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COMPUTERS AND EDUCATION: ARTIFICIAL INTELLIGENCE

Development of Intelligent and Affective Tutoring Platform for Mathematics – A case study for Primary Education

DOCTOR OF PHILOSOPHY DISSERTATION IN CO-SUPERVISION WITH THE UNIVERSITY OF THE PELOPONNESE

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Development of Intelligent and Affective Tutoring Platform for Mathematics – A case study for Primary Education

DIMITRIOS A. MASTORODIMOS

A dissertation submitted to the Neapolis University Pafos in partial fulfillment of the requirements for the degree of the Doctor of Philosophy in Co-Supervision with the University of the Peloponnese

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Δηλώνω υπεύθυνα ότι η διπλωματική εργασία είναι εξ ολοκλήρου δικό μου έργο και κανένα μέρος της δεν είναι αντιγραμμένο από έντυπες ή ηλεκτρονικές πηγές, μετάφραση από ξενόγλωσσες πηγές και αναπαραγωγή από εργασίες άλλων ερευνητών ή φοιτητών. Όπου έχω βασιστεί σε ιδέες ή κείμενα άλλων, έχω προσπαθήσει με όλες μου τις δυνάμεις να το προσδιορίσω σαφώς μέσα από την καλή χρήση αναφορών ακολουθώντας την ακαδημαϊκή δεοντολογία.

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Abstract in Greek

Τα μαθηματικά ήταν ανέκαθεν ένα από τα πιο σημαντικά θέματα διδασκαλίας σε όλα τα επίπεδα εκπαίδευσης. Εάν κάποιος καταλάβει τις θεμελιώδεις έννοιες των Μαθηματικών, τότε θα έχει διαρκές ενδιαφέρον για αυτές και θα τις μελετήσει για πάντα. Αλλά, αν συναντήσει δυσκολίες και δεν μπορεί να τις ξεπεράσει, αυτές οι δυσκολίες θα τον ακολουθήσουν στο μεταγενέστερο ακαδημαϊκό του μονοπάτι. Τα κλάσματα είναι ένας από τους βασικούς τομείς της άλγεβρας που διδάσκονται από τα πρώτα χρόνια της εκπαίδευσης ενός μαθητή. Ταυτόχρονα, τα κλάσματα είναι ένας από τους δύσκολους τομείς των Μαθηματικών, τουλάχιστον για μαθητές δημοτικού.

Η κατανόηση της έννοιας του κλάσματος είναι δύσκολη για τους μαθητές, καθώς είναι μια αφηρημένη έννοια. Είναι η πρώτη αφηρημένη έννοια στα Μαθηματικά που γνωρίζει ο νεαρός μαθητής. Η χρήση παραδειγμάτων από την καθημερινή ζωή είναι μια τεχνική που χρησιμοποιούν οι εκπαιδευτικοί στη διδακτική διαδικασία για την εισαγωγή της έννοιας της κλασματικής μονάδας. Ωστόσο, εάν οι εκπαιδευτικοί πρέπει να παρουσιάσουν πιο δύσκολες έννοιες των κλασμάτων, όπως οι βασικές πράξεις, οι συγκρίσεις και άλλες έννοιες, αυτά τα παραδείγματα δεν βοηθούν τόσο πολύ. Αυτό απαιτεί έναν καλύτερο και πιο λειτουργικό τρόπο για την αναπαράσταση των κλασμάτων. Αναπόφευκτα, η χρήση απτών εργαλείων, γραφικών αναπαραστάσεων και ψηφιακών εργαλείων είναι πολύ καλές πρακτικές για τη διδασκαλία και την κατανόηση των κλασμάτων.

Η ταχεία εξέλιξη της τεχνολογίας, η ανάπτυξη και εξάπλωση του Διαδικτύου καθώς και τα σύγχρονα ψηφιακά εργαλεία που διατίθενται στο Διαδίκτυο, παρέχουν τη δυνατότητα ανάπτυξης κατανοητών διαδραστικών αναπαραστάσεων για τη διδασκαλία και την κατανόηση των κλασμάτων και των βασικών λειτουργιών τους. Χρησιμοποιώντας όλα αυτά τα σύγχρονα τεχνολογικά μέσα, οι μαθητές έχουν τη δυνατότητα να κατανοήσουν αφηρημένες έννοιες και να δημιουργήσουν βασικές έννοιες σε κλάσματα. Η διατριβή συμβάλλει με διάφορους τρόπους προς αυτή την κατεύθυνση. Στην αρχή, διερευνήθηκαν οι δυσκολίες που αντιμετώπισαν οι μαθητές στα κλάσματά και στις βασικές τους πράξεις. Στη συνέχεια διερευνήθηκαν οι απόψεις των εκπαιδευτικών σχετικά με τις δυσκολίες που αντιμετωπίζουν οι μαθητές στη διδασκαλία κλασμάτων.

Τα αποτελέσματα της βιβλιογραφικής επισκόπησης για τα ψηφιακά εργαλεία που βοηθούν τους μαθητές σε κλάσματα, καθώς και η εμπειρική έρευνα που πραγματοποιήθηκε, οδήγησαν στον ορισμό των βασικών στοιχείων για το σχεδιασμό και υλοποίηση ενός νέου εκπαιδευτικού εργαλείου για τη διδασκαλία και την κατανόηση των κλασμάτων.

Λαμβάνοντας αυτό υπόψη, σχεδιάστηκε και αναπτύχθηκε το σύστημα Student's Knowledge and Affective level for Fractions in an Open System (SKAFOS).

Τα αποτελέσματα της εμπειρικής έρευνας που ακολούθησε έδειξαν ότι το SKAFOS βοηθά τους μαθητές να κατανοήσουν τα κλάσματα με την ενεργή συμμετοχή τους, καθώς και τον πειραματισμό τους με το λογισμικό. Οι μαθητές που χρησιμοποίησαν τις δυνατότητες του εργαλείου SKAFOS οδηγήθηκαν σε καλύτερη κατανόηση των κλασμάτων χρησιμοποιώντας πολλαπλές αναπαραστάσεις. Τα αποτελέσματα έδειξαν επίσης ότι τα δυναμικά χαρακτηριστικά του εργαλείου SKAFOS συνέβαλαν στην κατανόηση της κλασματικής μονάδας, στη δημιουργία επαρκών αναπαραστάσεων για την έννοια της κλασματικής μονάδας και στη σύγκριση δύο κλασμάτων με κοινά ή καθόλου κοινά συστατικά (ονομαστής ή παρονομαστής).

Abstract in English

Mathematics has always been one of the most important teaching subjects at all levels of education. If someone understands the fundamental concepts of Mathematics, then he will have a perpetual interest in them, and study them forever. But, if he encounters difficulties and he cannot overcome them, these difficulties will follow him on his later academic path. Fractions are one of the key areas of algebra taught since the early years of a student's education. At the same time, fractions are one of the difficult areas of Mathematics, at least for primary school students.

Understanding the concept of fraction is difficult for students, since it is an abstract concept. It is the first abstract concept in Mathematics that met the young learner. The use of examples from everyday life is a technique that teachers use in the teaching process in the introduction of the concept of fractional unit. However, if teachers have to present more difficult concepts of fractions, such as basic operations, comparisons and other concepts, these examples do not help so much. This requires a better and more functional way to represent fractions. Inevitably, the use of tangible tools, graphic representations and digital tools are very good practices for teaching and understanding fractions.

The rapid evolution of technology, the development and spread of the Internet as well as the modern digital tools available to the Internet, provide the possibility of developing understandable interactive representations for the teaching and understanding of fractions and their basic operations. Using all these modern technological means, students have the ability to understand abstract concepts and build basic concepts in fractions. The thesis contributes in a number of ways. At the beginning, the difficulties encountered by students in the fractions and basic operations were investigated. Then the views of teachers on the difficulties students encounter in teaching fractions were explored.

The results of the literature review on digital tools that help students in fractions, as well as the empirical research carried out, led to the definition of key elements for the design and implementation of a new educational tool for teaching and understanding fractions. Taking this into account, the Student's Knowledge and Affective level for Fractions in an Open System (SKAFOS) system was designed and developed.

The results of the empirical research that followed showed that SKAFOS helps students to understand fractions with the active participation of them, and also their experimentation with the software. Students using the capabilities of the SKAFOS tool were led to a better understanding of fractions using multiple representations. The results also

showed that the dynamic characteristics of the SKAFOS tool contributed to the understanding of the fractional unit, to the building of sufficient representations for the concept of fractional unit, and the comparison of two fractions with common or no common components (nominator or denominator).

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The completion of the trip leaves me with a bittersweet feeling. Sweet because after a while I completed an effort with pain and effort that had to teach me many useful things. Bitter because it completes the very good collaboration you had with some scientists but first of all with some people who helped you gain some knowledge that alone you might not be able to achieve and acquire supplies for your subsequent academic career.

I would like to express my gratitude to my supervisors Savvas Chatzichristofis, Assistant Professor from Neapolis University Pafos, and Athanassios Jimoyiannis, Professor from University of the Peloponnese. I want to thank both of them for providing excellent cooperation, scientific guidance, adoption of the scientific way of thinking, and supporting me whenever I needed it all these years. In addition, I would like to thank both supervisors for the human side they showed me during these years of our cooperation. Furthermore, I would like to thank Professor Halatsis Costas and Assistant Professor Klitos Christodoulou for the cooperation we had the first two years, as well as the members of my committee, Assistant Professors Zinon Zinonos and Andreas Masouras.

Of course, the whole effort to complete it definitely took time, which I somewhat deprived of my family, which I want to thank. I thank both my wife Panagiota and my two daughters Athanassia and Maria. I also want to thank my parents for the positive persistence and stubbornness they have instilled in me so that I always try to complete a process and not give up no matter how many difficulties I encounter.

