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Advantages and Risks Associated with Portfolio Maximization

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Abstract

The main purpose of the underlying thesis was to put in practice Markowitz Modern Theory in the Nasdaq 100 Index using the original Capital Asset Pricing Model (CAPM) as well as finding the advantages and risks associated with Portfolio Optimization. We employed monthly data from January 2010 to January 2020, 12 out of 103 stocks comprised in the NASDAQ 100 Index, and the 3-month US T-Bill rate. We firstly obtained the Expected Rate of Return for each stock by estimating a simple CAPM. Apple Inc (AAPL), Texas Instruments Inc (TXN), Microchip Technology Inc (MCHP), VeriSign Inc (VRSN), Western Digital Corp (WDC), Mondelez International Inc (MDLZ), Tesla Inc (TSLA), Starbucks Corp (SBUX), Amgen Inc (AMGN), United Airlines Holdings Inc (UAL), Fiserv Inc (FISV), and Amazon.com, Inc. (AMZN). Then, the values found were weighted to find the Minimum Variance Portfolio. We found that WDC had the highest expected return (24.76%), while TSLA had the highest standard deviation (15.26%). AMZN and AAPL on the other hand had the second and third highest expected return respectively (21.89% and 20.08%) with a relatively low enough risk (7.89% and 7.76%) respectively. Nevertheless, our analysis reveals that stock series with the high expected return (AMZN, APPL WDC, TXN) and stock series with relatively high standard deviation (TSLA) were given low to zero weights within the Minimum Variance Portfolio, while stock series with relatively medium to high returns and relatively low standard deviation were given higher proportion. FISV SBUX, and AMGN had expected returns of 11.88%, 9.27%, 11.17% respectively as well as standard deviation of 4.46%, 5.63%, 6.13% respectively. Specifically, our Minimum Variance Portfolio results suggest that we shall invest the 34.9% of our wealth to FISV, the 17.55% to SBUX, and the 18.85% to AMGN and 0.00% to AMZN, TXN and WDC. Finally, this thesis provides recommendations to investors.

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Chapter 1: Introduction

1.1 Background of the Study

Portfolio optimization is the procedure of selecting the investment portfolio, out of a set of portfolios, which comprises the optimal combination for achieving a specific objective. Maximize the expected return or minimize the financial risks of the investor or it can be a mixture of both (Civitanic, Polimenis, & Zapatero, 2008). To attain high expected returns, there is a need for more risk-taking in a way that the investors have to pursue the balance between the risk and the expected return (Chua, Krizman, & Page, 2009). Rehnman (2018) considered portfolio optimization as the procedure of choosing asset weights for achieving an optimal portfolio, depending upon the objective. Usually, the objective is maximizing expected return or minimizing financial risk, or both.

CAPM in theory indicates how investors could optimize a portfolio's return in relation to risk. This relation which eventually leads to the optimal risky portfolio, exists on the efficient frontier as presented in Table 5. The relation is also graphically illustrated in Figure 4 where the demand for higher expected returns (vertical axis) requests for higher expected risks (horizontal axis). As Modern Portfolio Theory (MPT) proposed, the expected return of a portfolio increases as the risk increases. Additionally, it is observed that any portfolio which lays on the Capital Market Line (CML) is preferable than any portfolio which falls to the right of the line, but on a particular point, a theoretical portfolio can be set up exactly on the CML producing the best return at a specified degree of risk. The CML and efficient frontier illustrate a major perception for investors which identifies the offset between increased return and increased risk. But since in real life it is not possible to create a portfolio that perfectly fits on the CML, a typical practice for investors is to take additional risks as they pursue additional return.

Similarly, as the CAPM assumes that a portfolio exists on the efficient frontier providing the maximum return for its level of risk or the minimum risk for its level of return this can only be calculated in theory and it is impossible to realize if it is applied in real life scenarios or not because future returns cannot be predicted. However, the illustration is producing a future forecasting based on the historical data that gives significant guidance on where an investor could route when selecting the stocks that would participate in the investment portfolio.

In the finance literature review, the results generated by the empirical test of the CAPM are classified as a single-factor CAPM and a multifactor CAPM. Lintner, 1965; Douglas, 1969; Miller and Scholes, 1972, who conducted the first studies of the CAPM model, reported the same pitfalls as they did on the basis of individual stock returns and stressed the risk-return correlation and their observational findings were not promising. Evidently, there was discovered that the intercept of risk and return generates larger values inconsistent with the risk-free rate, where as the beta coefficient shows a marginally smaller value.

However, this problem was resolved when other studies were using portfolio returns instead of individual stocks returns. Black et al. (1972) in their study, utilized the stocks from the New York Stock to create investment portfolios over the period 1931–1965. From their results they concluded that the expected excess return and its β are not necessarily proportional: "and we believe that this evidence…is sufficiently strong to warrant rejection of the traditional form of the model" [Black et al., (1972), p.2] given by Sharpe (1964).

The results of the existing literature during the earlier CAPM tests suggest that higher stock returns and higher betas are correlated and were taken as proof on the side of the CAPM, while results that challenged the appropriateness of the CAPM model did not demoralize eagerness for more extensive testing. Miller and Scholes (1972), Black et al. (1972), and Fama and MacBeth (1973) also reveal an understandable connection between beta and asset return. However, actual results show that the returns with higher betas have systematically lower value from those expected by the CAPM, whilst the smaller betas returns are systematically higher. Black (1972) responded with a two-factor model suggestion. Sharpe (1964) and Lintner (1965) add two key assumptions to the Markowitz model to identify a portfolio that must be efficient in terms of mean-variance. "First, we assume a common pure rate of interest, with all investors able to borrow or lend funds on equal terms. Second, we assume homogeneity of investor expectations: investors are assumed to agree on the prospects of various investments – the expected values, standard deviations and correlation coefficients (previous) described" [Sharpe, (1964), pp.433–434]. Also, Fama and French (2004) have described portfolio opportunities using CAPM factors (Figure 4).

1.2 Objective, Research Questions, and Significance

In light of the foregoing discussion, the current thesis attempts to find the Minimum Variance Portfolio using 12 randomly selected stocks from the NASDAQ 100 Index for the last 10 years. These stock series include Apple Inc., Texas Instrument Inc., Microchip Technology Inc., Verisign Inc., Western Digital Corp., Mondelez International Inc., Tesla Inc., Starbucks Inc., AMGEN Inc., United Airlines Holdings Inc., Fiserv Inc., and Amazon Inc. Moreover, our thesis tries to examine the advantages and risks associated with portfolio optimization. Even though there is extensive research around the topic of Portfolio Optimization, there has been little interest in employing simple models like CAPM as well as stating in one place the advantages and disadvantages associated with it. The empirical part is decomposed into three parts: at the first stage, we employ CAPM to obtain the Expected Returns for each stock, whereas, at a second stage, the results found at the first one are weighted in order to find the Minimum Variance Portfolio. At a third stage, taking into account the theoretical and the empirical results of the thesis, we discuss and analyse the advantages and disadvantages associated with Portfolio Optimization.

Overall, the goal of this thesis is to answer the following research questions:

1. What are the methods that can be used by the investors for achieving portfolio optimization, that is, attaining high expected return, minimizing costs and maintaining a balance amid the risk and the return?
2. What are the Expected Returns and standard deviation of the 12 Stocks using CAPM?
3. What is the Minimum Variance Portfolio Expected Return and Risk?
4. What are the advantages and risks associated with Portfolio Optimization?

1.4 Structure of the Study

The remainder of the current research study is organized as follows. Chapter 2 provides the theoretical background whereas Chapter 3 reviews existing literature on different methodologies used for obtaining Portfolio Optimization. Chapter 4 discusses the research approach in detail, data gathering methods, originality and limitations of the data, reliability and validity of the data, ethical issues related to the research and philosophical approach. Moreover, in Chapter 5 we presented and discussed the empirical results, including descriptive statistics of our data, estimation results from CAPM. Finally, Chapter 6 concludes, provides recommendations and feeds for future research.

Chapter 2: Theoretical Background

Many theories are aiming to explain investor behavior and portfolio optimization. In this chapter are laid down and briefly explained six major theories.

Portfolio theory is defined as the utilization of the decision-making tools for solving the issue of the management of the risky investment portfolio (Nawrocki, 1999). The major portfolio optimization theories comprise of Modern Portfolio Theory (MPT), Post-Modern Portfolio Theory (PMPT), Mean Absolute Deviation (MAD) Optimization, Feinstein-Thapa Modification, Mansini-Speranza Optimization and Beta Model. These theories and models are briefly explained below.

2.1 Modern Portfolio Theory (MPT)

Modern Portfolio Theory (MPT) also known as the Mean-Variance Analysis is a theory that was presented by Harry Markowitz during the early 1950s. According to this theory, the investors try to make rational and logical decisions and hence, they expect higher returns even in case of increased risk. The theory emphasizes on the construction of a portfolio that supports the maximization of the expected return at a particular level of risk. This kind of portfolio has the ability to attain an efficient frontier curve. A rational investor will not be willing to undertake extra risk if the investment does not yield higher returns. On the contrary, if the objective is to achieve higher returns, then the investor must necessarily undertake higher risk. A major approach in this theory is considering the return and risk of the assets together and not separately, as both the factors together constitute the overall risk and return of the portfolios (Markowitz, 1952). MPT has been criticized by many theorists and practitioners because it requires expected returns input, which basically calls for a future output forecasting by the investor. Practically it is achieved by examining the historical data. The forecasts usually fail to consider the new conditions or circumstances which lead to flawed forecasts. Furthermore, the risk measure of Modern Portfolio Theory is variance. Thus, the optimization model also becomes quadratic, as the variance is quadratic. In the case of large portfolios, it requires the model incompetent in the computational way. Moreover, Modern Portfolio Theory considers that the asset returns adopt the Gaussian distribution, leading to two critical inferences. First, the likelihood of large and substantial changes in the values of assets is underestimated. Secondly, it relies on the correlation matrix that limits its ability among assets to capture the associated portfolio structure (Rachev & Mittnik, 2006). These drawbacks restrict the practical usage of Modern Portfolio Theory.

