | 14,9622 | 15,0034 | 0,0413 |
| 30,0164 | 11 | 0,044 | 0,5004 | 0,0001 | 0,0036 | 14,9711 | 15,0166 | 0,0457 |
| 30,0279 | 12 | 0,04 | 0,5004 | 0,0000 | 0,0004 | 14,9768 | 15,0228 | 0,0461 |
| 30,1518 | 13 | 0,036 | 0,5019 | 0,0015 | 0,0452 | 15,0272 | 15,1297 | 0,1025 |
| 30,0786 | 14 | 0,032 | 0,5008 | -0,0011 | -0,0335 | 14,9992 | 15,0595 | 0,0605 |
| 30,0455 | 15 | 0,028 | 0,5002 | -0,0006 | -0,0176 | 14,9871 | 15,0255 | 0,0385 |
| 30,1649 | 16 | 0,024 | 0,5016 | 0,0014 | 0,0436 | 15,0360 | 15,1288 | 0,0930 |
| 30,2296 | 17 | 0,02 | 0,5023 | 0,0007 | 0,0215 | 15,0628 | 15,1825 | 0,1198 |
| 30,1956 | 18 | 0,016 | 0,5017 | -0,0006 | -0,0181 | 15,0502 | 15,1473 | 0,0972 |
| 30,2977 | 19 | 0,012 | 0,5029 | 0,0012 | 0,0365 | 15,0920 | 15,2349 | 0,1430 |
| 30,2070 | 20 | 0,008 | 0,5016 | -0,0013 | -0,0405 | 15,0571 | 15,1490 | 0,0920 |
| 30,3436 | 21 | 0,004 | 0,5033 | 0,0017 | 0,0504 | 15,1128 | 15,2678 | 0,1551 |
| 30,3400 | 22 | 0,000 | 0,5031 | -0,0002 | -0,0057 | 15,1126 | 15,2605 | 0,1481 |

### 4.1.2. Quantile hedging.

Tables 4.5 and 4.6 shows the procedure of quantile hedging, one of the method of imperfect hedging of financial options.

Technical difference between quantile hedging procedure and method of delta perfect hedging consists of:

1. As the ratio ($\mu$-r) / $\sigma$2 ($\mu$ - the average increase in the cost of the asset, r - interest rate, $\sigma$ - volatility) is less than the figure one in both cases (shares of Sberbank and the currency pair USD RUR), the structure of the hedging portfolio in the model of quantile hedging will be a combination of three options: two standard call options $C_T$ (K) and $C_T$ (h) with strike price K and h (upper limit of h is calculated from the formula described in the second chapter of this study) and the binary option $CoN_T$ (h) with a strike price h.
2. In tables below calculations are made with an upper limit of h, with the probability of success of 95%. To reach mentioned probability, the upper limit of h is equal to 98.8745 in the case of shares of Sberbank and 30.4991 in the case of the currency pair USD RUR.
3. Structure of payments of hedging portfolio in such a case will be described by the following formula:

$V_T = [S_T - K] * I(S_T \geq K) - [S_T - h] * I(S_T \geq h) - [h - K] * I(S_T \geq h)$, *where*

$[S_T - K] * I(S_T \geq K) - [S_T - h] = C_T(K)$,

$[S_T - h] * I(S_T \geq h) = C_T(h)$,

$I(S_T \geq h) = CoN_T(h)$

4. Ratio of the delta hedging portfolio is calculated using the formula:

$\Delta V_t = \Delta C_t(K) - \Delta C_t(h) - [h - K] * \Delta CoN_t(h)$

5. Thus, correction of hedging portfolio in the method of the quantile hedging is not done in accordance with coefficient $\Delta C$ (K) as in the method of classical perfect model, but with the coefficient of $\Delta V$

Based on the foregoing, in Tables 4.5 and 4.6 columns of delta factor are added.