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Introduction

Chapter One

1.1 The subject of thesis

Educating children is one of the essential obligations of the coordinate State and each family individually. The factors that influence education and training are varied and they can be found in different environments. In the first years of their lives, children follow the philosophy of their family environment, mainly from the mother and the father, for issues of education. Sometimes by other related persons who are responsible for raising children. In general, education and culture of parents can affect children's development both in cognitive and behavioral levels. Moreover, an important role in the initial education and the education of children can have, in addition to parents, grandmother with grandfather, since they play a key role in the Greek family. They come into contact with the children for many hours and spend a pleasant and creative time with them. Through games, fairy tales, narratives, and whatever they consider appropriate activities, parents and grandmothers and grandparents use them to set the basic foundations of children's education and character development. They play games with numbers, card games, finger counting, they say numbers out of order and other pleasant number games, and finally they engage in a game of learning mathematical concepts. This type of education and literacy that children receive is part of informal education.

This way, the students' first acquaintance with Mathematics is made. Children know informal Mathematics [151] before they even enter the elementary school, from their home and/or kindergarten, and usually make mental calculations for simple problems. There are not a few times when children even enroll in school know how to measure up to a certain number, they know how to make very simple additions and subtractions, and generally to handle Mathematics, without of course scientific background, concepts, and theories, through play and fun. This contributes to the great development of the technology that has been integrated into the games and the children are learning by playing games, even before enroll at school. However, the space that is appropriate to acquire abilities, skills, knowledge, and to learn Mathematics, as well as the other courses, is the school, which is institutionalized by the state. Thus, this necessary connection between informal and typical Mathematics makes them more intimate for children [151].

The school has a mission beyond shaping the children's character and conveying the cultural identity of each country. Also, it has as its main pillar the learning, knowledge, and education in the general condition. Children go to school like dough without shape and form, and at the end of their student journey, they have been carved and acquired education, knowledge, skills, attitudes, and behaviors. Courses are taught with a different subject, but some always remain constant from kindergarten to high school. One of these basic lessons taught over time, in addition to reading and writing, is Mathematics.

Mathematics is the core of the basic courses taught in all parts of the world, over time, from kindergarten, elementary, high school, to university. The National Mathematics Advisory Panel [197] states that the target for students of grades K-8 is the adequacy (proficiency) in fractions so that they do well in the next classes with algebra. Mathematics generally meets in every facet of human life and is important for every financial transaction of people. Every day people use in their dealings and in their activities without understanding mathematical concepts, make meaningless operations, and apply mathematical algorithms. However, despite the necessity of Mathematics in people's lives, few have of them done their math and understand from early on the terms and concepts of Mathematics. Usually, most people have gaps in mathematical concepts and did not catch up in the school's educational time to fill these gaps. Sometimes due to their omissions and misunderstandings, other times the teaching process was not properly completed by teachers and the education system. Even the parents' stereotypical perceptions of Mathematics can affect students' performance. Also, some people seem to encounter several difficulties since their childhood, and this is sometimes due to the informal education they received from their home before they even enter the school. The way of learning was not the right or wrong the informal education they received from people who did not have the appropriate pedagogical training. Sometimes students, especially in the first steps of their student journey, are confused and dilemma and do not know which method to use, the one they learned from the home or the one taught at school.

Mathematics is usually one of the most difficult courses in school and one of the lessons that make students face difficulties at all levels of education. Certainly, students are not as familiar with Mathematics as with other courses. Many students, from the very first classes of elementary school, face difficulties with Mathematics, both in simple mathematical operations and in their application in everyday life.

Research [48] showed that less than 10% of students, attending from first to sixth grade at elementary school, managed to solve a mathematical problem correctly (e.g. 8+4=_+5). The difficulties encountered by students in the problems of Mathematics are presented both in the pronunciation of answers with the spoken word and in written tests and exercises. Students are not able to understand the exercises, so they cannot respond to their solving mechanism. In other words, students have not developed a mechanism for converting pronunciations into a mathematical model so that they can analyze it and later be able to resolve it. Also, students feel fear and negative attitudes for the mathematics lesson [34]. Also, the confrontation of unusual problems that are not part of the teaching contract in mathematics, leads students in a hostile and inconvenient environment [92].

These difficulties that students may encounter at school during solving exercises of Mathematics, Vosniadou [36] has summarized them in the following categories:

a) Many errors in arithmetic operations are systematic and not accidental, which usually it states that students have not conquered the knowledge of arithmetic operations;

b) Frequent removal of the smallest number from the largest regardless of their position;

c) Difficulty in understanding the mathematical problem;

d) Difficulty in selecting arithmetic operations;

e) Multiplication by decimal numbers is difficult for students;

f) Incorrect models of representation of arithmetic operations from the daily experiences of students;

g) The language that Mathematics is written, it makes it difficult for students;

h) Problems representing associated proposals, and

i) Wrong representation of the problem;

Also, the teaching contract in Mathematics creates misconceptions among students and difficulties when faced with different problems than routine problems that have been learned to respond and create a stereotyping perception of students in solving mathematical problems [92]. The teaching contract in Mathematics also creates problems for secondary school students as mentioned in the above bibliographic research conclusions.

In addition, as many researchers mention [39], [53], [72], students encounter difficulties in fractions. Indeed in [260] it is reported that Chinese students consider fractions one of the most difficult fields of Mathematics. Fractions are difficult on their own because the student should often use their imagination to describe, visualize, and understand them. After all, fractions are an abstract concept. In other words, the student should be able to think about the problem and the solution. Many times these difficulties in fractions accompany students in the next grades of education, resulting in 1/3 of students fail to add fractions with different denominators [27].

Also, division is very difficult for many students [55]. What usually makes students difficult is the algorithm of the operation, which clearly it shows that the steps of the algorithm have not been understood by students. Repeating the basic steps until the operation is completed seems to create problems of understanding among students. In addition, students acquire a fear of Mathematics, arithmophobia, or mathematics phobia that several times leads them up and think to leave school [225].

Furthermore, in [260] it refers that several reasons contribute to the students having difficulties in fractions, but the most important are:

a) Fractions can be manufactured and represented in many ways, such as quotient, part to whole, ratio, measures, rate, and operations;

b) Previous knowledge of fractions can prevent the comprehension of fractions;

c) Inability of teachers to connect fractions to the actual experiences of students outside school;

d) Due to lack of educational Mathematics to train teachers for Mathematics;

e) Lack of knowledge of teachers to teach Mathematics; and

f) Many teachers lack basic knowledge of fractions;

By enumerating the accusations of problems in Mathematics, but by the difficulty of understanding the mechanisms of fractions, it is perceived that Mathematics is indeed a difficult chapter in the educational community, and should be treated with due care by teachers to help students in understanding these mathematical concepts. However, surveys show that teachers who teach Mathematics in elementary school do not feel as ready [21] for teaching this difficult by common scientific field confession. As a result, teachers have anxiety, which is called Mathematics Teaching Anxiety [219], [209], they are concerned about the way they teach and are afraid that they will confuse the students [209]. Also, a study [111] concluded that a large percentage of elementary teachers do not have the necessary knowledge to teach fractions and to understand students the meaning of fractions. Furthermore, teachers find it difficult to understand and teach fractions [167]. Moreover, pre-service teachers encounter difficulties in solving problems that contain fractions [275]. In a survey [168] that took place, it appeared that fewer than half of the teachers were able to use a suitable algorithm for dividing fractions and no one could explain the reasons for this operation. Besides, the level of knowledge in the Mathematics of students is significantly influenced by teachers' knowledge in Mathematics [259]. It is understood that teachers follow the curricula drawn up by the official state. Specifically, in Greece, teachers when teaching Mathematics they follow the framework established by the Inter-thematic Single Framework of Curricula and the Analytical Curricula of compulsory education as published in the official Sheet Newspaper of Government (number 304, and year 2003).

Teachers trying to implement Analytical Curricula use various supervisory means and technology to pass on knowledge to students. Sometimes they are indicated by curricula and other times by choosing their tools, which from their experience have seen that they are meaningful and reliable. The use of visual representations can help students to understand the sense of numbers, decimals, and fractions [69]. Through the multiple representations, the instructor tries to help all students and to commune knowledge. The fingers, the abacus, the chopsticks, the cubes, the folding paper, and other manipulatives play the role of the mean representation of a mathematical operation or become a tool for solving a problem. Except for the traditional way of learning, the teaching of Mathematics could be by using Information and Communications Technology (ICT). There is ready-made software for students to understand mathematical concepts. But it needs a specific strategy using technology to be efficient and meaningful in the classroom since the general strategies for the use of technology in the classroom are difficult to determine [126]. The teaching time, the classroom, the quality and the size of the classroom, and the level of computer knowledge of students are some obstacles that need to be overcome, so that the teacher can implement the instruction module using the ICT in the classroom. The multiple representations in the software can be represented by the pie, the area models, and other objects that are known to the students from their daily routine. For example, the circle is tied to the pizza or the traditional pie from the everyday life of the students, so it is easier for the student to think that he has a pizza and he has to distribute it to some friends rather than have a circular disc and he has to break up in some pieces.

In the current era, that everything become digital, students come into contact with technology from a very early age. Most students even before entering the school have acquired intimacy and skills with tablets and smartphones. It's usually done after their parents' motivation. They have spent too many hours playing games and using training apps that their parents usually choose. When children enroll in school, they still devote time to games. Research has shown that gamification is a learning incentive [143]. Additional environments with fun and entertainment elements have a positive impact on learning [16], [246]. Specifically, a survey [50] showed that students devote enough free time to their entertainment with video games. Apart from video games, students are engaged in educational activities that aim to improve learning and problem-solving, both in Mathematics and in other subjects, after their parents' motivation as well as school teachers. But, most educational activities have not been tested in the pedagogical directions and they are not designed by specialists, so they do not lead to the desired result. However, technology in general assists in training wherever computers are placed, such as in kiosks in the courtyard of the elementary schools in India [114]. Of course, there are several digital activities and digital platforms that have been created by academic institutions and meet educational criteria, and they are suitable for learning difficult concepts in Mathematics. For example, there is a digital platform called Fractions Tutor [238] that enables students to interact with the system, make mistakes, ask for help from the system, and visually see the solution to their problem. Fractions Lab [101], part of a collaborative European project that was implemented by a mix of academic and commercial partners, also has a digital platform that allows students to engage in fractions. It is an interdisciplinary program that involves the mechanical learning of user modeling, natural language processing, educational psychology, and the education of students. Another educational platform with training simulations and Mathematics is the PhET [220], a repository of training simulations, open source written in Java, Flash, or HTML5.