2.1.2 Efficient Frontier

The core of modern portfolio theory is the efficient frontier, and Harry Markowitz pioneered it in 1952. Both collections of optimal portfolios delivering the highest anticipated return for a given degree of risk or the lowest risk for a certain amount of expected return are on the productive boundary line. Portfolios that do not lie on the effective border line, on the other hand, are known as the sub-optimal. The successful boundary is typically represented as a hyperbola with the risk on the X-axis and the rate of return on the Y-axis as demonstrated in Figure 1 below.

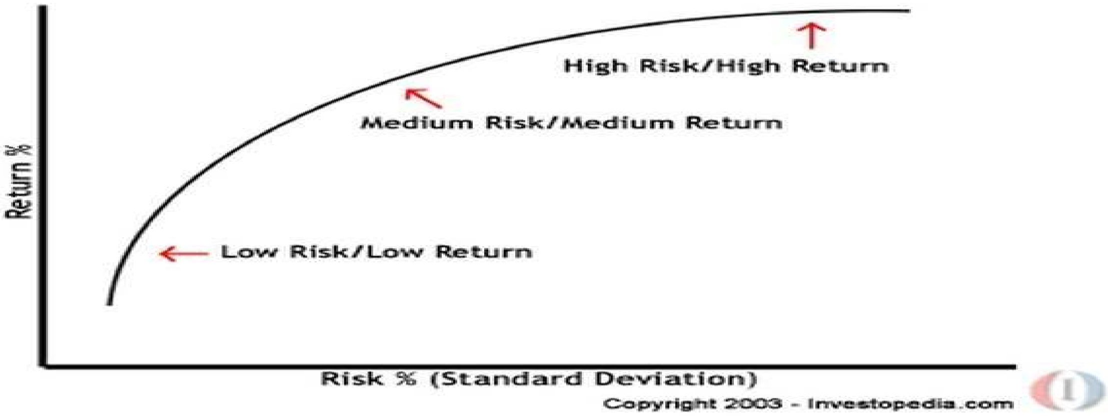

Figure 1: Efficient Frontier Explained under Modern Portfolio Theory

2.2 Post-Modern Portfolio Theory (PMPT)

In 1994, Rom and Ferguson (1994) suggested the Post-Modern Portfolio Theory (PMPT), which is based on Markowitz's Modern Portfolio Theory (1952), but which compensates for some of its disadvantages.

The PMPT uses the standard deviation of negative returns as a risk measure, while the MPT uses the standard deviation of returns. The approximation of the symmetric probability reflects the total standard deviation under the PMPT and the difference in the return on investment is considered to be the same as the high - risk and low deviations. Rom and Ferguson (1994) argued that for investors, as the positive deviations are helpful, it is contradictory instinctive. They observed that, from a realistic point of view, the danger was steeply distorted, with the greater worry for the slowdown.

In addition, Post Modern Portfolio Theory accepts that the investment risk and the investor 's particular goals should be completely interconnected and that the target return is referred to as the Minimum Appropriate Return (MAR). Goal return is indicated as the return needed to be reached in order to prevent failing to reach substantial financial targets. This calculation is specifically used in the computation of productive boundaries of the Post-Modern Portfolio Theory, which shows that there is a clear effective boundary for each of the Minimum Acceptable Returns. This would be in conversely to the Modern Portfolio Theory, in which the goals of the investors are not apparent.

In order to expose the inherent fragility of asset prediction, the optimization methodologies of both the Modern Portfolio Theory and the Post-Modern Portfolio Theory involve a statistical return distribution that is unique to each asset. The Modern Portfolio Theory allows only log-normal distributions and two-parameter normal distributions, while the Post Modern Portfolio Theory allows for a wider set of distributions, including asymmetrical distributions. Rom and Ferguson (1994) concluded that Post Modern Portfolio Theory optimization can typically yield more reliable results , allowing for a reasonable representation of the true form of the asset and empowering the optimization of heavily distorted investing strategies.

2.3 Mean Absolute Deviation (MAD) Optimization

Konno and Yamazaki (1991) suggested another alternative and effective model known as the Mean Absolute Deviation (MAD) to address the possible computational complexity and disadvantages of the Markowitz Quadratic Model (1952) and its derivatives The Mean Absolute Deviation Model leads to a linear programming model that has been confirmed to be equal to the Markowitz Model, but advantageously being more efficient in computational time. The MAD Optimization theory can be mathematically explained as follows:

$$
\begin{aligned}
&\min \sum_{t\in\mathcal{T}} p_t y_t,\\
&\text{s.t. } y_t + \sum_{j\in\mathcal{N}} w_j(r_{j,t} - r_j) \geq 0,\ t\in\mathcal{T},\\
&y_t - \sum_{j\in\mathcal{N}} w_j(r_{j,t} - r_j) \geq 0,\ t\in\mathcal{T},\\
&0 \leq w_j \leq u_j,\ j\in\mathcal{N},\\
&y_t \geq 0,\ t\in\mathcal{T},
\end{aligned}
$$

where

$\mathbf{p_t}$ is the probability of scenario t;

$\mathbf{y_t}$ is the new variable;

$\mathbf{r_{jt}}$ is the average expected return of the asset j in scenario t;

$\mathbf{w_j}$ is the proportion of the investors capital allocated to asset j = 1,…,N;

$\mathbf{r_j}$ is the average expected return of the asset j;

$\mathbf{u_k}$ is a limit set to ensure a greater portfolio diversification;

T is the number of time periods used to approximate the parameters of the assets return distributions.

2.4 Feinstein-Thapa Modification

Feinstein and Thapa (1993) in their paper *"Notes: A Reformulation of a Mean-Absolute Deviation Portfolio Optimization Model"* presented a modification on Konno and Yamazaki's model. Under **MAD optimization**, it is assumed that there is no limit up on the investment in an asset and that the number of non-zero assets in the optimal portfolio is at most 2T + 2. Under **Feinstein-Thapa Modification** there is a bound of T + 2 on the number of non-zero assets in the optimal portfolio. In other words, Feinstein-Thapa reduced the abovementioned model by defining two non-negative variables, that is, a_t and b_t and implementing them to the relevant constraints imposed. The final mathematical model is expressed below:

$$\min Z = 2\sum_{t\in T} b_t,$$
$$\text{s.t. } b_t + \sum_{j\in N}(r_{j,t} - r_j)w_j = a_t,$$
$$t \in T.$$

2.5 Mansini-Speranza Optimization

Mansini and Speranza (2005) a few years later proposed further additions of the Mean Absolute Deviation (MAD) Optimization). The goals of the modifications were to integrate the additional features of the specific portfolios into the model , i.e. the board exchanging of securities and the incorporation of the exchange costs. In particular, by using the mean risk model, the researchers considered the anticipated return and risk in the objective function of the portfolio.

2.6 Beta Model

Albuquerque (2009) proposed the Beta model which is regarded as an extension of the Mansini-Speranza Optimization model of 2005. The Beta model comprises of the factor of diversifiable and non-diversifiable risks in it. The diversifiable risk is taken under consideration when a minimum number of assets is implemented in the group of the optimal portfolio, while, the non-diversifiable risk is considered when the beta coefficient of the portfolios is employed.

Chapter 3: Literature Review

3.1 The Concept of Portfolio Optimization

There has been extensive literature in place trying to understand and explain investor behavior in selecting the "right" portfolio. Portfolio optimization plays an important role in making the right investment decisions, maximizing profit and minimizing risk. Dangi (2012) defines the portfolio optimization as the process that comprises the determination of an optimal organization of weights associated with the financial assets that are included in a portfolio. A portfolio is considered as an appropriate collection of investment that is held by the individuals/financial institutions. Rehnman (2018) regarded portfolio optimization as the process of choosing asset weights for attaining optimal portfolio, depending upon the purpose. Typically, the purpose is to maximize expected return or minimize financial risk or it can be an amalgam of both the purposes. The investment may involve government bonds, mutual funds, commodities, derivatives fixed income securities or any other type of financial instruments. The fund manager is responsible for making decisions relating to the investment kept in the fund by the financial institutions. Portfolio optimization is the procedure for choosing financial instruments that construct investment portfolios, thus assigning particular resources to the available funds, meeting specific pre-defined goals, suppressing financial risks and maintaining improved alert against unpredictable situations, and creating mathematical as well as numerical methodologies for portfolio optimization (in situations of short as well as long positions), and delivering steadiness in situations of fluctuations in the intra day and inter markets (Dangi, 2012).

According to DiBartolomeo (1993), the practitioners normally regard the optimization issues as a combination of two portfolios, firstly, the investor portfolio and secondly, the benchmark portfolio, while, the better formation comprises of three portfolios: (a) the investor portfolio, (b) the benchmark portfolio and (c) the difference portfolio. It thus consists of a collection of securities which, when complemented by an investment portfolio, provides a benchmark portfolio. DiBartolomeo (1993) also emphasized that the difference portfolio needs to be optimized. The optimal investor portfolio is just the portfolio that is a result of the subtraction of the optimal difference portfolio from that of the benchmark one. There is also a need to concentrate on the various portfolios in order to produce improved outcomes.