**Table 4.5 Procedure of quantile hedging of Sberbank shares.**

| The quotation | № | T | ΔC(K) | ΔC(h) | ΔCoN(h) | ΔV | Purchase | Cost of purchase | Amount of the loan | Value of an asset in the portfolio | Value of the portfolio |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 92,60 | 0 | 0,088 | 0,6035 | 0,0386 | 0,0360 | 0,1945 | 0,1945 | 18,010 | 16,4623 | 18,0099 | 1,5477 |
| 92,70 | 1 | 0,084 | 0,6104 | 0,0362 | 0,0338 | 0,2079 | 0,0134 | 1,2346 | 17,7027 | 19,2580 | 1,5554 |
| 92,75 | 2 | 0,08 | 0,6137 | 0,0329 | 0,0307 | 0,2224 | 0,0147 | 1,3499 | 19,0589 | 20,6161 | 1,5573 |
| 92,50 | 3 | 0,076 | 0,5809 | 0,0239 | 0,0223 | 0,2391 | 0,0168 | 1,5428 | 20,6084 | 22,1099 | 1,5016 |
| 93,00 | 4 | 0,072 | 0,6487 | 0,0313 | 0,0293 | 0,2484 | 0,0094 | 0,8638 | 21,4795 | 23,0931 | 1,6136 |
| 93,90 | 5 | 0,068 | 0,7595 | 0,0543 | 0,0513 | 0,2202 | -0,0283 | -2,6489 | 18,8384 | 20,6578 | 1,8194 |
| 92,85 | 6 | 0,064 | 0,6233 | 0,0195 | 0,0182 | 0,2804 | 0,0603 | 5,5910 | 24,4362 | 26,0242 | 1,5882 |
| 93,19 | 7 | 0,06 | 0,6713 | 0,0220 | 0,0207 | 0,2976 | 0,0173 | 1,6039 | 26,0488 | 27,7205 | 1,6719 |
| 93,60 | 8 | 0,056 | 0,7294 | 0,0266 | 0,0251 | 0,3123 | 0,0148 | 1,3801 | 27,4381 | 29,2195 | 1,7815 |
| 92,98 | 9 | 0,052 | 0,6400 | 0,0113 | 0,0106 | 0,3218 | 0,0096 | 0,8798 | 28,3277 | 29,9119 | 1,5843 |
| 92,80 | 10 | 0,048 | 0,6120 | 0,0070 | 0,0065 | 0,3229 | 0,0012 | 0,1044 | 28,4422 | 29,9647 | 1,5226 |
| 92,07 | 11 | 0,044 | 0,4659 | 0,0017 | 0,0015 | 0,2602 | -0,0628 | -5,7718 | 22,6807 | 23,9444 | 1,2638 |
| 91,55 | 12 | 0,04 | 0,3544 | 0,0005 | 0,0004 | 0,2012 | -0,0592 | -5,4055 | 17,2834 | 18,4011 | 1,1178 |
| 93,59 | 13 | 0,036 | 0,7466 | 0,0061 | 0,0058 | 0,4029 | 0,2018 | 18,878 | 36,1670 | 37,6906 | 1,5237 |
| 92,56 | 14 | 0,032 | 0,5475 | 0,0007 | 0,0006 | 0,3106 | -0,0924 | -8,5398 | 27,6403 | 28,7401 | 1,0998 |
| 93,07 | 15 | 0,028 | 0,6544 | 0,0008 | 0,0007 | 0,3713 | 0,0608 | 5,6476 | 33,2978 | 34,5397 | 1,2420 |
| 92,90 | 16 | 0,024 | 0,6267 | 0,0003 | 0,0003 | 0,3574 | -0,0140 | -1,2901 | 32,0198 | 33,2015 | 1,1818 |
| 92,80 | 17 | 0,02 | 0,6021 | 0,0001 | 0,0001 | 0,3440 | -0,0136 | -1,2504 | 30,7809 | 31,9154 | 1,1345 |
| 94,47 | 18 | 0,016 | 0,9408 | 0,0008 | 0,0008 | 0,5349 | 0,1910 | 18,040 | 48,8310 | 50,5286 | 1,6977 |
| 94,55 | 19 | 0,012 | 0,9660 | 0,0002 | 0,0001 | 0,5515 | 0,0167 | 1,5729 | 50,4214 | 52,1336 | 1,7122 |
| 94,65 | 20 | 0,008 | 0,9891 | 0,0000 | 0,0000 | 0,5652 | 0,0138 | 1,2963 | 51,7358 | 53,4794 | 1,7437 |
| 94,59 | 21 | 0,004 | 0,9991 | 0,0000 | 0,0000 | 0,5710 | 0,0058 | 0,5410 | 52,2953 | 53,9864 | 1,6912 |
| 94,65 | 22 | 0,000 | 1,0000 | 0,0000 | 0,0000 | 0,5715 | 0,0007 | 0,0538 | 52,3679 | 54,0801 | 1,7123 |