Such large repositories or educational platforms have been created also in Greece with the help of the State and the Ministry of Education. More specifically in the Aesop repository (http://aesop.iep.edu.gr) there are educational scenarios for various courses in primary and secondary education, as well as for Mathematics, after evaluation by a committee of experts. But in this repository, there are not so many issues for fractions in primary education and there is not much interactivity. Also, there is another educational platform, called Photodentro (http://photodentro.edu.gr/aggregator/), a repository with many autonomous applications for various topics implemented in various programming languages, as for Mathematics and fractions. But most of these digital scripts don't run or want special specifications to run properly. As a result, teachers do not use them in everyday educational activities in the classroom. This of course contributes both to the poor logistical infrastructure of most elementary schools, and to the possibility for the majority of teachers not to be able to use ICT in their daily lessons.

Additionally, the emotional state of the students affects many times the problem solving, except for the way and the medium that the mathematical scenarios will be available into the student. Both the initial mood and the condition in which the student is located in solving problems and exercises affects him both negatively and positively. The positive resolution of a problem raises the student's mental disposition to solve other problems, while failure to solve an exercise usually predisposes him negatively to try to solve other problems. This also refers in a research [152] whereby the performance of students without mathematical backgrounds is influenced by emotional changes. This requires continuous emotional activation of students, either by prompts or assisted (with text, sound, motion, or animation) to gain a positive mood and continue solving problems and not to abandon their efforts. However, in order to happen this, it should first have been imprinted on how students feel about the specific activities and exercises that they need to complete.

1.2 Necessity of the thesis

The difficulties that students encounter during fractions tasks were lead us to the investigation of this situation. We wanted to determine why students had these difficulties, their origins, and how we can try to eliminate this phenomenon.

The first step was to design research in fifth and sixth-grade students of a Greek elementary school to fractions comparison tasks. This research verified our concerning that indeed students face difficulties in fractions tasks, in the first place, in comparisons. So, we tried to find out how a teacher can help students who met challenges infractions. Except for the traditional way of drilling infractions, we thought a teacher could help students using the ICT.

The second step was to find out if there is a software to help students in fractions. Thus, we surveyed with specific characteristics. Two of the essential criteria were that the e-learning systems must be free and based on the web. We surveyed electronic research databases, and we found literature for a few of them around the globe that met our criteria. We studied these three e-learning platforms, and we noticed some basic features that an e-learning system for fractions could have. Some of these features we later embedded in our proposed approach for fractions.

Furthermore, in the literature, the students' emotions are rising during the mathematical task; they are among the reasons that a student could succeed or not in

Mathematics. Thus, we thought that we must find out the student's emotions during fractions' tasks. First of all, we have to research if there were any mathematics systems for recognizing students' emotions during mathematical studies.

The third step was to research systems that could recognize students' emotions. The recognition of the student's emotional state during teaching or the resolution of exercises can be identified by an Intelligent Tutoring System (ITS) called Affective Tutoring System (ATS). From the literature, it appears that ITSs have been developed that can recognize the student's emotional state using a camera or other devices when involved in activities or exercises. From the search in the international literature, such a system or similar to the recognition of students' emotions through an image, is not developed in the field of fractions. There is not such a system in Greece and in the Greek language.

The fourth step was to conduct an interview using the focus group method in Greek elementary teachers, in order to discuss the techniques and strategies they use in class when they teach fractions. The research concluded with some interesting findings that it would be interesting for a new e-learning system for fractions.

Therefore, the difficulties that students face when they solve fractions tasks, the need to recognize students' emotions when solving fractions exercises, and to help students better in their understanding, the lack of ATS or e-learning system that could realize emotions during fractions tasks, and the opinions of Greek elementary teachers when they teach fractions, has led us to the design of a digital web platform that can recognize the students' emotions when solving fractions tasks, and specific comparisons of fractions. The recognition of emotions is done with the help of a Convolutional Neural Network (CNN). So the teacher using this tool will know what kind of exercises the student is struggling with and will be able to assign each student separately those exercises with the appropriate graphical representations in the future. In this way, teachers can help students and improve the student's cognitive level of fractions.

In addition to research conducted, there appears to be no such platform or accessible free of charge and open throughout the educational community in Greece for Mathematics and especially for fractions. Also, the fact that both primary education teachers and students have a problem with fractions helped create this tool that would support and provide assistance to the learner in understanding the problem and solving it. Also, the existing software for Mathematics available free of charge is tailored to the countries' mentality and traditions that have been created so that they do not find the same interest from the Greek elementary students. For example, understand and show great interest in the Greek student examples of Mathematics for Thanksgiving when he does not know this day and the corresponding tradition. It is easier to have mathematical examples for the money they receive from Christmas Carols since students are more familiar and know these celebrations. It also applies to Mathematics that applies to the rest of life: to love something, you must first understand it. Also, the pronunciation, instructions, and observations are usually English, which makes the students of the Greek elementary schools even more difficult since their native language is not English.

In conclusion, the difficulty encountered by students in fractions, but the lack of recognition systems of the student's current emotional state when solving fractions exercises led us to the thought of creating a web learning tool for fractions, which can be used by the classroom teacher to help students understand fractions. The possibility of recognizing the student's emotions when solving exercises with a specific form of representation of fractions, through the particular tool, will help the teacher to provide the student with tasks with representations that create positive emotions. The proposed mechanism will essentially offer the opportunity to the teacher or the system itself through multiple graphical representations of fractions (i.e. pie, bar, and combination of them) to keep the student emotionally energized, so that has a cheerful disposition to be able to continue solving problems infractions.

1.3 Structure of the thesis

The chapters of the current doctoral thesis are structured in such a way that they smoothly lead the reader to the understanding of the subject with which this work is negotiated. The first chapter, titled Introduction, briefly presents the general concern, the basic framework of work, and the main reasons that led to this choice of work. It generally describes the situation in the subject in question so that the reader can understand what this thesis is about. In the second chapter, titled Theoretical Framework, the main pillars of support for this thesis are presented, such as the didactic of Mathematics, the didactic of fractions, Information and Communication Technologies in education, Intelligent Tutoring Systems and existing Affective Tutoring Systems. In the third chapter, titled Literature Review, a review of the bibliography on research and publications that have been carried out and have led to some results for the use of Information Technologies and Communications in both Mathematics and fractions. It also refers to the need to create a specific thesis, the purpose of the research, and the research questions that have been asked for the investigation. In the fourth chapter, titled Research Design, it is presented the design of the current research. Specifically, they are described the phases and the methodology of the current thesis. In the fifth chapter, titled

Survey on Students, it is presented the survey that was conducted to fifth and sixth-grade students in comparing pairs of fractions. It is presented the related work, the method, the results, and the findings of this survey. In the sixth chapter, entitled Survey on E-learning Systems for fractions, it is presented the method, the results, the analysis, and the findings of e-learning systems for fractions. In the seventh chapter, entitled Survey on Affective Tutoring Systems, it is presented the method, the results, the analysis, and the findings of ATSs for Mathematics. In the eighth chapter, entitled Teachers' Focus Group, there is presented a survey on elementary in-service teachers using the focus group method, the results, the analysis, and discussion about the findings of this interview. In the ninth chapter, titled Proposed System, the proposed interactive multimedia environment is presented. It is described briefly how the architecture, the representations, and interactivity with students is. Furthermore, indicative activities will be presented in the web proposed system as well as the activities that will be developed in the basic acts of fractions. Also CNN and its function are described. In the tenth chapter, entitled Survey on proposed system, it is presented the proposed system for fractions, and the study that was conducted in students who used this system with multiple representations for fractions. Also, it is presented the method, the results, the analysis, and the findings of this study. In chapter eleven, titled Discussion, the findings of this thesis are presented through research conducted by both teachers and elementary school students, the literature review study on e-learning systems for fractions and ATSs, as well as proposals for future exploitation of proposed tool, an online learning environment for fractions. The twelfth chapter, titled Published Work, are presented the papers that were published during this doctoral thesis. At the end of the chapters follows the section of References, both the Greek and the foreign literature are listed, which was studied for this thesis.

Theoretical framework

Chapter Two

2.1 Introduction

The second chapter of the current thesis presents the theoretical framework in general of the didactic of Mathematics, and in particular of fractions, Information and Communications Technology in education, Intelligent Tutoring Systems, Affective Tutoring Systems, emotion recognition with the use of Artificial Intelligence, and emotional state of the student. Each of the above concepts is a piece of the puzzle that if they unite together, they are the foundations of the present work. Without them, the reader cannot understand nor the logic of the thesis or any other concepts introduced in the following chapters.

2.2 Mathematics

Mathematics is not a new science, but it keeps its roots from very old, almost from man's original presence on Earth. Many peoples dealt with Mathematics, but those who put them down were the Ancient Greeks, so the names and theorems they introduced are known and are still used today. It is no coincidence that many mathematical symbols come from the Greek Alphabet. The Ancient Greeks were among the first communities to deal with mathematical science and managed to put new data in the scientific field. They had the Mathematics in a very high position, and in [226], is reported that (translated from Greek):

> Plato argued that no other lesson had as much educational power as mathematics. According to Plato, mathematics wakes the man who, by his nature, is sleepy and makes him receptive to learning, memory, and sharpness of spirit. He considered the knowledge of Mathematics as a basic prerequisite for attending higher education in Philosophy. So he puts at the entrance to the Academy of the famous inscription: *μηδείς αγεωμέτρητος εισίτω* (do not enter anyone who does not know mathematics).