The portfolio may include various classes of assets, earnings and liabilities. The Modern Portfolio Theory proposed and developed by Harry Markowitz (1952) introduces the Mean-Variance Efficiency (MVE) which was later refined by Sharpe (1963). The purpose of using MVE is to find the most favourable combination of securities generating minimum risk or maximum return (i.e., efficient portfolio), and the factor analysis to model how the security prices fluctuations behave and how the correlation among security returns is working. A factor model, as Sharpe (1984) described, is the process of producing the stock expected return which encompasses the security prices behavior. The aim is to set out, among security returns, the major sources of correlation. It emphasized on maximizing the portfolio expected returns acknowledging any associated risk. This suggests that, in order to obtain a high return on the portfolio, investors are forced to take more risk. They need to trade in the anticipated return and risk. Portfolio maximization can occur either through the optimization of the weights of the assets for the purpose of holding (for instance, choosing the proportion held in equity versus the bonds) or for the objective of optimizing the weights of assets within a similar group of assets (for instance, choosing the proportion of the sub-portfolio stock that has been placed in the stocks) (Humphrey, Benson, Low, & Lee, 2015). These methods assist in the diversification of the portfolios and elimination of the non-systematic risk.

Further to the above, investing in non-standard investments involves a high project risk for firms and requires financing. Firms fear the idea of not securing enough funds, thus, they are reluctant in taking up such projects. (Finkelstein & McGarry, 2006). On the cumulative level, it will have a negative effect on innovation, economic progress and employment. Under informational asymmetry, ex-post it has been estimated, that just the investing firm and not the credit extending bank can perceive the real amount of profit at no cost. It happens in case of a moral hazard with the hidden knowledge.

Neff (1998) mentioned in his article entitled “Asymmetric Information, Credit Rationing and Investment” that asymmetric information in the credit market as well in the other financial markets leads to an inconsistency of the internal as well as external financing of the investments. The return of the investment project and the market value of the assets are both high in good situations and low in bad circumstances. If the firm chooses to (externally) finance the investment project by the issuance of new shares, the price of the new shares will be equal to the addition of the investment return and the expected values of assets.

Jothimani, Shankar and Yadav (2015) in their article “A Big Data Analytical Framework for Portfolio Optimization” presented a framework for incorporating structured as well as unstructured data for portfolio optimization purposes. The portfolio optimization process comprises of asset selection, asset weighting and asset management. These researchers proposed the framework for obtaining the first two processes by utilizing a five stages methodology, that is, (a) shortlisting of the stocks by making use of \Data Envelopment Analysis (DEA), (b) integration of the qualitative factors by using the text mining process, (c) stock clustering (the greater the clusters, the higher will be the diversification), (d) stock ranking (usually done through Artificial Neural Network (ANN)) and (e) optimizing the portfolio by employing the optimization heuristics. The framework presented by these researchers can assist the investors in selecting the suitable assets for making a portfolio, investing in them for minimizing the risk and maximizing the returns and finally monitoring their performance.

Moreover, Chua, Krizman, & Page (2009) mentioned in their article entitled "The Myth of Diversification" that high expected returns can be obtained but is demanded by the investor to take more risk in order to tackle the trade-off amid the risk and the expected return. The risk and expected return relationship is represented by the curve which is known as the efficient frontier and portray the points of a well diversified and efficient portfolios. The mathematical tools and techniques used for the portfolio maximization or optimization include (a) quadratic programming, (b) mixed inter programming, (c) stochastic programming, (d) deterministic global optimization, (e) copula-based methods, (f) meta-heuristic methods, (g) nonlinear programming, etc. (Low, Faff and Aas, 2016).

Further to the above, Kuutan (2007) in his thesis entitled "Portfolio Optimization Using Conditional Value at Risk and Conditional Drawdown at Risk" made a detailed investigation on how risk measures influence the position sizing of a portfolio. The risk measures employed for generating the optimal allocation of the portfolio assets were Conditional Value-At-Risk (CVaR) and Conditional Drawdown-At-Risk (CDaR).

Low, Faff and Aas (2016) mentioned in their article entitled "Enhancing Mean-Variance Portfolio Selection by Modeling Distributional Asymmetries" that the mathematical tools and techniques used for the portfolio maximization or optimization include quadratic programming, mixed inter programming, stochastic programming, deterministic global optimization, copula-based methods, meta-heuristic methods, nonlinear programming, etc.

3.2 Methods for Achieving Portfolio Optimization

Portfolio performance can be calculated in different ways. Different investors think differently about measuring the portfolio performance. Some of them consider the rate of return to be the most significant measure of portfolio performance, by considering the riskiness of assets or portfolio volatility (Kuutan, 2007). Other investors regard risk-adjusted return as the most suitable measure of portfolio performance. There are three strategies for achieving portfolio optimization, including achieving high estimated returns, minimizing costs and ensuring a balance of risk and return which are explained in later Chapter.

3.2.1 Attaining High Expected Return

Mayambala (2015) mentioned in this thesis entitled "Mean-Variance Portfolio Optimization: Eigendecomposition Based Models" that Modern portfolio theory is about deciding how to distribute wealth among available securities in such a way that, for a given level of risk, the expected return is maximized, or for a given level of return, the associated risk is minimized. In Markowitz's seminal work in 1952, variance was used as a metric of risk that contributed to the well-known mean-variance portfolio optimization model. While other medium-risk models have been introduced in the literature, the mean-variance model continues to be the backbone of modern portfolio theory and is still widely used.

Investors seeking high risk returns can face substantial financial damages if they are unable to retain their investment effectively. Silva, Alem and Carvalho (2017) in their article entitled "Portfolio Optimization Using Mean Absolute Deviation (MAD) and Conditional Value at Risk (CVaR)" evaluate the effects of traditional portfolio optimization models when financial asset returns are extremely unpredictable, such as during the financial crisis era. The researchers have developed alternative optimization models that constitute a mixture of Conditional Value at Risk (CVaR) and Mean Absolute Variance (MAD) in order to minimize low-return inefficient or high-risk portfolios. They also calculated the possibility of historical asset returns using the three methodologies. They evaluated the success of their suggested strategies by using the Brazilian stock exchange in the years 2004 and 2013. The results showed that traditional models provided portfolios with high yields, but their proposed model produced low-risk portfolios that could be considered attractive in volatile markets. They also found that models that do not use equiprobable scenarios are producing improved cost and return outcomes.

In its report entitled "Portfolio Optimization and Monte Carlo Simulation", Pedersen (2014) used the Monte Carlo simulation of a simple equity growth model. It included a re-sampling of past financial results to determine the probability distributions of earnings, potential equity and payouts of some firms, including Wal-Mart, Coca-Cola, S&P 500 Stock Price Index and McDonald. The simulated equity was then used with the historical distribution of P or Book to predict the probability distributions of potential asset values. To produce optimal portfolios using the Geometric Mean (i.e. Kelly) and Mean-Variance (i.e. Markowitz) methodologies, return distributions were also used. The variance as a measurement of investment risk was seemed to be an inaccurate, since the mean variance of optimal portfolios was not shown to help minimize risk. This downside is considered valid for distributions of returns. Kelly portfolios are correctly optimized for investment risk and long-term returns, but portfolios are usually concentrated in just a few assets and are therefore responsive to the estimation errors that occur in the return distributions.

3.2.2 Minimizing Cost and Maximizing Wealth

Levy and Ritov (2001) mentioned in their article “Portfolio Optimization with many Assets: The Importance of Short Selling” investigated the properties of the mean-variance efficient portfolios in cases where the assets are large. Empirical evidence revealed that the cost of no short selling constraints enhances with the increase in the assets. When the number of assets reaches 100, the Sharp ratio doubles and the constraint gets removed. The results can be applicable to the policy related to the limitations of short selling.

Clark and Mulready (2007) in their research thesis entitled “Portfolio Optimization with Transaction Cost” suggested a way for limiting the transaction costs by controlling the portfolio turnover amid the specific time periods. Portfolio change is an absolute change/alteration which is considered as a fraction of the book size. The outcome is a multiperiod optimization issue comprising of the quadratic objective function as well as non-smooth restraints. The resulting portfolios performed better than the benchmark portfolios in both the expected value and the real portfolio value.

Lai, Yang and Wu in their article entitled “Short-term Sparse Portfolio Optimization Based on Alternating Direction Method of Multipliers” proposed use of sparse portfolios to accomplish short-term portfolio optimisation. It maximizes the aggregate wealth of the overall investment by first focusing wealth on the proportion of limited assets The short-term portfolio optimization requires flexibility for responding to the changing financial environments. The researchers suggested the use of the portfolio optimization by means of the machine learning systems that are based on the optimization strategies and financial principles. Machine learning approaches are reliable and remove human mistakes and biases.

3.2.3 Maintaining a Balance between Risk and Return

There is a need to maintain a balance between the risk and the return in portfolio management. The financial risks involved in a portfolio include high volatile movements in the asset's prices, economic imbalance, financial crisis and algorithmic trading (Balbs, 2007). Risk-based allocation strategies are used for mitigating these financial risks.

Chua, Krizman, & Page (2009) mentioned in their article entitled “The Myth of Diversification” that to achieve high expected returns it is necessary to take more risk in a way that the investors have to tackle the risk and the expected return relationship. The relationship is represented by the curve known as the efficient frontier. The efficient portfolios are well diversified and are depicted by a point on the efficient frontier.

The portfolio efficient frontier is best illustrated by the CAPM Model developed by Harry Markovitz in 1952. According to the equilibrium theory between risk and return of any given investment, Markowitz structured a theory for asset assumptions. In any investment project, there are many uncertainties involved. Hence, the return should be specified by calculating the net present value average or the internal rate of return which is measured by means of a number of iterations large enough to approach the average result to the expected value of the investment or project Risk is considered to be the dispersion of the results of the measure of return.