**Table 4.6 Procedure of quantile hedging of a currency pair USD RUB.**

| The quotation | № | T | ΔC(K) | ΔC(h) | ΔCoN(h) | ΔV | Purchase | Cost of purchase | Amount of the loan | Value of an asset in the portfolio | Value of the portfolio |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 30,1140 | 0 | 0,088 | 0,9964 | 0,0524 | 0,0521 | 0,5280 | 0,5280 | 15,8989 | 15,7442 | 15,8989 | 0,1547 |
| 30,0775 | 1 | 0,084 | 0,9862 | 0,0137 | 0,0136 | 0,5527 | 0,0248 | 0,7456 | 16,4955 | 16,6252 | 0,1297 |
| 30,0695 | 2 | 0,08 | 0,9790 | 0,0065 | 0,0064 | 0,5543 | 0,0016 | 0,0468 | 16,5482 | 16,6675 | 0,1193 |
| 30,1710 | 3 | 0,076 | 0,9993 | 0,0742 | 0,0738 | 0,5124 | -0,0420 | -1,2658 | 15,2884 | 15,4583 | 0,1699 |
| 30,1593 | 4 | 0,072 | 0,9986 | 0,0382 | 0,0379 | 0,5405 | 0,0281 | 0,8481 | 16,1420 | 16,3001 | 0,1581 |
| 30,1580 | 5 | 0,068 | 0,9982 | 0,0237 | 0,0236 | 0,5517 | 0,0112 | 0,3374 | 16,4852 | 16,6367 | 0,1515 |
| 30,0499 | 6 | 0,064 | 0,9252 | 0,0002 | 0,0002 | 0,5286 | -0,0231 | -0,6936 | 15,7975 | 15,8836 | 0,0860 |
| 29,9599 | 7 | 0,06 | 0,5445 | 0,0000 | 0,0000 | 0,3112 | -0,2174 | -6,5135 | 9,2897 | 9,3226 | 0,0329 |
| 30,1234 | 8 | 0,056 | 0,9885 | 0,0009 | 0,0009 | 0,5641 | 0,2530 | 7,6204 | 16,9135 | 16,9939 | 0,0804 |
| 29,9255 | 9 | 0,052 | 0,2393 | 0,0000 | 0,0000 | 0,1368 | -0,4274 | -12,7898 | 4,1298 | 4,0924 | -0,0374 |
| 29,9968 | 10 | 0,048 | 0,5763 | 0,0000 | 0,0000 | 0,3293 | 0,1926 | 5,7770 | 9,9083 | 9,8792 | -0,0291 |
| 30,0164 | 11 | 0,044 | 0,6331 | 0,0000 | 0,0000 | 0,3618 | 0,0324 | 0,9728 | 10,8847 | 10,8584 | -0,0263 |
| 30,0279 | 12 | 0,04 | 0,6442 | 0,0000 | 0,0000 | 0,3681 | 0,0064 | 0,1913 | 11,0799 | 11,0538 | -0,0260 |
| 30,1518 | 13 | 0,036 | 0,9912 | 0,0000 | 0,0000 | 0,5664 | 0,1983 | 5,9790 | 17,0628 | 17,0783 | 0,0155 |
| 30,0786 | 14 | 0,032 | 0,8297 | 0,0000 | 0,0000 | 0,4741 | -0,0923 | -2,7763 | 14,2926 | 14,2606 | -0,0320 |
| 30,0455 | 15 | 0,028 | 0,5559 | 0,0000 | 0,0000 | 0,3177 | -0,1564 | -4,7002 | 9,5976 | 9,5447 | -0,0529 |
| 30,1649 | 16 | 0,024 | 0,9938 | 0,0000 | 0,0000 | 0,5679 | 0,2502 | 7,5469 | 17,1480 | 17,1297 | -0,0183 |
| 30,2296 | 17 | 0,02 | 1,0000 | 0,0000 | 0,0000 | 0,5714 | 0,0035 | 0,1070 | 17,2612 | 17,2733 | 0,0121 |
| 30,1956 | 18 | 0,016 | 0,9995 | 0,0000 | 0,0000 | 0,5711 | -0,0003 | -0,0083 | 17,2591 | 17,2454 | -0,0137 |
| 30,2977 | 19 | 0,012 | 1,0000 | 0,0000 | 0,0000 | 0,5714 | 0,0003 | 0,0089 | 17,2742 | 17,3126 | 0,0384 |
| 30,2070 | 20 | 0,008 | 1,0000 | 0,0000 | 0,0000 | 0,5714 | 0,0000 | -0,0002 | 17,2802 | 17,2607 | -0,0196 |
| 30,3436 | 21 | 0,004 | 1,0000 | 0,0000 | 0,0000 | 0,5714 | 0,0000 | 0,0002 | 17,2866 | 17,3389 | 0,0523 |
| 30,3400 | 22 | 0,000 | 1,0000 | 0,0000 | 0,0000 | 0,5714 | 0,0000 | 0,0000 | 17,2929 | 17,3371 | 0,0442 |