It is understood that since antiquity, people considered that Mathematics was an essential knowledge, which was the passport on several occasions for higher education, but the personal success of a person.

The data in Mathematics has not changed even today, since Mathematics is the basis, the foundations, the skeleton for a series of sciences, such as Computer Science, Mechanics, Economics, Robotics, etc. To enable a student to follow these sciences should have gained fundamental knowledge of Mathematics. Otherwise, the tower of knowledge that will build will be demolished since it will not be based on solid foundations.

It is understood that students' knowledge should not be superficial and acquired by mechanical acts, but to have a thorough understanding of the essential functions of Mathematics and be able to build upon them. In other words, students must have mastered the mechanism of mathematical operations and phenomena, and have various strategies for solving problems at any time. However, to do so, students should be actively involved and not only passive receivers.

It is mentioned in [226] that Mathematics learned with students' active participation, which generally agrees with Piaget's pedagogy that one learns by acting. Both in teaching the course should show interest and solve the exercises not to abandon their effort at the first difficulty. Thus, for this active participation of students to be carried out in the teaching of Mathematics, the teacher should have the aim to teach the development of positive attitudes towards Mathematics, expression-communication through the Mathematics, the incorporation of technology in the teaching of Mathematics, the cross-thematic approach-unification of lessons, and the solution of the problem ([226]). With the breakthrough of new technologies in all aspects of life, this may facilitate the above objectives to make Mathematics more understandable to students. Students can use both tablets and smartphones to have continuous access to the software, even from their homes without a technology expert's help. Of course, the transmitter (teacher) and the receiver (student) must have developed a communication channel without loss, but there is a two-way relationship. Trust is often leading the student to success since the student trusts the teacher's instruction and is taught methodologies, strategies, and problem-solving methods correctly.

2.2.1 Purpose of Mathematics

From the personal experience of the author as a student, it is deposited the view that many instructors taught Mathematics and stood only in the memorization of mathematical formulas to resolve exercises, without being consumed in the more in-depth analysis of concepts and the development of a specific methodology and analysis of the algorithms for solving the exercises. In this way, the main objectives of the Mathematics mentioned in the literature were not observed.

More specifically, in [226] is mentioned that there are six purposes of Mathematics:

a) The student understands concepts, mathematical procedures, facts, and principles;

b) The student to perform operations and to follow procedures with understanding, accuracy, and speed;

c) The student to be able to solve problems;

d) The student to understand the logical structure of a mathematical proof;

e) The student to develop positive attitudes and aesthetically enjoy Mathematics, thus driven to self-confidence, initiative, interest, curiosity; and

f) The student to develop efficient ways of learning and communicating in Mathematics, as well as study and search habits of knowledge for autonomous progress;

In [83], it is stated that the general purposes of teaching Mathematics are (a) to contribute to the education of variable objectives of education, (b) to contribute to the attainment of the temporal objectives of education, and (c) the multiple supplies to humans. Moreover, in [83] it is mentioned that at a conference of the Organization for Economic Co-operation and Development (OECD) concluded that in general education, the purposes of teaching Mathematics are:

a) Educational, humanistic;

b) Utilitarian, auxiliary for life;

c) Preparatory for professional goals; and

d) Preparatory for university studies;

Also, the National Council of Teachers of Mathematics of the United States (NCTM), as mentioned in [83] it is noted that the purposes of teaching Mathematics are:

a) The development of the ability for pure thought;

b) The acquisition of mathematical knowledge practices that will serve in life;

c) The acquisition of skills in the execution of operations;

d) The acquisition of basic mathematical knowledge for further studies; and

e) The creation of love for Mathematics;

By the years, some of the purposes changed and the new principles for school Mathematics as mentioned in 2016 from NCTM [196] are:

a) Equity, high expectations and support for all students;

b) Coherent curriculum, focused on essential Mathematics, well-articulated across the grades;

c) Effective Mathematics teaching;

d) Learning Mathematics with understanding, building new knowledge from experience and previous knowledge;

e) Assessment that supports the learning of important Mathematics; and

f) Technology is essential for teaching and learning Mathematics;

The goals of Mathematics in the current era should be to improve the quality of human life by improving learning, emphasizing the creative and critical thinking of students [226]. Moreover, excellent Mathematics training is essential for both students and the workforce for a strong economy and national defense [243].

So, it seems that students learning Mathematics, apart from knowledge of types and proofs, should begin to apply Mathematics in their everyday life, thus developing creative and critical thinking. This, coupled with their computing thought development, will make students ready-made citizens for the new era of technology that has already begun.

2.2.2 Teaching Mathematics

The teaching of Mathematics and the other courses at the school have specific aims and goals defined through the analytical learning programs known as curricula, which are determined by the leadership of the respective ministry responsible for education. In [83], it is mentioned that the teaching is the constituent of three components (a) art, (b) science, and (c) technique. Furthermore, the term science, according to [83], consists of (a) the scientific material that will be taught to students, (b) the psychology of the teacher, and (c) the knowledge of modern pedagogical theories.

According to the above, it is understood that for the proper teaching of the course of Mathematics, certain conditions must be met, mainly on the part of the teachers. Indicatively, some of the necessary prerequisites for teachers are the syllabus of each course that must be taught, the proper preparation of the courses, the knowledge of the students' interests, the knowledge of the students' mental level, the scientific knowledge of teaching methods and the way of building knowledge.

The planning of the teaching of the courses according to [83] should have to follow three stages (a) the planning of the mathematical material to be taught during the school year,

(b) the planning of teaching each module, and (c) the planning of the daily lesson. The above opinion for the planning of the teaching of Mathematics and the lesson's design to be taught finds us consistent. But there are supporters of the contrary view that the planning of teaching binds the teacher, student and deprives the initiative of independent action [83].

For the most effective planning of teaching a course of Mathematics should be taken into account, according to [83]:

a) Students;
b) The conditions of order;
c) Content and purpose of the course of the day;
d) Particular objectives;
e) Strategy;
f) Evaluation teaching; and
g) Work at home;

The indicative strategic steps that could be proposed, the control of the prior knowledge, the connection of this section with previous sections, and the control of understanding of the new knowledge.

In order the teaching to have a result should be followed by the analytical curricula that the coordinate state draws up, announces, and allocates, and clearly, its didactic objectives are clear. In [90] it is considered that the objectives can be developed in five categories (a) intellectual skills, (b) cognitive strategies, (c) verbal information, (d) attitudes, and (e) motor skills. In contrast, in [33] mentioned that the teaching goals could be classified in the three axes (a) cognitive, (b) affective, and (c) psychomotor.

2.2.3 How to teach Mathematics

The basic principles of teaching Knowledge building theory (constructivism) in the learning theory of Mathematical concepts are based on [226] (a) principle of functionality, (b) principle of connection, (c) principle mutant, (d) participation principle, and (e) promotion authority.

As researchers say, [75], [246] learning is built through the experiences that make sense for students. In Mathematics specifically, the student's experiences can be from contact with the environment or contact with various models [226]. In any case, models that simulate real situations and phenomena that are more familiar to students have more opportunities to be accepted. It is no coincidence that fractions are used for the example of pizza (circular disc), which is very familiar and dear to children. With the significant development of ICT, several mathematical models have been developed to understand mathematical concepts in the current era better. Animation plays a significant role and helps to understand key concepts better. In other sciences such as Computer Science, there are complementary views on the visualization of algorithms [44], [51], [154], [286].

In Mathematics, the introduction of mathematical concepts is done both through experiences and the introduction of definitions. Of course, if empirical knowledge is not combined with the definitions, it will be tough to understand the definitions. Understanding mathematical concepts can be done in other ways, such as observation, reading, and practicing. The mathematical concepts affect other factors, such as the student's level of knowledge, student's motivation, the family's environment, and students' spiritual maturity to reach the student at the desired level of understanding. There are vital factors that determine the degree of understanding of mathematical concepts. Finally, the student's encouragement from the teacher plays a crucial role, since many times, students show indifference to a lesson because of the disharmony of his relationship with the teacher.

2.2.4 Ways of understanding mathematical concepts

Mathematics is a difficult lesson that has many abstract meanings. Thus, to enable students to understand abstract mathematical concepts, teachers should link them in some way with daily applications and activities. In other words, teachers must transform the scientific concept into a concept that is understandable to students to approach it. In [226] it is suggested by authors that the teachers: a) to associate the activities of school mathematics textbooks with daily applications; and b) to associate the teaching instruments and models of instruction with mathematical concepts and mathematical symbols.

As mentioned in [110], the teacher's role is essential for connecting the particular to the abstract stage. Usually, for this connection, some models of representation of mathematical concepts are used. The number line is used to represent the natural numbers [109].

Moreover, Μιχαηλίδου in her thesis [187] tries to study whether the educator can use the number line for the concept of fraction in the context of learning fractions (equivalent fractions and addition of fractions) students of the class of the elementary school. That is, she uses the number line as a learning model for fractions.

Of course, nowadays, there is a plethora of supervisory tools, hardware, and computer software, and students can take appropriate guidance from their teachers to understand the difficult mathematical concepts. They can see mathematical functions, graphs, and other mathematical concepts by representing models on the computer in two-dimensional and three-dimensional spaces.

In a research [301] showed that through computers and appropriately designed software increases students' ability to express mathematical concepts, understand the mathematical equations of other classmates, and understand mathematical thinking of others. In another survey [165] showed that an online collaborative learning environment for Mathematics positively influences students' attitudes towards Mathematics.