Moreover, Ban, Karoui and Lim (2016) in their research article entitled “Machine Learning and Portfolio Optimization” made use of the two machine learning methods, that is, cross-validation and regularization for attaining portfolio optimization. Performance-Based Regularization (PBR) was used by them for constraining the sample differences of the expected risk and return of the portfolio. This paves the resolution to the one that is associated with less estimation error in the portfolio performance. Performance-Based Regularization was utilized for mean-variance as well for mean Conditional Value-at-Risk (CVaR) issues. In the case of the mean-variance issue, a "quartic polynomial constraint" was introduced by the Performance-Based Regularization for which two convex estimations were made, firstly, based on rank-1 estimation and secondly based upon convex quadratic estimation. The rank-1 approximation Performance-Based Regularization added biases to the optimal allocation, whereas, the convex quadratic approximation Performance-Based Regularization shrank the sample covariance matrix. In the case of the mean Conditional Value-at-Risk problem, the Performance-Based Regularization model is a combinatorial optimization issue, but the Quadratically Constrained Quadratic Program (QCQP) which is the convex relation is fundamentally strong. The researchers revealed that Performance-Based Regularization models can be cast as robust optimization issues with new uncertainty sets and can set up asymptotic optimality of not only Sample Average Approximation (SAA) as well as Performance-Based Regularization resolutions but also of the related efficient frontiers. For calibrating the right-hand sides of the Performance-Based Regularization restraints, the researchers developed novel k-fold cross-validation algorithms. A detailed empirical examination of Performance-Based Regularization can be carried out by utilizing these algorithms in contradiction to Sample Average Approximation, L1 regularization, L2 regularization and the equally weighted portfolio. It was revealed that Performance-Based Regularization had domination over all other benchmarks for 2 out of 3 of the data sets of Fama-French.

Rahnama (2016) mentioned in his thesis entitled “A Portfolio Optimization Model” that a diversified portfolio has less risky behavior and also yields less difference in the expected return. A well diversified portfolio tends to minimize the uncertainty in portfolio returns when the price of the whole asset does not move in the same direction and at a similar rate.

Blomgren (2016) in his thesis "A Mean-Variance Portfolio Optimizing Trading Algorithm Using Regime Switching Economic Parameters" presented a model of algorithmic trading. The purpose of that model was the creation of an optimal investment portfolio comprising of both a risk-free asset and a risky asset. The risky asset was in the shape of a stock generated by utilizing the regime-switching constraints with a Markov chain depicting the situation of the economy. The optimization of the portfolio was done under specific assumptions and rational restraints on the transaction costs, risk and the amount that was traded. The restraint on the financial risk was applied via the acknowledged mean-variance condition by balancing the expected value of the portfolio against the difference of the portfolio after a specific time interval. The algorithm was implemented by utilizing the quadratic programming methods in Matlab. By using different parameters of the model, a sensitivity analysis was carried out. Simulated situations and the behavior of the algorithm were depicted in graphs. The algorithm was found to be logical and performed better than a static portfolio in each circumstance.

3.3 Risks Associated with Portfolio Optimization

The portfolio optimization involves challenges such as mathematical calculations, business restraints, limitations and complicated financial tools. Dangi (2012) found that the empirical nature of data is one-sided which reflects both the upside and downside developments with recurring and not easily identifiable cyclic behaviors which are potentially the result of high-frequency volatile exchanges in the trading of the assets. The process of portfolio optimization in such situations is theoretically as well as computationally difficult.

According to Moyer, McGuigan and Kretlow (2006), a portfolio is exposed to two types of risks, namely systematic risk and unsystematic risk. The systematic risk which is also known as the market risk arises from the overall condition of the economy and the industry. This risk is inbuilt in the nature of the business. Systematic risk is beyond the control of the management and it cannot be eliminated. The unsystematic risk is an outcome of miss-management, inadequacy in the planning process or in decision making, low forecasting precision or any kind of risk which can be eliminated through logical or appropriate decision making. As a result of which, unsystematic risk is under the control of the management, though it is not possible to totally eliminate this risk but it can be significantly lessened.

Bonami and Lejeune (2009) in their article entitled *"Exact Solution Approach for Portfolio Optimization Problems under Stochastic and Integer Constraints"* have mentioned the use of mean-variance method for studying the ways through which risk evasion investors can create optimal portfolios by considering the trade-off amid the market volatility and the expected returns. They proposed the usage of a portfolio optimization model which takes the shape of a probabilistically restrained optimization model along with the random technology matrix.

AlMahdi (2015) has discussed in his article "*Smart Beta Portfolio Optimization"* that conventionally the portfolio managers have been discouraged from timing the market, for instance, the equity managers have been forced to comply sternly to a benchmark with static or comparatively steady components, for example, Russell 3000, S&P 500, etc. It means that the exposure of the portfolio to all the risk factors should be expected to be close to the related exposures of the benchmark portfolios. The major risk factor involved in the market itself. A long portfolio would be restrained to possess a beta of 1. Managers are currently provided greater discretion to regulate their portfolios risk exposures dynamically for matching the beliefs of the managers about future performance of the risk factors themselves, specifically the Beta in their portfolios. These strategies are called the smart beta strategies. Beta should be adjusted dynamically by the portfolio managers, that is, to enhance the exposure when one predicts that the market trend will rise, and to lower it when one expects that the market trend will fall.

In their article "Multiperiod Portfolio Optimization with Multiple Risky Assets and General Transaction Costs," Mei, DeMiguel and Nogales (2016) evaluated the optimum portfolio strategy for a multi-period mean-variance investor facing multiple risky assets, including typical transaction costs. In the case of proportional transaction costs, the researchers presented a closed-form expression for a non-trade area shaped as a multidimensional parallelogram and showed how an ideal portfolio strategy can be effectively measured for a variety of risky assets by a single quadratic program. The optimal portfolio policy can be easily calculated by addressing a an one of polynomial program for issues affecting several volatile securities. In the case of market impact costs, it has been shown by the researchers that for every time span it would be beneficial to trade to the margin of an eventual reliant re-balancing area. Moreover, the re-balancing region congregates to the Markowitz portfolio as soon as the investment begins to expand. Researchers have found, by empiric data, that losses linked with disregarding costs of transacting and acting short-sightly can be high.

Chapter 4: Research Methodology

The current chapter comprises of the research approach used, data gathering methods, originality and data restrictions, reliability and validity of the data, ethical issues related to the research and philosophical approach.

4.1 Data

4.1.1 Data Gathering Methods & Data Description

There are two types of data gathering methods that are used in the research, i.e.; primary source and secondary source. The present study comprises the use of both primary as well as secondary sources for data collection. Both sources used in this research are briefly discussed below:

4.1.1.1 Primary Source

For the empirical analysis, we employed data from the NASDAQ 100 Benchmark for a 10-year period. The underlying index comprises of 103 stocks out of the 100 largest non-financial companies listed on the Nasdaq stock market. The assets (103 stocks) included in the actively managed portfolio are companies from sectors, such as Consumer Goods, Consumer Services, Telecommunications, Health Care, Industrial, Technology and Utilities Sectors. Out of these 103 assets, 12 Companies have been selected in a **semi-randomly way** and cover all the sectors mentioned. Specifically, for the empirical analysis of this thesis were used 1560 monthly data of 12 randomly selected companies for the period February 2010 to

January 2020. The companies selected are **Apple Inc** (AAPL), **Texas Instruments Inc** (TXN), **Microchip Technology Inc** (MCHP), **VeriSign Inc** (VRSN), **Western Digital Corp** (WDC), **Mondelez International Inc** (MDLZ), **Tesla Inc** (TSLA), **Starbucks Corp** (SBUX), **Amgen Inc** (AMGN), **United Airlines Holdings Inc** (UAL), **Fiserv Inc** (FISV), and **Amazon.com, Inc.** (AMZN)). Moreover, we employed for the same period, monthly data for the relevant US 3-month interest rate as a proxy for the risk-free rate.

4.1.1.2 Secondary Source

In-depth objective review of the literature allows to collect authentic information applicable to the research. Open and previously collected data that is valuable to the researcher is known as secondary data (Saunders et al., 2009). There are unclear places in which the researcher is unable to gather ample solid evidence from the data set, and the primary data are not adequate to the researcher's precise understanding, and thus, in those situations, a secondary source is required to gain in-depth information from historical facts. According to Wrenn (2006), secondary data can allow the researcher to consider the methods used by other researchers and to find the right method that can help address study questions. The secondary source of this study is the analysis of papers from magazines , books, corporate studies, authoritative websites, etc.

4.1.2 Originality & Data Restrictions

On the grounds of originality, the application and development of a specific research technique includes several innovative areas that have not yet been studied. It involves bringing novel evidences to an old problem. It also comprises of the use of new methods that have not been implemented before. Originality also includes presenting context that is based on the researcher's own research and explained in his/her own words. In the present study , the purpose of which is to ensure the originality of the scope, attention is given to the of the data collection and their applicability to Portfolio optimization; methods used by the investors for achieving portfolio optimization, factors responsible for attaining the portfolio optimization and advantages and risks that are associated with the portfolio maximization. Data is limited to the topics listed above.

Limitations are occurrences or difficulties that emerge in studies and are beyond the influence of the researcher. They limit the degree to which analysis can continue and typically affect the result of the research study. The restriction of the existing study relies on the data frequency. Particularly, in the current study, we use monthly rather than quarterly data in order to maximize the number of observations and therefore the accuracy of the estimation. As we move from annual, to quarterly, monthly and higher frequencies of data, the accuracy of expected returns measures increases. However, even though daily data could be used over monthly (for higher accuracy), time limitations against large dataset handling (advanced methods and computational programming) render daily and higher frequency data analysis impossible.

4.1.3 Reliability and Validity of the Data

The reliability of the analysis includes the extent to which the findings of the research can be repeated or used in another sample by using common research methodologies and practices in a particular area. As Marshall and Rossman (1999) concluded, the reproduction of the findings of the qualitative research was very complicated, considering that the research data were obtained at the time of the study and that the changes may have emerged from a shift in the circumstances. The consistency of the analysis often relies on how the findings are evaluated and presented in such a manner that useful results are extracted and the original thesis is expressed in them. In this analysis, the findings of the study can be repeated and extended to other portfolio optimization studies.

According to Bell and Bryman (2007), a valid research is accomplished when the process built upon the integrity of the results obtained. It is the capability of finding the cause and effect. It is a verification of the appropriateness of the research design. In this study, the researcher has carefully analyzed the secondary sources for deriving useful findings.