### 4.1.3. Expected shortfall hedging.

The algorithm of the method of expected shortfall hedging is not different from the algorithm of quantile hedging, since methods are quite related. However, theoretically, it differs in the structure of payments of hedging portfolio, but in this case they are equal, as the average growth rate is lower than the bank interest rate. There is also a difference in the calculation of the upper limit of h, because, instead of the probability of success, the expected loss is used. To reach the level of expected losses in the value of 5%, upper limit of h is equal to 95.2841 in the case of shares of Sberbank and 30.6901 in the case of the currency pair USD RUR.

The cost of hedging portfolio in the method of expected shortfall hedging is calculated as follows:

$Vi(x) = S_i * \Phi(d_1(K)) - K * e^{-rT} * S_i * \Phi(d_2(K)) - S_i * \Phi(d_1(h)) - - K * e^{-rT} * S_i * \Phi(d_2(h)) - (h - K) * e^{-rT} * S_i * \Phi(d_2(h),$ где

$$\Phi(d_1) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(d_1-\mu)^2}{2\sigma^2}},$$

$$\Phi(d_2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(d_2-\mu)^2}{2\sigma^2}},$$

Tables 4.7 and 4.8 shows the procedure of the expected shortfall hedging. Accordingly, Table 4.7 shows the method of delta of expected shortfall hedging for the underlying asset - shares of Sberbank, and Table 4.8 - for the currency pair USD RUR.