2.2.5 Didactic/mathematical objectives through taxonomy

As mentioned in [226], all human activities can be classified into three categories (a) the gnoseological sector, (b) the emotional sector, and (c) the psychokinetic sector. These three categories are the classification model of the teaching goals proposed by Bloom [83]. The exact terms used by Bloom and its collaborators were (a) cognitive domain, (b) affective domain, and (c) psychomotor domain [33]. The first sector, the cognitive domain, refers to how students develop knowledge and directly concerns Mathematics. The second sector, the affective domain, concerns developing a value system for reactions to stimuli and emotions that they accept. The third sector, the psychomotor domain, concerns the development of students' mobility skills in the face of everyday life. To identify the objectives of school activities, a classification of learning and pedagogical objectives is needed.

The same applies to the Mathematics field; the teaching/mathematical objectives need to be defined. In the cognitive domain, the teaching goals, according to Bloom, can be classified into six primary stages (a) knowledge, (b) comprehension, (c) application, (d) analysis, (e) synthesis, and (f) evaluation [145]. The revised classification of Bloom [145] substantially replaced with verbs, and now the six stages are (a) remember, (b) understand, (c) create, (d) apply, (e) analyze, and (f) evaluate. The affective domain objectives are classified according to the taxonomy proposed by Krathwohl and his associates in 1964 [146], in five stages (a) receiving, (b) responding, (c) valuing, (d) organization, and (e) characterization by a value or value set. No textbook was published for any taxonomy from the same initial group in the psychomotor domain, but other researchers suggested various classifications [145]. In the classification proposed by Simpson in 1966 [266], the teaching objectives were classified in the following five stages (a) perception, (b) set, (c) guided response, (d) mechanism, and (e) complex overt response. Then added two more stages, adaptation and origination [296], [265], so that the psychomotor domain finally has seven stages.

Apart from the classification of Bloom, other taxonomies of mathematical objectives were developed. One of them is the SOLO taxonomy [32], in which the teacher can understand the level of knowledge of the students in Mathematics and help them appropriately to rise to the next level of knowledge of the particular taxonomy. SOLO taxonomy was proposed by Biggs & Collis in 1982 [32] as a tool for classifying learning outcomes. With this tool, teachers can evaluate the work of students. But students on their side can use it to improve their performance. The SOLO taxonomy consists of five levels which are:

a) Pre structural, it is the stage at which students are owned without knowledge of the subject they are studying;

b) Uni structural, it is the stage to which students have a few knowledge of the subject they are studying;

c) Multi structural, it is the stage to which students with a lot of knowledge in the subject are studying, but they cannot combine them;

d) Relational, this is the stage at which students can understand the subject they are studying and can combine the information between them; and

e) Extended abstract, it is the last and highest stage in which students can combine the information between them and integrate them into a broader whole;

According to [83], Bloom's taxonomy is more helpful in mathematics didactic.

2.2.6 Difficulties in Mathematics

In the preface of his book [83], Εξαρχάκος says that generations and generations of mathematicians and teachers experiment in the classroom, searching anxiously, most of the time blindly, to discover roads their teaching work. Instructors when teaching Mathematics in elementary schools do not feel ready [21], a lot of them are not sufficiently trained [111], have difficulties with fractions [167], [168], [275], have anxiety [209], [219], and think that they will confuse students [209]. This shows that Mathematics is not an easy way for teachers, let alone students who do not have the knowledge, training, and technique to assimilate concepts of Mathematics. Furthermore, the teachers' knowledge of Mathematics influences the students' knowledge in Mathematics [259].

Many students have difficulties in understanding Mathematics and fail in oral and written examinations. Students find it difficult to understand basic terms and abstract concepts, and they cannot reproduce the mathematical concept with ease. The abstract and typical character of Mathematics taught in schools makes it difficult for students, since they are very different from the Mathematics they already know [110]. In other words, they cannot form the mathematical concept and represent it in their imaginations.

Many students have not understood the mathematical symbols they use and do not understand the procedures they use to manage the symbols [110]. It is understood that students cannot connect the pieces of the puzzle, although they may have all the pieces that form it and perhaps know their meaning. Thus, if students are helped in understanding the symbols and recognizing that the solutions of problems should make sense, they will be able to associate the form with understanding to a large extent [110].

Several times students have not understood mathematical terminology and have difficulty in solving problems. The lack of linguistic comprehension is the leading cause of failure of students to solve the problem [226]. Also, students often do not have a clear picture of the mathematical concepts taught to them by the teacher. It should have been a correlation between mathematical concepts.

This difficulty in Mathematics reflects the size of the students involved in the following levels of education. From surveys, it seems that fewer and fewer students in high school have been studying higher Mathematics in recent years [81], [168].

Moreover, students start not interested in Mathematics because they are taught by teachers and their perception of them that they are difficult [186]. During mathematical tasks, a lot of negative emotions have been identified. Some of them are panic, dislike, anxiety, bewilderment, fear, fright, terror, stupidity, frustration, and a fear of looking stupid [45].

The difficulty in Mathematics is often transformed into a fear of Mathematics called arithmophobia or mathematicophobia [225]. The development of this negative emotion of the student against this subject may even lead to extreme situations, such as school leakage of students from schools.

Furthermore, the students have anxiety when they learn or perform Mathematics, which is called mathematics anxiety [209] and is a passive mathematics learning experience [166]. The mathematics anxiety is also accompanied by some adverse physiological reactions (e.g., perspiration of the palms, dry lips, pale face) [166], [209]. Also, mathematics anxiety is related inverse to mathematics achievement [169]. All these difficult situations lead students to failure and to have lower achievement in Mathematics.

In other surveys have shown that during the teaching of Mathematics, students have a concern and feel they will not succeed [244]. This stress that students have on Mathematics leads to low-performance [306]; indeed, as the stress of the student's increases, their performance in Mathematics and motivation is reduced so that they can do things related to the Mathematics. Also, students feel fear and negative attitudes toward the course of Mathematics [34].

In a research carried out, as mentioned in [92], elementary school students do not face the problem, and the teaching contract in Mathematics that they have concluded with their teacher forces them to answer every problem. Of course, following the teaching contract, students find it difficult to cope with problems apart from what they have learned to solve. They have to automate some problems and do not apply their thinking when confronted with different problems.

In another research [116] showed that stress on Mathematics usually is created from the early years of kindergarten and sometimes takes up to twenty years to be eliminated. Sometimes, students' stress comes from teaching Mathematics in elementary school, and a large percentage of students had their first traumatic experience in Mathematics in the 3rd and the 4th grade. Also, the fractions created them unusual stress.

2.2.7 Methods for changing students' attitudes in Mathematics

It is mentioned in [226] that nowadays, what is meant by Mathematics education is quality in learning and not quantity. Besides, students gain a positive attitude towards Mathematics when using cooperative learning, inquiry, and investigation [242]. Indeed, when the activities are creative and do not tire, as is the playful form of the exercises mainly in the small classes, the students' interest is highlighted and shows more significant interest in the exercises [226].

Moreover, as one of the learning motives is mentioned, gamification [277] is also available. However, for playmaking to obtain positive attitudes towards Mathematics, specific rules and educational objectives should be set to make the activities based on gamification. In [144] it has proposed such activities for the development of computational thinking, which is closely related to algorithmic and mathematical thinking. With this guide and appropriate modifications, someone can respond to the Mathematics of such activities.

In another survey [16] showed a significant difference in students' mathematical achievements exposed to strategy games and those exposed to a modified conventional teaching method.

Also, students develop a positive attitude towards math problems when they have successes; they can respond to the difficulties of problems and do not fear continuing failure. Building a positive attitude is very important to start from an early age. The student can love Mathematics and not feel fear and insecurity when the time of Mathematics comes.

Moreover, the use of technology in conjunction with the free initiative of students helps in education. It even improves students' performance in Mathematics, as shown by research on using computers in kiosks with the name "Hole-In-The-Wall" [114].

Finally, in the study [92], teachers should tailor their lessons on Mathematics to break with the teaching contract, and thus achieve the learning of Mathematics.

2.2.8 Multiple representations

Many teachers use only the representation part/whole infractions, resulting in students' inability to understand (misunderstanding) fractions in different situations [260]. To enable students to understand Mathematics, teachers try to use various supervisory and multimedia tools. They use the abacus, the bricks, the ruler, and other supervisory means, but now the teachers use computers, tablets, and smartphones. In recent years, there has been a feast of introducing innovative technological tools that the teacher cannot follow many times, let alone implement a strategy of using these tools in his classroom. Moreover, the general strategies for using technology in the classroom are challenging to define [126].

In addition to understanding the mathematical terms, even the simplest, commonly used different representations. For example, in addition to the third grade of elementary school, a teacher can use the hand's fingers, the abacus, the chopsticks, bean bags, etc. It is understood that the teaching of basic concepts of Mathematics is done with multiple representations. With new technologies, the educator has the ability to use many ready-made programs (if using a computer) and applications (if using tablets and smartphones) to teach basic mathematical concepts. Of course, when the representations that will be used are connected, the transition from a known representation system to another representation system less well known to the student is smoother [126], [127].

2.2.9 Representational models

Teachers advocate using various models for the necessary acts of Mathematics, such as set models, length models or linear models, and surface models [253].

In more detail, the set models consist of independent shapes such as foam cubes, bricks, two-shape counters, etc. (http://mathwire.com/fractions/ fracmodels.html). To make a representation based on this model, the numbers to be processed are presented as sets, as shown in Figure 1.

The length models or linear models usually use the numbering line model, such as rulers, scoops, measures, line with numbers, etc. (http://mathwire. com/fractions/fracmodels.html).

The surface models or area models typically use pizza, colored wheels, patterned blocks, or geoboards (http://mathwire.com/fractions/fracmodels. html).