4.2 Research Approach

There are two main types of research: (a) qualitative or deductive approaches and (b) quantitative or inductive approaches to research. The deductive approach is considered to be the most significant and widespread approach used by researchers to conduct their research (Bell and Bryman, 2007). Compared to the deductive approach, most scholars consider the inductive approach to be the tool used in constructing hypotheses by the findings of the study carried out (Anderson, 2004). Hackley (2003) emphasized that the inductive methodology lets researchers identify particular variables in a multi-case population and the continues to identify what needs to be examined in the study. Current research has used mixed methodology, i.e. Qualitative Research Methodology (Deductive Approach) and Quantitative Research Methodology (Inductive Approach). The two methodologies are explained below.

4.2.1 Qualitative Research Methodology

Qualitative science requires an in-depth understanding of human actions and the reasoning underlying these activities. It is analytical and uses content analysis methodologies at the level of communication content selected. It is a comprehensive and intensive approach that examines the specifics of critical and challenging topics. It provides a more detailed explanation of attitudes, actions, behaviors and motivation. The method of thinking used in qualitative analysis consists of perceptually adding together sections in order to construct wholes. The sense is taken from this method. Since perception differs with individuals, there is a possibility of different meanings (Burns & Grove, 1993). This method provided a comprehensive study of the current research topic and helped to demonstrate the factors, benefits and risks associated with portfolio optimization and the methods required to achieve portfolio optimization. The report also addressed the examples and case studies of UK companies that have accomplished portfolio management either by generating high projected returns, reducing costs, or ensuring a balance between risk and return. Taking into account the definitions and explanations mentioned above, it can be concluded that the use of the qualitative research methodology for this research is justified.

4.2.2 Quantitative Research Methodology

According to Sanchez (2006), quantitative analysis is useful for the verification of conclusions from qualitative research. It exams and validates the hypothesis using either mathematical or statistical methods.In the underlying study, the Quantitative research relies on the Modern Portfolio Theory, particularly the Capital Asset Pricing Model (CAPM) and the approach of the Efficient Frontier and tries to find the optimum portfolio using NASDAQ

100 index. Then, a comparison is made between the actively managed portfolio with the passive index benchmark portfolio.

Taking things from the beginning, as mentioned in Chapters 2 and 3, Harry Markowitz (1952) proposed The Modern Portfolio Theory and introduced the Mean-Variance Efficiency (MVE) which was simplified some years later by Sharpe (1963) . The purpose of using MVE is to find the most favourable combination of securities generating minimum risk or maximum return (i.e., efficient portfolio), and find the factor analysis model to interpret how the security prices fluctuate and how the correlation among security returns is working. A factor model is the process of generating the stock expected return which underlies the behavior of security prices, as Sharpe (1984) described. The aim is to set out, among security returns, the major sources of correlation. It emphasized on maximizing the portfolio expected returns acknowledging any associated risk. This suggests that, in order to obtain a high return on the portfolio, investors are forced to take more risk. They need to trade in the anticipated return and risk.Portfolio maximization can occur either through the optimization of the weights of the assets for the purpose of holding (for instance, choosing the proportion held in equity versus the bonds) or for the objective of optimizing the weights of assets within a similar group of assets (for instance, choosing the proportion of the sub-portfolio stock that has been placed in the stocks) (Humphrey, Benson, Low, & Lee, 2015). These methods assist in the diversification of the portfolios and elimination of the non-systematic risk.

4.2.2.1 Capital Asset Pricing Model (CAPM)

The single-factor model did not meet with the MVE implications and assumptions as it could not describe or extract the equilibrium rates of return on shares exchanged on stock markets.

Thus, Sharpe (1964), Lintner (1965), and Mossin (1966) independently developed the Capital Asset Pricing Model (CAPM) to fill the gaps. CAPM is a market equilibrium model and has the following assumptions. To begin with, under the CAPM model, there is ***perfect competition*** in the markets which implies that investors act as though they have no market influence over prices. Secondly, it is assumed that ***markets are frictionless***, that there are no transaction costs, taxes, or restrictions on the assets traded and that the underlying assets are marketable and indefinitely divisible. Thirdly, all investors express ***uniformity in their beliefs*** and Bayesian preferences and access the same useful information that affects the market prices (information symmetry). Fourthly, the investors are engaged in their own risk-expected return appraisal method and they have their own ***individual preferences***. Fifth, all investors are utility maximizers expressing ***rational expectations***. Last but not least, investors can ***borrow and lend*** at the risk-free rate and ***no abnormal profits*** occur assuming that an efficient market exist.

The model is mathematically expressed as follows:

$$E[r_j] \ = \ r_f + \beta_j(E[r_m] - r_f) \qquad (1)$$

where

$E[r_j]$ = expected rate of return of each stock j;

$E[r_m]$ = expected rate of return on market portfolio;

r_f = rate of return on a risk-free security, i.e., interest rate; β_j = $\frac{Covariance(r_j, r_m)}{Variance(r_m)}$. The r_j is the rate of return for each stock j and is calculated using the logarithmic formula instead of the arithmetic one which is $\ln\frac{P_t}{P_{t-1}}$, where P_t are the prices of stock j at time t;

$(E[r_m] - r_f)$ = market risk premium.

According to the Mathematical Expression (1), the Expected Return of each stock j, hence of the actively managed portfolio is the risk-free rate plus a sensitivity parameter $\boldsymbol{\beta_j}$ multiplied by the market risk premium. As mentioned above, the $\boldsymbol{\beta_j}$ parameter is a sensitive measurement as to how the expected return movement of each security is aligned with the movement of the expected return on the market portfolio. As the market risk premium is positive due to the risk-return trade-off, if $\boldsymbol{\beta_j}$ is 1.0, the security return movement would exactly suits the market return movement. If it is greater (less) than 1.0, the return on the security is projected to change faster (slower) than the return on the market. It should be noted, however, that CAPM does not require the two returns to be linearly dependent (linear relation). The security j expected return extracted from CAPM is linked to the $\boldsymbol{\beta_j}$ along the so called the ***Security Market Line*** (SML) rather than to its variance or standard deviation along the ***Capital Market Line*** (CML) as specified by the MVE. Investors may be believed to have expectations more than the market risk premium as they extract their desired return on security.

4.2.2.2 Finding the Optimal Risk Portfolio

There are two steps in finding the Optimal Risk Portfolio. The first step is to find the expected returns of the portfolio. To do so, all the expected returns and the betas found in the expression (1) should be summed up. Then, the optimal weight should be given in each stock of the portfolio in order to create the optimal portfolio, which has the minimum variance but provides relatively high returns. For the purposes of the underlying study, the optimal weight is calculated using Solver in Excel.

The model is mathematically expressed as follows:

$$E(R_p) = \sum_{j=1}^{n} w_j \ E(R_j) \qquad (2)$$

where

$E[R_p]$ = expected return of portfolio p;

$\sum_{j=1}^{n} w_j$ = Sum of weights of each asset in the portfolio

w_j;$E[R_j]$ = expected return of each stock j;

4.2.2.3 Efficient Frontier

The Efficient Frontier is characterized as a sequence of optimized portfolios providing the highest expected return for a particular level of risk or the smallest risk for a given level of expected return.

Returns are dependent on the investment combinations that make up the portfolio. The lower covariance between portfolio securities and the market benchmark results in lower portfolio standard deviation (i.e. risk). Successful optimization of the return against risk paradigm should place a portfolio along the efficient frontier line. Optimal portfolios that comprise the efficient frontier tend to have a higher degree of diversification.

The following steps are necessary for the composition of the efficient frontier:

I. Determine the lower risk given a minimum investment and its respective return;

II. Measure the maximum risk associated with the portfolio;

III. Measure the maximum return for the degrees of risk between the minimum and maximum (calculated on the previous steps);

IV. Connecting the points through a smoothed curve, the efficient frontier is plotted.

4.2.2.4 Sharpe Ratio

There are numerous of risk-adjusted return ratios that support investors in evaluating existing or potential investments. These ratios can be more supportive than simple investment return metrics that do not take the level of investment risk under consideration.

The Sharpe ratio measures how well an investor is compensated for the risk they've taken in an investment. When comparing divergent investment portfolios in opposition to their related market benchmark, the portfolio with the higher Sharpe ratio provides a higher return for the same amount of risk or the same return for a lower risk than the other asset. It is considered as one of the most common ratios used to calculate the risk-adjusted return. Sharpe ratios greater than 1 are preferable; the higher the ratio, the better the risk to return scenario for investors.

Sharpe Ratio is mathematically expressed as follows:

$$Sharpe\ Ratio = \frac{R_p - R_f}{\sigma_p} \tag{3}$$

where

$\boldsymbol{R_p}$ = return of portfolio

$\boldsymbol{R_f}$ = risk-free rate

$\boldsymbol{\sigma_p}$ = standard deviation of the portfolio's excess return

It is noted that the Sharpe ratio adjusts a portfolio's past or future expected performance, for the excess risk that was taken by the investor. Moreover, a high Sharpe ratio is good when compared to similar portfolios or funds with lower returns.

4.2.2.5 Capital Market Line

The capital market line (CML) portrays portfolios that optimally align risk and return. The distinctive aspect of the CML and the productive frontier is that it includes risk-free investment. Conceptually, it defines the optimum mix of the risk-free rate of return with the volatile asset market portfolio and describes the most optimal portfolios. CML is a variation of the Capital Allocation Line (CAL) which makes up the allocation of risk-free assets and market portfolio for an investor (CAL uses the risk portfolio). Thus, the CML slope is found to be the Sharpe ratio. The resulted most efficient portfolio, also called the tangency point, is exhibited as the intercept point of CML and efficient frontier. Generally, it is implied that investors buy stocks if the Sharpe ratio is above the CML and sell if the Sharpe ratio is below the CML.