**Table 4.7 The procedure of the expected shortfall hedging of shares of Sberbank.**

| The quotation | № | T | ΔC(K) | ΔC(h) | ΔCoN(h) | ΔV | Purchase | Cost of purchase | Amount of the loan | Value of an asset in the portfolio | Value of the portfolio |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 92,60 | 0 | 0,088 | 0,6034 | 0,0193 | 0,0180 | 0,2097 | 0,2097 | 19,423 | 17,875 | 19,4232 | 1,5476 |
| 92,70 | 1 | 0,084 | 0,6103 | 0,0180 | 0,0168 | 0,2164 | 0,0067 | 0,6226 | 18,504 | 20,0605 | 1,5559 |
| 92,75 | 2 | 0,08 | 0,6136 | 0,0164 | 0,0153 | 0,2228 | 0,0064 | 0,5919 | 19,103 | 20,6610 | 1,5578 |
| 92,50 | 3 | 0,076 | 0,5808 | 0,0119 | 0,0111 | 0,2220 | -0,0008 | -0,0741 | 19,035 | 20,5379 | 1,5020 |
| 93,00 | 4 | 0,072 | 0,6486 | 0,0156 | 0,0146 | 0,2407 | 0,0187 | 1,7369 | 20,779 | 22,3858 | 1,6061 |
| 93,90 | 5 | 0,068 | 0,7594 | 0,0271 | 0,0256 | 0,2543 | 0,0136 | 1,2811 | 22,068 | 23,8739 | 1,8056 |
| 92,85 | 6 | 0,064 | 0,6232 | 0,0097 | 0,0091 | 0,2475 | -0,0068 | -0,6350 | 21,441 | 22,9794 | 1,5382 |
| 93,19 | 7 | 0,06 | 0,6712 | 0,0110 | 0,0103 | 0,2649 | 0,0174 | 1,6178 | 23,066 | 24,6789 | 1,6122 |
| 93,60 | 8 | 0,056 | 0,7293 | 0,0133 | 0,0125 | 0,2835 | 0,0186 | 1,7448 | 24,819 | 26,5296 | 1,7098 |
| 92,98 | 9 | 0,052 | 0,6399 | 0,0056 | 0,0053 | 0,2673 | -0,0162 | -1,5044 | 23,324 | 24,8551 | 1,5308 |
| 92,80 | 10 | 0,048 | 0,6119 | 0,0034 | 0,0032 | 0,2615 | -0,0058 | -0,5373 | 22,795 | 24,2750 | 1,4796 |
| 92,07 | 11 | 0,044 | 0,4658 | 0,0008 | 0,0007 | 0,2047 | -0,0569 | -5,2340 | 17,569 | 18,8396 | 1,2701 |
| 91,55 | 12 | 0,04 | 0,3543 | 0,0002 | 0,0002 | 0,1569 | -0,0478 | -4,3702 | 13,205 | 14,3609 | 1,1553 |
| 93,59 | 13 | 0,036 | 0,7465 | 0,0030 | 0,0028 | 0,3225 | 0,1656 | 15,496 | 28,706 | 30,1787 | 1,4722 |
| 92,56 | 14 | 0,032 | 0,5474 | 0,0003 | 0,0003 | 0,2424 | -0,0801 | -7,4146 | 21,302 | 22,4352 | 1,1329 |
| 93,07 | 15 | 0,028 | 0,6543 | 0,0003 | 0,0003 | 0,2898 | 0,0474 | 4,4071 | 25,717 | 26,9610 | 1,2440 |
| 92,90 | 16 | 0,024 | 0,6266 | 0,0001 | 0,0001 | 0,2782 | -0,0116 | -1,0743 | 24,652 | 25,8491 | 1,1970 |
| 92,80 | 17 | 0,02 | 0,6020 | 0,0000 | 0,0000 | 0,2675 | -0,0107 | -0,9938 | 23,667 | 24,8275 | 1,1604 |
| 94,47 | 18 | 0,016 | 0,9407 | 0,0003 | 0,0003 | 0,4170 | 0,1495 | 14,126 | 37,801 | 39,4002 | 1,5986 |
| 94,55 | 19 | 0,012 | 0,9659 | 0,0001 | 0,0001 | 0,4291 | 0,0121 | 1,1405 | 38,955 | 40,5657 | 1,6100 |
| 94,65 | 20 | 0,008 | 0,9890 | 0,0000 | 0,0000 | 0,4396 | 0,0105 | 0,9914 | 39,961 | 41,5957 | 1,6346 |
| 94,59 | 21 | 0,004 | 0,9990 | 0,0000 | 0,0000 | 0,4440 | 0,0044 | 0,4200 | 40,395 | 41,9893 | 1,5938 |
| 94,65 | 22 | 0,000 | 1,0000 | 0,0000 | 0,0000 | 0,4444 | 0,0004 | 0,0418 | 40,451 | 42,0622 | 1,6104 |