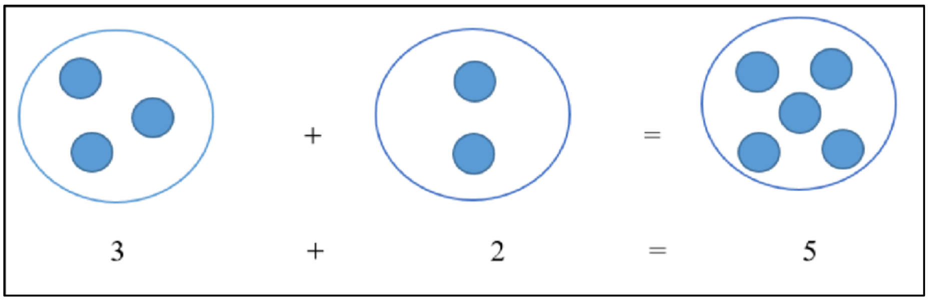

Figure 1: Representation of addition using sets of objects

Basic principles for representational models' construction

Designing a mathematical representation model must meet certain criteria and correspond precisely to some basic rules. According to [253] for the modeling of a mathematical concept, the following questions should be answered:

a) How appropriate is the design model for students' features?

b) How relevant is the model for the student's experiences?

c) How effectively does the planned model depict the mathematical idea?

d) How relevant is the model for the student's future real experiences? and

e) How well can the model be generalized in mathematics so that the student can study it in the future?

Using models in Mathematics in Elementary School

The use of models in Mathematics in elementary school is quite widespread. For integer multiplication in the third grade of elementary school, models of total and surface models [253] are used. The surface models, mainly with tiles, are used to multiply two-digit numbers in the fourth and fifth grades of the elementary school. They can also be used in the fifth grade of elementary school to represent problems with decimal numbers. Additionally, models are used for fractions containing the separation of circular or rectangular areas, such as the use of pie or cake [17].

Utility of surface models

The Area models, of the five questions raised by Schultz in [253], answer mainly the last two questions and are mainly used for future experiences in real life. For example, when someone doesn't know how many cans of paint he needs to paint a particular surface in his house. An even more everyday example is a family in an afternoon outing having to distribute the remaining pieces of pizza equally to the underage family members not to complain. It becomes apparent that through the daily examples of Mathematics, the surface models in fractions can be real supplies to people for their future daily and not only occupations and activities.

Use of geometric surface models in probabilities

Armstrong [17] notes that the numerical solutions that can be given to problems with probability concepts are complicated even for mature students, let alone students of small classes. Thus, he proposes a surface model for probability, for intermediate students that meets the following:

a) To be possible so that it can handle several complex problems of probability;

b) Based on the mathematical skills of students who have already conquered;

c) Be consistent with students' current perceptual ability for probability concepts; and

d) To support possible development of more advanced technical solutions;

It even states that most probability cases include fractions and that a common standard for fractions contains the separation of circular or rectangular areas (pies or cakes). This led to the creation of a geometric model that satisfies the above criteria. The geometric model it proposes, a parallelogram that is considered a unit, is divided into areas so that the regions' region is proportional to the situation's possibilities. To prove that this model has pedagogical and mathematical characteristics, it defines three activities for students (a) marriage by chance, (b) the problem of points, and (c) game archery.

Significance of surface models for dividing

The surface models are very good at generalizing arithmetic issues for divisions, decimal numbers, probabilities, algebra, and Mathematics at a higher level [253]. Moreover, as mentioned in [218] students between the first and the third grade can understand the meaning and symbols of fractions when these ideas are related to regions. Indeed in [253] stated that area models could be used in fraction models and later in equivalent fractions. Also, in [270], it was mentioned that rectangular surface models are better at modeling fractions than circular models.

Schultz [253] concluded in his research that surface models are useful for understanding high level Mathematics in small classrooms, while in larger classrooms the surface models are used to build on previous experiences of students. He also mentioned that the surface model could be used in higher Mathematics, such as calculus. Furthermore, students can benefit from multiple representations, including circular models, in preparing for real-world situations.

Importance of surface models in future life

The variety of representation models, such as models for representing fractions with parallelograms and circles, can help students cope with future situations in their lives [253].

2.3 Fractions

In the field of education and the teaching of Mathematics, students usually encounter difficulties in fractions. This also shows surveys that students in the elementary school find it difficult on Mathematics and specifically in fractions [39], [53], [72].

Indeed, as mentioned in [83], Mathematics taught in the old classical schools limited to basic operations with numbers (addition, subtraction, multiplication), and the division was taught in Universities. This separation of operations shows that the division, represented by fractions, was considered for many centuries to be a difficult concept. The students should first have understood the other three basic operations at school until they study at University and learn the division. As the Behr & Post mentioned in [27], while most 13 and 17 years old students easily add fractions with the same denominator, they find it very difficult to add fractions with different denominators. In particular, 2/3 of students 13 years old and 1/3 of 17 years old students fail. This conclusion shows that students have difficulties in fractions from an early age, do not solve them, and these difficulties follow them in the rest of their student and adult lives.

One of the difficult thematic sections of Mathematics is division [107], and is difficult for many students [55]. Students find it difficult to understand both the concepts and their application algorithms. The different mode of representation of fractions by integers makes it difficult for students to learn new algorithms.

Given the difficulty of understanding fractions, several researchers and scientists suggested different ways of understanding them. In particular, the Behr & Post in [27] proposed for the exact numbers of six interpretations:

a) As a comparison of the part and the whole;

b) As a decimal;

c) As a ratio;

d) As quotient;

e) An operator; and

f) As a measure;

Since the explicit numbers can be represented as fractions, a/b with a, b integer numbers and $b \neq 0$, then these interpretations can be extended to fractions.

The Γαγάτσης et al. [93] follow the proposal of other researchers suggesting five dimensions of the fraction:

a) Fractions as part of the whole;

b) The fraction as a ratio;

c) The fraction as a measure;

d) The fraction as division; and

e) the fraction as a multiplier;

2.3.1 Fraction as part of whole

Presenting the fraction as part of the whole usually, the instructor presents a surface of a geometric shape (usually rectangular or circle). He divides it into equal parts, and he outlines or paints what he wants to choose as part. Another way is when educator have a set of objects (e.g. sticks or marbles) and divide them into equal parts. This form of representation of fractions is more familiar to students since many are the examples of everyday life of the associative student that comes to mind (e.g. pizza divided into equal pieces, chocolate bar divided into pieces).

2.3.2 Fraction as decimal

This representation of the fraction arises as a division of the numerator and the denominator with zero balance. For example, the fraction 3/4 is equivalent to the decimal number 0.75, since if 3 is divided by 4, this decimal number occurs.

2.3.3 Fraction as a ratio

This representation arises when comparing two quantities; for example, there is a class with 7 students. Two of them are girls, and five are boys. Then the ratio of girls in the class is 2:7 or 2/7. Additionally, the ratio of boys in the class is 5:7 or 5/7. Finally, the ratio of girls to boys is 2:5 or 2/5.

2.3.4 Fraction as division/quotient

The fraction can be presented as dividing two numbers, the numerator and the denominator. It is difficult as a concept, since students need to understand the importance of numerator and denominator or as usually called the divisor and dividend. As numerator or dividend is the number to be divided as a denominator or divisor, it will divide into equal portions of the divisible. For example, there is a bag with fifteen candies (dividends) and there are three students (divisor). They want to divide them evenly, then each one will take five candies (quotient), 15/3 = 5, i.e., one-third of the original candies.

2.3.5 Fraction as operator/multiplier

In this form of representation, the fraction is found as part of a multiplication. For example, if there is a multiplication 5 x 2/3, then the result will be (5 x 2)/3 = 10/3. In this case, the original amount (which is 5) is multiplied by the numerator (which is 2), and divided by the denominator (which is 3). In research [297] held in students of the sixth-grade elementary school, it concluded that fractions could be interpreted both as operators in fractions multiplication.

2.3.6 Fraction as a measure

Integers can be represented on the numbering line as distinct points, and the fraction can be represented as a point on it. It isn’t easy to understand the fraction in this representation because it involves an abstract concept, and elementary school pupils are not so familiar with the abstract way of thinking. Also, the teachers seem not to use the number line for comparing fractions [276], but the students also have difficulty using it in fractions [105].

2.3.7 Difficulties in understanding fractions

Fractions are generally a vague concept for students, and it makes them difficult to quite heavily. Students make mistakes about fractions, and they have misunderstandings [2]. Indeed, in a survey conducted by Yang and Tsai [300], the comparison of fractions 8/9 and 11/12 challenging the students in the elementary school class.

Other researchers report that this multifaceted representation of fractions makes it challenging to teach and understand fractions [42], [53]. Most of the time, students have not imprinted some kinds of fraction representation, so they are taught another type of representation to be confused and have difficulty understanding them. They have not fully understood a form of representation and they have to understand another form of representation; they are confused about solving a problem. Also, the many representations and interpretations of fractions are among the many factors that fractions are a difficult topic for students [135]. The fraction 3/4 has five sub-constructs (part/whole, ratio, quotient, measure, and operator) [213]. As a result, students cannot understand that 3/4, 6/8, 0.75, and 0.750 are the same number [98].

One of the reasons that fractions are too tricky for students is their structure and the teaching methods of them [271]. As a result of that, many students can solve division tasks in fractions, but they cannot explain the process of division and why they must invert the second fraction [2].

Also, these difficulties in fractions lie in the fact that students follow "the temptation to deal with fractions in the same manner as natural numbers." [271]. Thus, they use a fraction as two independent natural numbers [30], and as a result, when they have to add 1/2 and 1/3, 30% of students added the numerators and denominators [231].

Some of the systematic errors that students usually do will disappear with the age and instruction level, but some of the themes are remarkably persistent [279].