CML is mathematically expressed as follows:

$$\boldsymbol{R_p = R_f + \frac{R_T - R_f}{\sigma_T}\sigma_p} \qquad (4)$$

where

$\boldsymbol{R_p}$ = return of portfolio

$\boldsymbol{R_f}$ = risk-free rate

$\boldsymbol{R_T}$ = return of the market

σ_T = standard deviation of the market's excess return

σ_p = standard deviation of the portfolio's excess return

4.2.3 Ethical Issues Related to the Research

Blumber Cooper and Schindler (2005) defined ethical issues as moral codes, ethical beliefs, or standards of behavior that guide moral selection of human behavior and relationships with others. The researcher will not publish any information that might threaten the reputation of a ny individual or financial firm. In order to avoid plagiarism, the researcher correctly cited all sources.

4.2.4 Philosophical Approach

According to Stiles (2003), sociology science is made up of diverse philosophies. They include symbolic interactionism, positivism, enthomethodology, idealism, realism and phenomenology. The Interpretivism philosophy has been used in this research which focuses on explaining the specific contexts and the subjective value of the research study rather than searching for generalizations (Levers, 2013). The Interpretivism Methodology incorporates human interest in science. Thus, this kind of approach uses a Qualitative research method for attaining important facts relevant to the research. This approach is focused on the relationship between the researcher and the subject. Interpretivists argue that there is no single or specific method needed to attain knowledge. They claim that truth can be accomplished by social constructions such as mutual beliefs and opinions. The presumption in the Interpretivism approach is that theories are not either correct or incorrect, but that the cognitive processes of the research participants are given importance.

Chapter 5: Data Analysis and Findings

The empirical analysis in the underlying research relies on the NASDAQ 100 benchmark, in which the actively managed portfolio will be compared with. The NASDAQ 100 index comprises of the 103 stocks of 100 of the largest non-financial companies listed on the Nasdaq[1] stock exchange based on market capitalization. The assets selected in the actively managed portfolio are companies from sectors such as Consumer Goods (7%), Consumer Services (23%), Telecommunications (7%), Health Care (7%), Industrial (5%), Technology (55%) and Utilities (1%). These 103 assets are weighted in the most efficient manner, assisted by Modern Portfolio Theory for achieving a tangent portfolio on the efficient frontier.

Due to time restrictions, 12 out of the 103 the mentioned assets are selected in a semi-random way, taking into account the capital weighting of sectors included in the index. In the table below are presented the symbol of each stock as traded in the stock exchange, its sector as well as the proportion of each sector in the final portfolio.

Sector	Weight	Company Symbol
Technology	55%	AAPL - TXN - MCHP - VRSN – WDC - FISV
Consumer	25%	MDLZ - TSLA - SBUX - AMZN
Health Care	10%	AMGN
Industrial	10%	UAL

Table 1: Summary of the Stocks Series used in the thesis.

[1] https://indexes.nasdaqomx.com/Index/Breakdown/NDX

5.1. Descriptive Statistics

	Number of Observations	Mean	Variance	Standard Deviation	Median	Min	Max	Skewness	Kurtosis
AAPL	119	0.0210	0.0053	0.0727	0.0257	-0.2034	0.1792	-0.4262	0.2543
TXN	119	0.0155	0.0035	0.0593	0.0204	-0.1448	0.1646	-0.2151	-0.0568
MCHP	119	0.0133	0.0048	0.0695	0.0142	-0.2217	0.1857	-0.3510	0.6041
VRSN	119	0.0192	0.0045	0.0671	0.0228	-0.2727	0.1614	-0.8772	2.3869
WDC	119	0.0063	0.0125	0.1119	0.0119	-0.3173	0.2660	-0.2640	0.2820
MDLZ	119	0.0113	0.0051	0.0712	0.0152	-0.4358	0.3803	-0.9278	17.7666
TSLA	119	0.0241	0.0233	0.1526	0.0038	-0.4313	0.5937	0.5493	1.8966
SBUX	119	0.0182	0.0032	0.0563	0.0200	-0.1634	0.1441	-0.3383	0.5758
AMGN	119	0.0129	0.0038	0.0613	0.0157	-0.1670	0.1438	-0.2069	0.1355
UAL	119	0.0124	0.0096	0.0979	0.0213	-0.2675	0.2049	-0.3576	0.0830
FISV	119	0.0192	0.0020	0.0446	0.0186	-0.0950	0.1480	0.1217	0.4471
AMZN	119	0.0238	0.0062	0.0789	0.0258	-0.2259	0.2297	-0.0853	0.4534
NASDAQ 100	119	0.0134	0.0018	0.0424	0.0186	-0.0933	0.1226	-0.3051	0.1350

Table 2: Summary of the descriptive statistics of Stocks Returns

Our analysis starts by investigating the characteristics of our data. Table 2 shows a summary of the descriptive statistics for all the variables. Our variables were converted into logarithmic returns since simple or arithmetic ones involve a positive bias. The two of the most important statistics shown in the above table are (a) the ***Skewness*** and (b) ***Kurtosis*** which provide information regarding the shape of the distribution of our data. From one side, Skewness quantifies how symmetrical the shape of the distribution is. If a distribution is symmetric, the skewness will be zero. If values are greater than 1, the distribution is positively skewed demonstrating a long tail in the positive direction. If values are less than -1, the distribution is negatively skewed with a long tail in the negative direction. As we can see from the above table, all variables demonstrate negative values near to 0, which means that most of our series are almost symmetrically distributed except MDLZ and VRSN series where their values were close to -1 (-0.87 and -0.92) implying that the data are slightly negatively skewed. On the other side, kurtosis tells us the height and the sharpness of the central peak as well as the fatness of the thickness of the tails. If a distribution is normal, then the kurtosis coefficient would take the value of 3. Higher values would indicate extreme leptokurtic (peaked distributions) while lower values would indicate extreme platykurtic (flat-looking distributions). Based on the above statistics, most of the stock series seem to be extremely platykurtic as at values extremely below 3, except MDLZ and VRSN series. In VRSN series the value is 2.3 which is below but close to 3 meaning that our data is close to normal distribution while in MDLZ series the value is 17.7 which implies an extreme leptokurtic distribution.

Based on the above analysis, we can conclude that a slightly negative skewness along with extremely low values of kurtosis are evidence that our variables are not normally distributed but rather platykurtic. Since we employed monthly data, we should have expected that stock market returns would be close to normal distribution. Based on empirical observation, the higher the frequency of sampled data, the more deviated from normality while the lower the frequency the higher the symmetry and closer to normality. Kumar and Dhankar (2011) in their study verified these findings as they found that the Indian stock market returns of daily and weekly frequency were not normally distributed, while monthly and annual data were normally distributed.

5.2 CAPM Results

	Average	Variance	Standard Deviation	Covariance	Beta	Expected Returns
AAPL	2.100%	0.528%	7.266%	0.0021	1.1576	20.08%
TXN	1.553%	0.351%	5.929%	0.0018	0.9904	17.20%
MCHP	1.333%	0.483%	6.948%	0.0017	0.9723	16.88%
VRSN	1.921%	0.450%	6.711%	0.0015	0.8136	14.15%
WDC	0.629%	1.251%	11.187%	0.0026	1.4291	24.76%
MDLZ	1.126%	0.508%	7.124%	0.0012	0.6823	11.88%
TSLA	2.409%	2.329%	15.260%	0.0015	0.8483	14.74%
SBUX	1.816%	0.317%	5.627%	0.0010	0.5306	9.27%
AMGN	1.293%	0.376%	6.128%	0.0011	0.6407	11.17%
UAL	1.238%	0.959%	9.792%	0.0012	0.6879	11.98%
FISV	1.921%	0.199%	4.457%	0.0012	0.6823	11.88%
AMZN	2.379%	0.623%	7.890%	0.0023	1.2627	21.89%
NASDAQ 100	0.629%	1.241%	11.140%	0.0026	1.4291	24.76%

Table 3:Expected Returns & Beta Calculations for Each Stock using CAPM

Before finding *The Optimal Portfolio* with the lower risk, we calculated the betas and the expected returns of each stock individually. The respected results are illustrated in Table 3 and Figure 2. As we can observe, WDC has the highest expected return (24.76%), while TSLA has the highest standard deviation, hence risk (15.26%). AMZN and AAPL on the other hand have the second and third highest expected return (21.89% and 20.08%) with a relatively low enough risk (7.89% and 7.76%) respectively.