**Table 4.8 The procedure of the expected shortfall hedging on the example of the currency pair USD RUB.**

| The quotation | № | T | ΔC(K) | ΔC(h) | ΔCoN(h) | ΔV | Purchase | Cost of purchase | Amount of the loan | Value of an asset in the portfolio | Value of the portfolio |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 30,1140 | 0 | 0,088 | 0,9964 | 0,0262 | 0,0260 | 0,4267 | 0,4267 | 12,8507 | 12,6960 | 12,8507 | 0,1547 |
| 30,0775 | 1 | 0,084 | 0,9862 | 0,0068 | 0,0068 | 0,4341 | 0,0074 | 0,2217 | 12,9223 | 13,0568 | 0,1345 |
| 30,0695 | 2 | 0,08 | 0,9790 | 0,0032 | 0,0032 | 0,4331 | -0,0010 | -0,0300 | 12,8970 | 13,0233 | 0,1263 |
| 30,1710 | 3 | 0,076 | 0,9993 | 0,0371 | 0,0369 | 0,4213 | -0,0118 | -0,3560 | 12,5456 | 12,7115 | 0,1659 |
| 30,1593 | 4 | 0,072 | 0,9986 | 0,0191 | 0,0190 | 0,4321 | 0,0108 | 0,3252 | 12,8754 | 13,0316 | 0,1562 |
| 30,1580 | 5 | 0,068 | 0,9982 | 0,0119 | 0,0118 | 0,4364 | 0,0043 | 0,1288 | 13,0088 | 13,1597 | 0,1509 |
| 30,0499 | 6 | 0,064 | 0,9252 | 0,0001 | 0,0001 | 0,4112 | -0,0252 | -0,7573 | 12,2561 | 12,3553 | 0,0992 |
| 29,9599 | 7 | 0,06 | 0,5445 | 0,0000 | 0,0000 | 0,2420 | -0,1691 | -5,0675 | 7,1931 | 7,2509 | 0,0578 |
| 30,1234 | 8 | 0,056 | 0,9885 | 0,0004 | 0,0004 | 0,4391 | 0,1970 | 5,9351 | 13,1308 | 13,2256 | 0,0948 |
| 29,9255 | 9 | 0,052 | 0,2393 | 0,0000 | 0,0000 | 0,1064 | -0,3327 | -9,9557 | 3,1799 | 3,1830 | 0,0031 |
| 29,9968 | 10 | 0,048 | 0,5763 | 0,0000 | 0,0000 | 0,2562 | 0,1498 | 4,4932 | 7,6742 | 7,6838 | 0,0096 |
| 30,0164 | 11 | 0,044 | 0,6331 | 0,0000 | 0,0000 | 0,2814 | 0,0252 | 0,7566 | 8,4336 | 8,4454 | 0,0118 |
| 30,0279 | 12 | 0,04 | 0,6442 | 0,0000 | 0,0000 | 0,2863 | 0,0050 | 0,1488 | 8,5854 | 8,5974 | 0,0120 |
| 30,1518 | 13 | 0,036 | 0,9912 | 0,0000 | 0,0000 | 0,4406 | 0,1542 | 4,6504 | 13,2389 | 13,2832 | 0,0443 |
| 30,0786 | 14 | 0,032 | 0,8297 | 0,0000 | 0,0000 | 0,3688 | -0,0718 | -2,1594 | 11,0842 | 11,0916 | 0,0074 |
| 30,0455 | 15 | 0,028 | 0,5559 | 0,0000 | 0,0000 | 0,2471 | -0,1217 | -3,6557 | 7,4325 | 7,4237 | -0,0088 |
| 30,1649 | 16 | 0,024 | 0,9938 | 0,0000 | 0,0000 | 0,4417 | 0,1946 | 5,8698 | 13,3050 | 13,3231 | 0,0181 |
| 30,2296 | 17 | 0,02 | 1,0000 | 0,0000 | 0,0000 | 0,4444 | 0,0028 | 0,0832 | 13,3931 | 13,4348 | 0,0417 |
| 30,1956 | 18 | 0,016 | 0,9995 | 0,0000 | 0,0000 | 0,4442 | -0,0002 | -0,0065 | 13,3914 | 13,4131 | 0,0217 |
| 30,2977 | 19 | 0,012 | 1,0000 | 0,0000 | 0,0000 | 0,4444 | 0,0002 | 0,0069 | 13,4032 | 13,4653 | 0,0622 |
| 30,2070 | 20 | 0,008 | 1,0000 | 0,0000 | 0,0000 | 0,4444 | 0,0000 | -0,0001 | 13,4078 | 13,4250 | 0,0171 |
| 30,3436 | 21 | 0,004 | 1,0000 | 0,0000 | 0,0000 | 0,4444 | 0,0000 | 0,0001 | 13,4128 | 13,4858 | 0,0730 |
| 30,3400 | 22 | 0,000 | 1,0000 | 0,0000 | 0,0000 | 0,4444 | 0,0000 | 0,0000 | 13,4176 | 13,4844 | 0,0668 |

### 4.1.4. Comparison of the results.

Thus, the above mentioned tables of procedures of various hedging techniques allow to conclude, that methods of imperfect hedging, such as quantile hedging and expected shotfall hedging can reduce the cost of the hedging portfolio in a Russian derivatives market, by using controllable risk.

Chart 4.1 illustrates the dynamics of the cost of hedging in the three methods of hedging - perfect delta hedging, quantile hedging and the expected shortfall hedging on the example of Sberbank shares. The x-axis on this graph shows the day of the hedge and the y-axis - the cost of hedging portfolio.