2.3.8 Misconceptions and mistakes in fractions

Students encounter many difficulties in fractions, often make mistakes, and make misconceptions. Authors of [93] categorize the mistakes and misconceptions of students in the following categories:

a) Mistakes-misconceptions of epistemological nature;

b) Mistakes-misconceptions of conceptual nature;

c) Mistakes-misconceptions of symbolic nature;

d) Mistakes of the students in recognizing the fraction as speech;

e) Mistakes of the students in recognition of the fraction as a measure;

f) Mistakes of the students in the identification of fractions as division; and

g) Mistakes-misunderstandings of a didactic nature;

Students make many mistakes during operations in fractions. Also, during the addition and subtraction of fractions, the students do not find the common divisor [43]. The research division in the study [198] showed that students were trying to reverse the dividend and not the divisor when they were dividing fractions. Also, students encounter difficulties when they have to order a pair of fractions [76]. Besides, the students' prior knowledge of the whole numbers can lead to a misunderstanding of the pupils in the comprehension of fractions and their representation in the number line, since they think they are two numbers [91].

2.3.9 Multiple representations help the learner

It is known that there are different types of learning styles in education, and students learn differently. One way of teaching a subject is useful for students, but other students do not understand it. Also, teachers in order to attract students, they often use different techniques and incorporates differentiated learning methods. Thus, teachers need to know the subject well and use a wide range of tools and techniques to have many representations [303], in order to teach mathematics effectively to students.

It has also been accepted as a good idea to use natural supervisory tools, called physical manipulatives, for teaching Mathematics [127], [157]. However, the manipulatives are generally not acceptable because when applied in practice, they encounter problems such as (a) difficulties in managing order, (b) cognitive difficulties in management, and (c) pressure of computational skills from the program [127]. Of course, suppose the teacher is appropriately scientifically qualified, prepared for the module to be taught, and has the appropriate supervisory tools. In that case, they will undoubtedly have less chance of failing than someone who does not.

On the other hand, as mentioned in [303], many scientists acknowledge and argue that when knowledge is represented in many ways, it helps in the education of Mathematics. The representation can be done with text, shapes, images, numbers, objects, graphs, animations, videos, etc. Multimodal representation is undoubtedly a tool of the instructor in teaching mathematical concepts. With the help of technology and ICT, the instructor now has too many tools to recreate mathematical concepts, such as fractions, with different representations and various electronic devices (tablets, smartphones, and laptops).

A research [276] held in 656 students fourth and fifth grades of elementary schools in the United States of America (USA), it was intended to investigate whether the representations of fractions with a circle, a rectangle, and a number line are related with the knowledge of the students for fractions and vice versa. The research questions were: (a) how students perform parallel problems with knowledge of fractions using graphical representations (circle, rectangle, and number line) as a framework in their written assessments, and (b) what kind of relationships can be found between multiple representations (e.g., circle, rectangle, and number line) in the same concepts of fractions. Research findings have shown that students have the same behavior in fractions when using the circle and rectangle shapes to represent the part/whole problems. But they do not do as well when they have to respond to the same problems using the number line to represent. When problems have to represent a unit when given the appropriate quantities of fractions, they do better with a rectangle's representations. Research findings have also shown that students have difficulty with any graphical representation in difficult fractional concepts, such as improper fractions. What might help them is the use of continuing linear quantities.

Another survey [297] held in students of the sixth-grade elementary school showed that multiple representations in the multiplication of fractions help students understand. Specifically, the students initially used only the number line to multiply integer positive numbers and succeeded. But when they were given an exercise to multiply two fractions, they didn't do so well with the number line. So it took teachers to show them that with a paper and following an algorithm, they could make the multiplication of two fractions. The investigation concluded that the sixth-grade elementary school students could (a) understand multiplication both as an iterative process and as a separation process, and (b) how fractions can be interpreted both as operators and parts of a set.

2.3.10 Usefulness of fractions to students

The knowledge acquired by the students is not static and does not apply only once, just when they have understood it. On the contrary, mathematical concepts can be used in the students' subsequent academic courses. Research findings [261] showed that knowledge of fractions and division with integers from elementary school students determines later the degree of success of students in Mathematics in high school, regardless of other factors such as verbal and nonverbal ability to analyze and solve problems, working memory, family income and the level of education of the family. Also, fractions are essential for various occupations related to physical, biological, and social sciences. For example, it is important to nurses [112] to calculate the dosage of medicines or automotive mechanics [257] to measure some materials.

2.4 ICT in education

Technology has evolved rapidly in recent decades, and new avenues have been opened to exploit it in many scientific fields. Technology is a significant influence in the field of education. The National Council of Teachers of Mathematics of US mentions in its official website (http://www.nctm.org/Standards-and-Positions/Principles-and-Standards/Principles,-Standards,-and-Expectations/) for mathematical education that technology is one of its main pillars and that: "Technology is essential in teaching and learning Mathematics; It influences the Mathematics that is taught and enhances students' learning" [196].

2.4.1 The technology helps in learning

In recent years and especially since 1990, there is a large and massive effort to integrate ICT in the classroom and benefit from students' use. Students use cutting-edge technologies, such as tablets, from kindergarten to university, and through the available applications, either charged or free, to broaden their learning horizon. The tablets can be used for teaching purposes and bring benefits to the teaching-learning process [125]. Other research conducted showed positive results, encouraging thoughts, and better performance [13] with the use of portable devices, smartphones, tablets, etc. Some other surveys were neutral [136] without presenting any negative elements to their use. In Greece, in the last years, individual surveys are done using tablets in education by researchers (i.e., [194]).

The use of new technologies, mainly through computer games, helps students acquire skills in many areas [40]. Additionally, surveys were carried out in different parts of the world, using various game applications for cognitive calculations in Mathematics in both Primary [149], [148], [185], [304], [295] and Secondary Education [314]. Finally, game applications have been developed for students with special needs [129], [215], [292].

Moreover, mentioned in [273]that according to the new curriculum, in the Greek educational system, the familiarization of students with computers is one of the objectives of mathematical education. Thus, the join of Mathematics and Informatics, through the connection of mathematical concepts with representations to the computer, constitutes necessary action of modern education requirements.

Βοσνιάδου [37] reports that learning environments that support ICT are of great interest and increase students' motivation for learning. Of course, to continue being the case and not ignore the students and experience boredom during learning, there should be a constant positive mood from dealing with ICT. This is usually achieved by new innovative activities incorporating image, sound, video, and interaction. A sterile learning environment can initially excite the learner, but it will become an obsolete tool and be abandoned or discredited over time. But if there is constant feedback so that it keeps the student's mood positive, it will become a useful tool, and the student will search for it all the time to use it. Indicative is an example mentioned by [37] that "there was a waiting list for children who wanted to use one of the three computers in their class to solve arithmetic problems during the break, although that meant they wouldn't have any free time to play.".

In another survey [52], it seems that educational technology is essential for improving Mathematics in cognitive areas related to surface and area in the third-grade school students of Taiwan. Another survey [227] applied to students of the fourth grade elementary school in Cyprus showed positive results for Mathematics, attitudes towards Mathematics, and attitudes towards learning Mathematics via computer.

Also, the use of technology can be done in places that most people cannot easily imagine. Specifically, in India, a project called "Hole-In-The-Wall" was created, according to which the schoolyard placed a kiosk with one or two computers for students to use freely. Research conducted on students at a school in India to explore the usefulness of this movement showed that it helps in education and even improves student performance in Mathematics [114].

A research [164] was held in 97 students from eighth to twelfth grade of schools of Australia (respectively second grade of high School and third grade of Lyceum of Greek School). The purpose of the research was the epistemological attitudes for using the Internet in the teaching of Mathematics. The research questions raised were: (a) what students feel when using the Internet to teach Mathematics, and (b) how students consider using the Internet in teaching Mathematics. The survey results showed that the online interactive objects in Mathematics that have been animated and managed better and provide feedback to students could involve and motivate students better than web pages containing only data or information.

In another survey [160] held in potential primary education teachers, the results showed that computers help in teaching and learning Mathematics.

2.4.2 Synchronous and asynchronous education

With the advent of technology in education, new structures and possibilities were created, allowing students to choose courses from a distance with the teacher's simultaneous presence or without. Thus, synchronous education and asynchronous learning were created.

The term synchronous education means access to a course with the simultaneous presence of learners and teachers. That is, there are interactions and feedback in real-time students and teachers. In other words, there is a virtual room accessible to all stakeholders, students, and educators, and by using conference calls, they are all involved in the discussion if and when needed. Besides, the course can be presented and exchange of files through the platform. A free application for modern education is the Big Blue Button, which can be integrated into Moodle type environments.

The main advantages of synchronous education are:

a) Direct connection of new students in the virtual classroom;

b) View of presentations;

c) Sending files;

d) Ability to follow the lecture in real-time; and

e) Participation in discussion during the lecture;

On the other hand, in synchronous education there are some disadvantages such as:

a) A lot of time is required on the part of the teacher for the preparation of the course;

b) Lack of infrastructures for the creation of these classes and simultaneous access of all students;

c) Collaborative learning is not developed; and

d) Continuous Internet connection;

With the development of technology, education incorporated another weapon into its quiver, students' ability to study theory, solve exercises, ask questions without the teacher's simultaneous presence, and follow their learning rhythm. This type of learning is called asynchronous training.

In asynchronous education, students are given the opportunity to access teaching materials that are available continuously throughout the day and the possibility of asynchronous communication with the instructor. In other words, the student can leave a message to the teacher when he has questions about the theory or exercises he wants to resolve. The advantages of asynchronous training are:

a) Continuous access to the material;

b) Remote access to education;

c) Saving time;

d) Participation in discussion with the teacher;

e) Participation in discussion with other students; and

f) Possibility finding the learning rate of a module;

But, apart from the positives offered by asynchronous education, some problems occur due to its nature. Precisely:

a) Inability of the learner to send questions directly to the teacher ;

b) Isolation of the student from his peers;

c) Inability to direct the student's encouragement from the teacher;

d) Continuous connection to the Internet to access the web platform;

2.4.3 Digital platforms and repositories

A few years ago, there was no mass space to find ready-made exercises, suggested teaching approaches, worksheets, and digital tools. As a result, each teacher is forced to prepare exercises, worksheets, lesson plans, digital applications to be better prepared in his class and to be able to respond to the requirements of the order. With the significant increase and spread of the Internet, there were many digital tools, and the situation changed. Now, a lot of digital platforms are provided with animated applications more friendly to users.