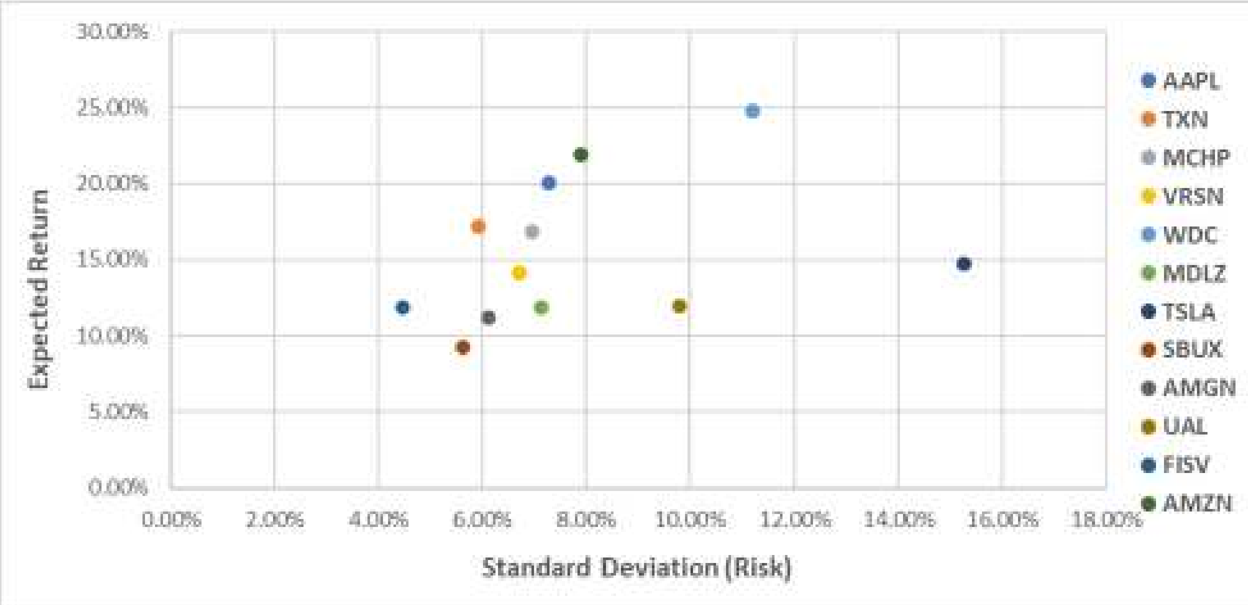

Figure 2: Risk/ Expected Return Relationship between the Assets

Further to the above, beta coefficients are given for each stock. Based on the calculations performed, most of the stocks included in our portfolio are defensive. This means that the beta coefficient is below 1 and implies that in case the market crashes, our stocks would drop less than the market. Same implies if the market goes up. Our stocks would get an increase less than the one of the market. For example, SBUX has the lowest beta (0.53). This means that if the market drops by 1% then SBUX will drop only by 0.53% and vice versa. On the other flip, AMZN, AAPL and WDC are classified as aggressive stocks as they experience betas higher than 1. This means that if the market crashes, all three stocks would drop more than the market and vice versa. For instance, the beta coefficient of AMZN is 1.26. In the event of a market crash, AMZN returns would drop 1.26% more than the market's drop, while in the event of a market upward, the returns of the underlying stock would increase by 1.26% more than the market one.

It is obvious that the higher the beta coefficient the higher return but the higher the risk as well. In a portfolio, it is wise to have both defensive and aggressive stocks, therefore, in the event of a market crash your portfolio losses are lower as possible and in the event, the market moves up the returns are higher as possible.

5.3 Minimum Variance Portfolio Results

Generally, investors have a given wealth and a number of assets available in the market. Thus, they have to decide how to allocate wealth amongst available assets in order to achieve the best possible result.

We are now interested in finding the Minimum Variance Portfolio (MVP), the Optimal Portfolio with the Minimum Variance (hence, risk). In order to construct MVP, investors need to be attached with low-volatility investments or create *a well-diversified investment portfolio* that carries assets with low correlation to each other. Low correlation investments are those that perform differently compared to the prevailing market and economic environment and by diversifying a portfolio, investors are aiming to reduce volatility.

After having the Expected Returns of the individual stocks calculated, a proportion of our wealth was allocated in each one of them using Solver to find the MVP. The respective proportion, otherwise called the weights assigned to the respective assets, are subject to two constraints:

1. The weight of each stock in the selected portfolio should be equal or greater than 0 ($w_j \geq 0$) and
2. The sum of weights of each asset must equal to 1 (i.e. 100%).

Having these constraints for all assets implies that each stock was given the same importance level in the portfolio. Both small-cap companies and large-cap companies have the same chances of participating in the Optimal Portfolio Selection. Historically, small-cap stocks are considered to be riskier than large-cap stock, and therefore they tend to perform with higher potential return compared to large-caps. In theory, giving greater weight to the smaller-cap names of an index in an equal-weight portfolio should increase the return probabilities of the portfolio.

In Table 4 are presented the results for the Minimum Variance Portfolio. The Results suggest that the best portfolio with the minimum risk relies on a 3.6% risk and a respective 12.26% Expected Return. Moreover, as we can observe the weights of each company either small or large are distributed accordingly in order to generate the optimal result. For example, large-cap companies such as Texas Instruments Inc (TXN), and Amazon.com, Inc. (AMZN) are left out from the portfolio selection, or Apple Inc. (AAPL) participates with a very small percentage of 4,91% and on the other hand, smaller-cap companies such as Fiserv Inc (FISV) has a significant participation weight of approximately 34%.

Average	Variance	Standard Deviation	Covariance	Beta	Expected Return
1.7097%	0.1315%	3.6269%	0.0013	0.7041	12.2599%

Assets	AAPL	TXN	MCHP	VRSN	WDC	MDLZ	TSLA	SBUX	AMGN	UAL	FISV	AMZN
Weights	4.9113%	0.0000%	6.3446%	6.4729%	0.0000%	7.0510%	3.5989%	17.5491%	18.8532%	1.1331%	34.0858%	0.0000%

Table 4: Minimum Variance Portfolio Results

Once the MVP was found, different Optimal Portfolios were created (see Table 5) in order to draft the Efficient Frontier. Figure 3 shows all the Optimal Portfolios available along the Efficient Frontier Curve**.**

							Weights											
Asset/Port.	Avg	Var.	St.Dev.	Covariance	Beta	ER	AAPL	TXN	MCHP	VRSN	WDC	MDLZ	TSLA	SBUX	AMGN	UAL	FISV	AMZN
MVP	1.71%	0.13%	3.63%	0.13%	0.70	12.26%	4.91%	0.00%	6.34%	6.47%	0.00%	7.05%	3.60%	17.55%	18.85%	1.13%	34.09%	0.00%
Port.1	1.80%	0.15%	3.93%	0.15%	0.86	15.00%	15.55%	5.29%	10.39%	6.77%	0.01%	4.93%	3.73%	4.06%	13.41%	1.16%	24.89%	9.80%
Port.2	1.85%	0.22%	4.68%	0.19%	1.04	18.00%	23.82%	14.45%	6.89%	4.36%	6.05%	0.00%	2.30%	0.00%	7.74%	0.00%	12.76%	21.63%
Port.3	1.86%	0.29%	5.37%	0.21%	1.15	20.00%	30.15%	20.60%	3.89%	1.29%	10.70%	0.00%	1.04%	0.00%	2.64%	0.00%	0.31%	29.37%
Port.4	1.71%	0.45%	6.72%	0.23%	1.30	22.50%	21.54%	0.00%	0.00%	0.00%	34.85%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	43.61%
Port.5	0.79%	1.08%	10.38%	0.25%	1.41	24.50%	0.00%	0.00%	0.00%	0.00%	90.97%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	9.03%
M	0.63%	1.24%	11.14%	0.26%	1.43	24.76%	0.00%	0.00%	0.00%	0.00%	100.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Sharpe Ratio	1.83%	0.18%	4.30%	0.17%	0.96	16.67%	19.89%	10.48%	8.60%	5.64%	3.13%	2.53%	3.07%	0.00%	10.36%	0.76%	19.33%	16.19%

Table 5: Results

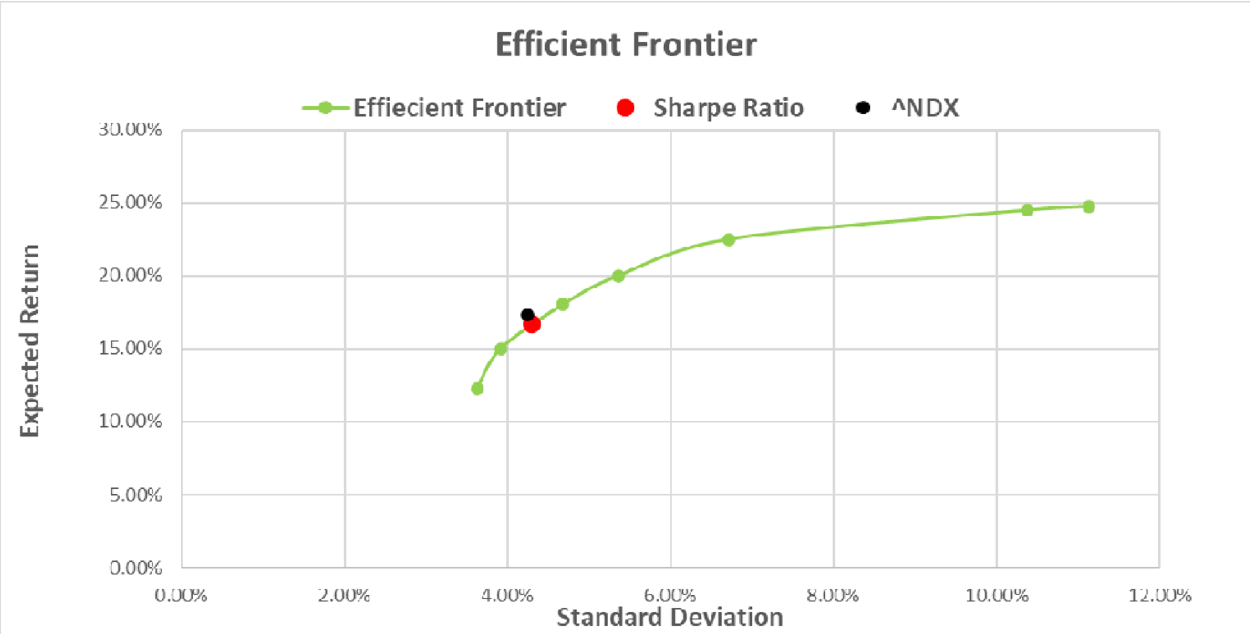

Figure 3: The Efficient Frontier

The above figure shows the return and its corresponding standard deviation for the 12 assets. The weights of these 12 Assets Portfolio generate the lowest standard deviation for the given return and is regarded as an efficient portfolio. Every efficient portfolio is then represented as a data point which results in the efficient frontier. Each point presents a variant but efficient portfolio. By investing in an efficient portfolio, investors can attain the lowest risk for a given return. Additionally, the weight for attaining the maximum ***Sharpe ratio*** has been given in Table 5 and presented in Figure 3.

Moving forward, we construct the optimal risky portfolio which indicates the tangency point of CML and the efficient frontier as mentioned in Section 4.2.2.5 above. As we observe in Figure 4, the optimal risky portfolio has an expected return is 16.67%, Standard Deviation 4.30% and a Sharpe ratio of 3.85.