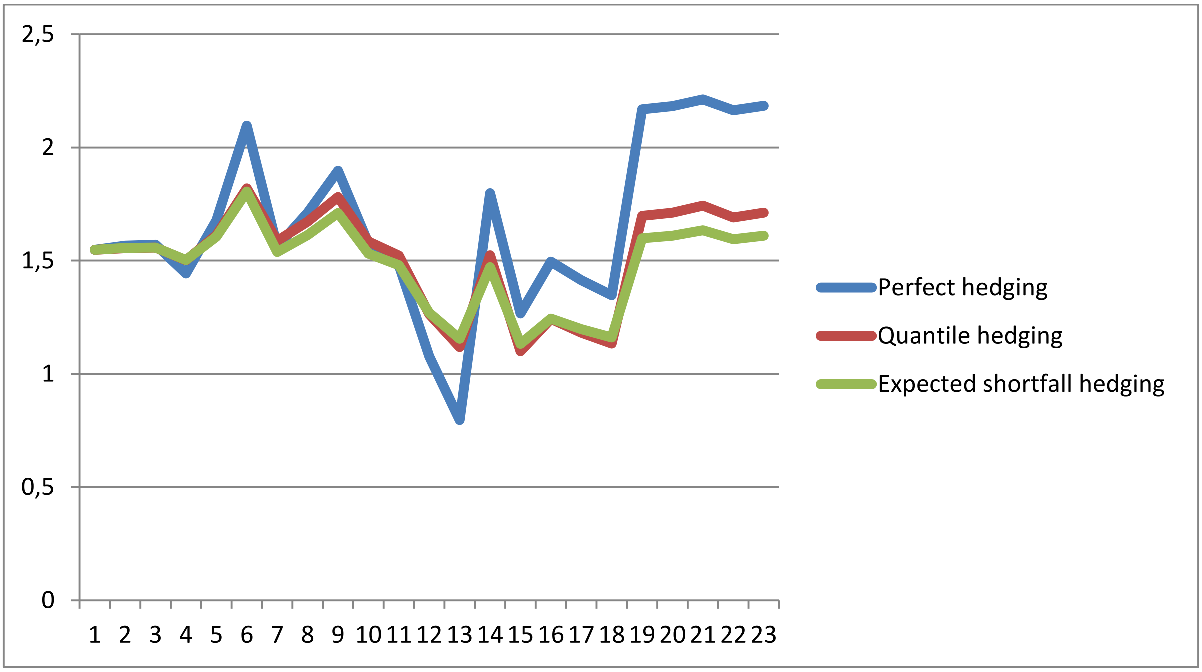


**Chart 4.1 Hedging of Sberbank shares with different methods of hedging.**

Thus, the cheapest method of hedging in this case is the method of the expected shortfall hedging, with a level of expected losses equal to 5%. Method of quantile hedging is little bit expensive, with a probability of success 95%. Perfect hedge in this case is expensive; it proves the theory that methods of imperfect hedging can reduce the cost of hedging.

Chart 4.2 illustrates the dynamics of the cost of hedging in the three methods of hedging - perfect delta hedging, quantile hedging and the expected shortfall hedging on the example of the currency pair USD RUB.