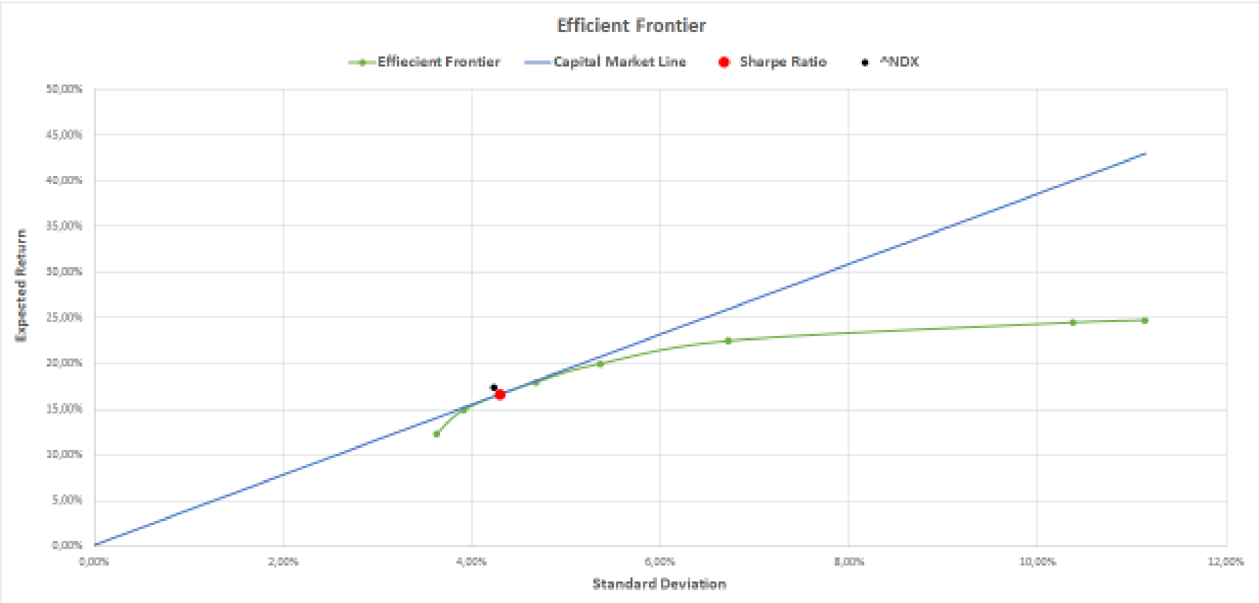

Figure 4: Optimal Risky Portfolio

5.4 Advantages of Portfolio Optimization

Bearing in mind the results of the literature review , the following benefits of portfolio optimization are presented below:

1. Portfolio optimization offers the benefit of a sustained, frequent and objective analysis of the portfolios of the company.

2. Portfolio optimization allows to identify acceptable metrics and to establish a method to address internal prejudices.

3. Allows to develop realistic portfolio optimization scenarios and to relate them to value generating metrics.

4. Portfolio management approaches diversify investments and eliminate systemic risks.

5.Post-Modern Portfolio Theory facilitates the optimization of extremely biased investing techniques.

6. Deep learning approaches for portfolio management are reliable and minimize human error.

5.6 Overall Findings

The literature review and data analysis have proven that portfolio optimization includes increasing expected returns or mitigating financial risks, which could have both functions. CAPM indicates how security prices function to offer a framework for investors to measure the effect of the proposed security expenditure on the total portfolio risk and return. It also indicates that securities values are calculated in a particular manner that shows that the risk premium or excess return is proportional to the systematic risk suggested by the beta coefficient. The model is used to evaluate the risk-return consequences of owning securities. Overall, CAPM refers to the manner in which shares are priced in accordance with their predicted uncertainties and returns. Different portfolio theories have their own benefits and drawbacks and are undertaken by having the business situation / circumstances in mind.

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Chapter 6: Conclusions

6.1 Summary

Decision making is very important when choosing the correct investment or group of investments. Portfolio Optimization aids in the selection of the optimal combination, e.g. asset selection, asset weighting and asset management for attaining the highest return for a given level of risk. The goal of portfolio optimization is to optimize projected returns or minimize financial costs, or it can contain all goals. The main purpose of the underlying study was to put in practice Markowitz Modern Theory in the Nasdaq 100 Index using the original Capital Asset Pricing Model (CAPM) as well as finding the risks associated with Portfolio Optimization. To date, there is an extensive literature on Portfolio Optimization Theories including Modern Theory, Post Modern, Mean Absolute Deviation Theory, etc. as well as on the factors responsible for attaining optimal investment.

Present literature results have led us to conclude that the successful boundary curve represents a combination of expected return and risk for unpredictable asset portfolios that minimizes risk at a certain level of expected return. The trade-off between probability and expected return on low-volatility investments is recognizable. An investor who wishes to achieve high expected return (point M) has to be able to engage in high volatility. At the rate of Sharpe Ratio, the investor would have an intermediate expected return with lower uncertainty. If no risk-free borrowing or loans (Rf) is given, the most effective are just portfolios beyond MVP and across the efficient frontier. By adding a risk-free rate, the curve turns the efficient portfolio into a straight line (CML) [Fama and French, (2004), p.27].

In order to achieve optimal portfolios of risk-free investing and lending, one draws a line from Rf point up and left as far as possible to the tangential point of CML and the efficient frontier line: all optimal combinations of risk-free assets and a single volatile tangential portfolio are then exhibited as optimal portfolios (Fama and French, 2004). The result is established by Tobin's (1958) separation theorem. The result of the CAPM is now much clearer. In absolute consensus on the allocation of returns, both investors understand the same incentive and incorporate the same volatile tangential portfolio in Rf. Since all investors seek the same portfolio of risky assets, it must be a value-weighted portfolio of risky assets in the market. Explicitly, the overall market value of all outstanding assets measured by the total market value of all risky assets would be the weight of each risky asset in the tangential portfolio.

For the underlying study, we employed monthly data from January 2010 to January 2020, 12 out of 103 stocks comprised in the Nasdaq 100 index, and the 3-month US T-Bill rate. We firstly obtained the Expected Rate of Return for each stock by estimating a simple CAPM. Then, the values found were weighted to find the Minimum Variance Portfolio. We found that WDC had the highest expected return (24.76%), while TSLA had the highest standard deviation (15.26%). AMZN and AAPL on the other hand had the second and third highest expected return (21.89% and 20.08%) with a relatively low enough risk (7.89% and 7.76%) respectively. Nevertheless, we found that stock series with the high expected return (AMZN, APPL WDC, TXN) and stock series with relatively high standard deviation (TSLA) were given low to zero weights within the Minimum Variance Portfolio, while stock series with relatively medium to high returns and relatively low standard deviation were given higher proportion. FISV SBUX, and AMGN had expected returns of 11.88%, 9.27%, 11.17% respectively as well as standard deviation of 4.46%, 5.63%, 6.13% respectively. Our Minimum Variance Portfolio results suggest that we shall invest the 34.9% of our wealth to FISV, the 17.55% to SBUX, and the 18.85% to AMGN and zero to AMZN, TXN and WDC.

6.2 Recommendations

Reflecting the facts of the literature review and the data interpretation, the following recommendations have been concluded:

1. Portfolio Optimization using the Productive Frontier is useful for a thorough study of investment prospects and for more rational decision-making. Using other approaches such as Mean Absolute Variance (MAD) and Conditional Value at Risk (CVaR) is useful for producing low-risk portfolios that can be found attractive in dynamic markets.

2. In order to achieve a higher return on portfolios, investors need to take more risk, which requires a trade-off between the expected return and the risk.

3. Models that do not use equal probabilities simulations have been described as producing improved cost and return outcomes.

4. Study has demonstrated that the use of Model Predictive Control generate computational positive effects as estimations of potential returns are modified once a new observation becomes available.

5. It has been shown that variance is an incorrect indicator of investment risk, as the mean variance of optimal portfolios has not been defined to help minimize risk.

6. Empirical evidence from the researchers has said that the cost of no short-selling constraints increases with increasing in assets.

7. Portfolio optimization accomplished by machine learning techniques should be used as reliable and should remove human error and bias.

8. A well diversified portfolio tends to minimize the uncertainty in portfolio returns when the price in the whole asset does not change in the same direction and at the same rate.

9.Beta should be adjusted dynamically by the portfolio managers, that is, to enhance the exposure when one predicts that the market trend will rise, and to lower it when one expects that the market trend will fall.

10.There is a need to work on the portfolio gap in order to produce stronger portfolio optimization performance. The difference portfolio consists of a set of securities which, when added to the investment portfolio, are assigned to the benchmark portfolio.

11.Regulations and taxes should be well managed by corporations for removing the hurdles in portfolio optimization.

6.3 Limitations of the Study and Suggestions for the Future

The underlying empirical study has a number of limitations. Few of them rely on data, timing, software and methodology issues. To begin with, data were gathered from Yahoo Finance for US corporations and indexes due to a lack of direct collection of information related to portfolio optimization from other markets. Therefore, future studies could expand their scope to the collection of data from European Markets such us FTSE 100 in England, DAX in Germany, or even from Emerging Markets such as India or China Stock Markets. Secondly, prices were collected for 12 stocks due to limitations of the software and technology usage. The use of Excel created limitations on the analysis as the usage of advanced programming languages and or software, such as MATLAB, would provide more accurate results. Future studies could be based on more advanced software where large datasets could be handled. Last but not least, the underlying study emphasizes on the classic CAPM model. There are various variations of this model in which future studies could use for estimating the Expected Returns of their portfolios and hence finding their optimal ones. Moreover, this study introduces the theoretical background of other models such as the Mean Absolute Deviation Optimization, and the Feinstein-Thapa Modification. Future studies could also be conducted utilizing the aforementioned models.

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