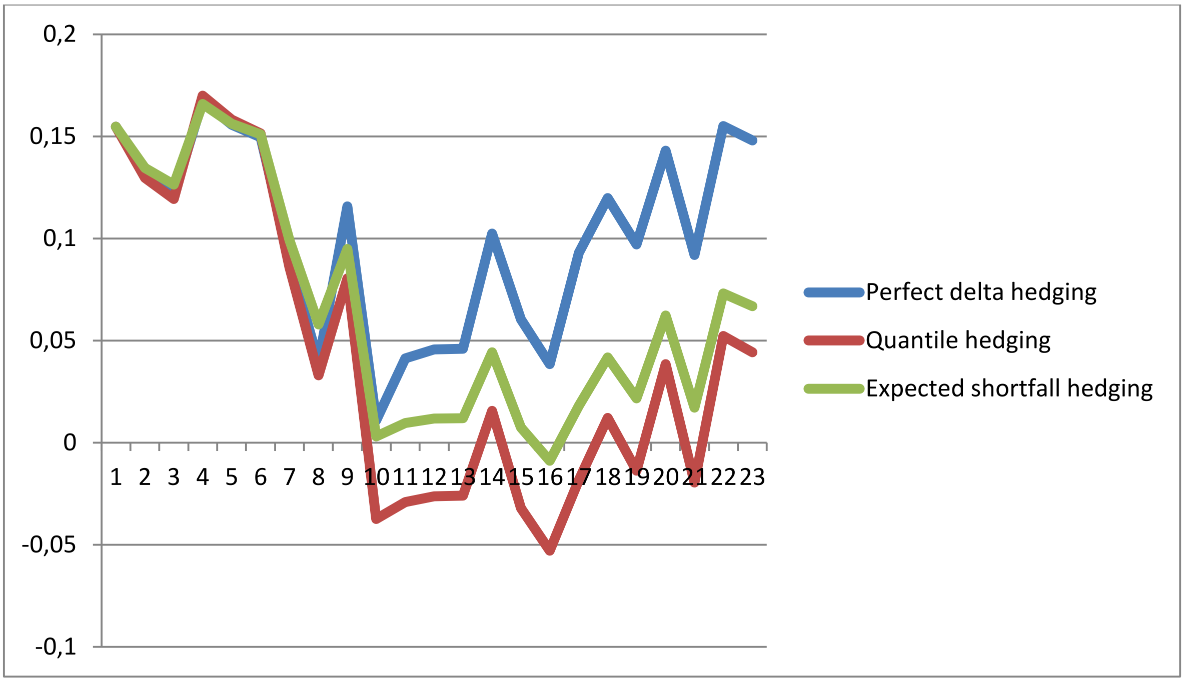


**Chart 4.2 Hedging of the currency pair USD RUB with different methods of hedging**

In the case of the currency pair USD RUB imperfect hedging techniques were cheaper as well. The cheapest method of hedging in this case was the method of quantile hedging. In theory any of the presented methods of imperfect hedging cost less than perfect hedge, but these methods are more risky.

## 4.2. The Summary of Chapter.

This empirical study gives understanding of the methods of an imperfect hedging of financial instruments, the method of quantile hedging and method of expected shortfall hedging can be integrated into the concept of dynamic hedging in discrete time. Even the choice of such a simple method as a delta hedging of the Black-Scholes model, demonstrates the possibility to reduce the cost of hedging portfolio within a controlled level of risk. With a sufficient degree of probability it can be assumed that with the use of improved models of dynamic hedging, the accuracy of the results can be improved.

# CONCLUSION

In the research the objective was to explore new approaches of pricing and hedging of financial options, as well as a comparison of these methods with conventional methods in the financial industry and the analysis of the use of such methods in the Russian derivatives market. The objective of the study was achieved.

Conclusions of the study in the final qualifying work are presented below:

1. Analysis of the theory of the methods of hedging of financial options allowed to create a working classification of the main directions of foreign researches in the field of imperfect hedging methods of financial options. In the study, it was revealed that the methods of imperfect hedging of financial options are a completely new approach in financial risk management positions of such derivative instruments as options. This approach takes into account the expectations of the holder's urgent position as well as its attitude to risk. Also it takes into account characteristics of the hedger's strategy in the construction of protection against risk.
2. a) In the research it has been shown that the methods of imperfect hedging of financial options reduce the cost of hedging portfolio due to potential losses.

   b) The set of successfull hedging depends on the type of urgent position, the selected measure of risk, as well as parameters such as volatility, the average growth rate and bank interest rate.

   At the same hedging strategy may be optimal either for one of the criteria (quantile or expexted short-fall), or for both criteria at once under certain conditions.

   c) Method of quantile hedging method has some negative aspects. The biggest critique is the fact that when it is used it is only taken into account the risk of loss, rather than their size. Regarding to this fact, is better to use the method of the expected shot-fall for the practical application of imperfect hedging of financial risk protection, because this method takes into account both the likelihood and magnitude of potential losses.

   d) It was revealed that the cost of hedging of the positions as a rule is not equal to the sum of values of its components, as in the numerical examples, it was found that the methods of imperfect hedging of financial options, such as quantile hedging and expected short-fall hedging, there is no additive property.

3. During the study were made calculation in the spreadsheet application Microsoft Excel, due to them can be obtained an algorithm to hedge various methods of perfect and imperfect hedging and draw conclusions about the necessity of use of a method of hedging financial options.

4. During research was compiled algorithm of hedging strategy implementation by techniques of imperfect hedging of financial options:

Step 1: Finding the structure of the set of successful hedging based on the type of the option and the parameters such as volatility, the average growth rate and bank interest rate.

Step 2: Determination of hedging strategy payments on the set of hedging strategies.

Step 3: Select the method of hedging strategy payments replication (for example, in the third chapter of research was chosen delta method of hedging).

5. In the construction of protection against the risk using the methods of quantile hedging and hedging of expected losses hedger may adjust the hedging portfolio and the hedging strategy depending on their expectations.

6. The current development of the derivatives market in Russia allows use effective new methods of imperfect hedge that has been proven in the third chapter of research by the example of the shares of Sberbank and the currency pair USD RUR. Conducted an empirical study that has proven the possibility of reducing the cost of protection against the risk by adopting a probability of loss.

Further research in the field of imperfect methods of hedging financial options should be directed to the possibility of combining different methods, and also it is important expose them in a more advanced analysis models.

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