Thus, individual, collective, web state-owned spaces were created for teachers to place their creations, and also to have access to these sources other colleagues, parents, and students. So, the trend in recent years in developing applications and putting them in large digital repositories. Digital applications, digital scripts, or lesson plans can be developed by anyone and stored in these repositories to be available throughout the global education community. They are usually independent applications or embedded within web pages, with continuous access.

The following paragraphs will be presented digital repositories that have been developed and accepted by the educational community, both in Greece, in Europe, and the rest of the world and have in their classrooms digital applications related to Mathematics.

Photodentro repository

After invitations from the Ministry of Education, several educational software tools were developed in Greece, distributed to schools via optical discs (CDs) for teachers to use. However, for teaching time to be held in these software tools, the essential prerequisite was that the teacher should have access to computers. In public schools, the right space for computers exists only in the computer lab and, in some cases, in some classes with smart boards. In the computer lab, to intervene, the software should have been installed initially by the computer science teacher and checked if it worked properly.

Other software, developed and accessible through the repository called Photodentro. Using this repository, every teacher can only use a web browser to access the database of these software tools. Of course, a prerequisite to run several software is to initially install the application locally on a computer and then use it to the teacher with his students. Today, in the repository Photodentro, there are almost all the software has been developed with State and European funding, and everyone can access them.

Search by keyword "fractions" and selection of the educational tier "elementary", displays 122 results (the search took place in November 2016), as shown in Figure 2.

From the examples shown when searching in Photodentro, we chose the first one related to this work to present its structure, execution, and usability. The name of the program is called ΚΛΑΣΜΑΤΑ, aimed at elementary school students, used as a teaching approach to exploratory learning, and as a basic didactical objective conceptual knowledge, always according to the data they are listed in the repository.

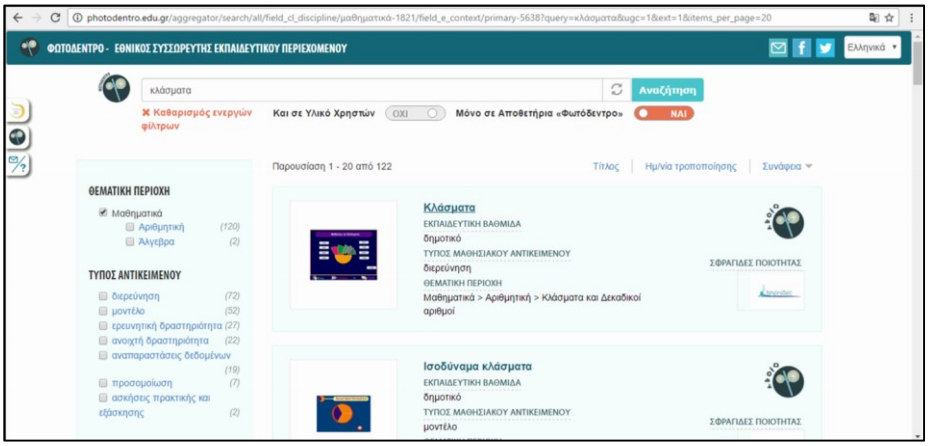

Figure 2: Search by keyword "fractions" in Photodentro

From the study of this example's technical elements, it seems that there is a difficulty running on all web browsers, which is why it is given direction for selecting some specific web browsers. Also, the installation of specific software, MicroWorlds Pro, is needed to reproduce this scenario. There is a difficulty for the students to run this scenario since they need specialized knowledge to install the software. Therefore it is not so very functional for small grades, where the student does not have so much knowledge of ICT and is quite time-consuming in the installation since if someone has does not have the specific web browser, he will have to install it. It can be done to all the computers used for teaching needs, and the whole delay creates concern for the students.

To execute the ΚΛΑΣΜΑΤΑ app, we had to download, following instructions in Photodentro, the MicroWords Pro software, and install it on the computer. The file was large, 128 MB, and it took almost three minutes to download it. After that, the MicroWords Pro program was installed to run the specific application for fractions. Figure 3 shows the result when the application runs.

It does not seem very easy to use in terms of application and does not seem to be incredibly helpful to the learner, since the many buttons on the toolbars can confuse him. The application can also not run on a tablet and certainly needs excellent familiarity with the program MicroWords Pro before using it in the classroom. Indeed it is an application that was good for the time created but not appropriate for the current tablet and the Internet.

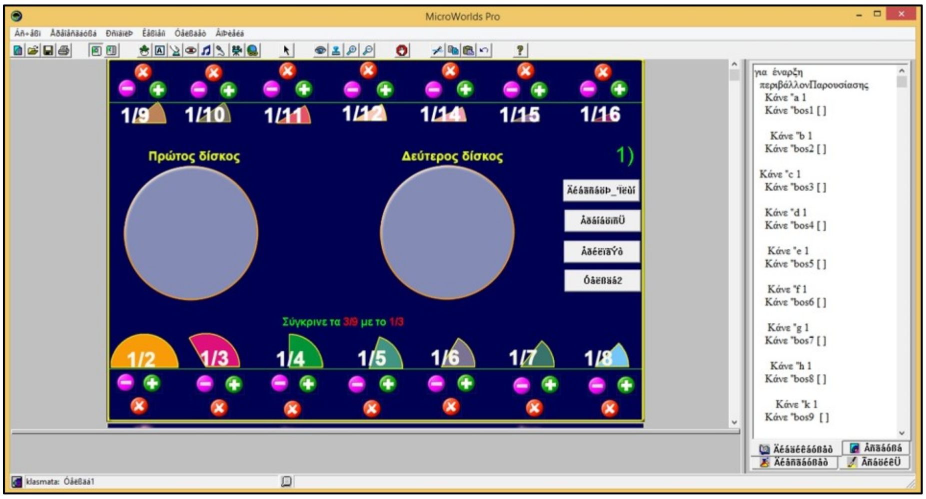

Figure 3: A screenshot of application ΚΛΑΣΜΑΤΑ in Photodentro

Repository Aesop

Aesop is a digital repository of all education levels' digital scenarios and covers subjects from kindergarten to high school. There are 771 digital interactive didactic scenarios (the search took place in November 2016), scientifically and pedagogic certified. Divided into three categories the exemplary (268 scenarios), implemented by scientific committees of experts, the best (331 scenarios), implemented by teachers and are evaluated by two evaluators, and the adequate (172) Implemented by teachers with an average score of 70 to 100 units and are evaluated by two teachers with an average score from 50 to 69.5 units.

In Mathematics, there are 14 exemplary scenarios in secondary education, five exemplary scenarios in primary education. Interestingly, in secondary education, a total of 46 scenarios were deposited from the maximum number of 250, i.e., a complete number of 18.4%. Of these, 17 were optimal, 14 sufficient, and 15 inadequate. In Mathematics for primary school, seven scenarios were submitted at a replacement rate of 3.59%, of which were optimally two, no adequate, and five were inadequate. In elementary school, there is a reference for fractions of only one digital script. This is exemplary, i.e., it has been developed by the scientific support team of the project. There is no application in this repository that can be universally applied to understand fractions and their necessary actions.

Everyone can understand that Mathematics is an intricate part of education, even in primary school teachers, and avoid taking thorough.

The PhET platform

The PhET, acronym of Physics Education Technology, is a platform based on exploring interactive educational simulations of natural sciences and Mathematics. Carl Edwin Wieman developed it at the University of Colorado Boulder in 2002. It provides a repository of educational simulations, open source written in Java, Flash, or HTML 5.

For primary education and Mathematics, there are three simulations titled "Built a Fraction", "Fractions Intro", and "Fraction Matcher".

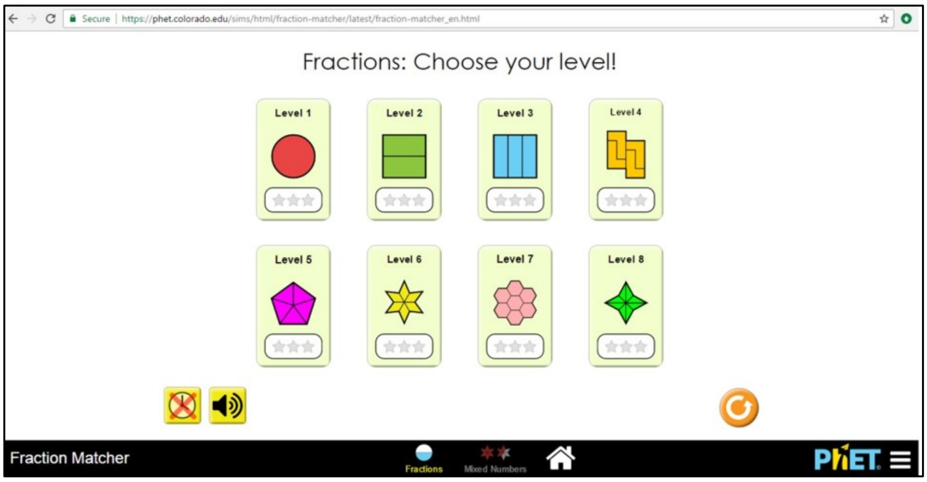

Figure 4: A screenshot of application "Fraction Matcher" of PhET

In this repository, there are a total of 48 simulations (the search took place in

November 2016) and for Mathematics, and especially for fractions, only three. It seems that this difficulty in fractions is facing it in other countries than Greece. From the specific repository we chose the application "Fraction Matcher", as shown in Figure 4, the version for HTML5. This application has several options, selecting layers, time, sound, and repetition. We choose the first level.

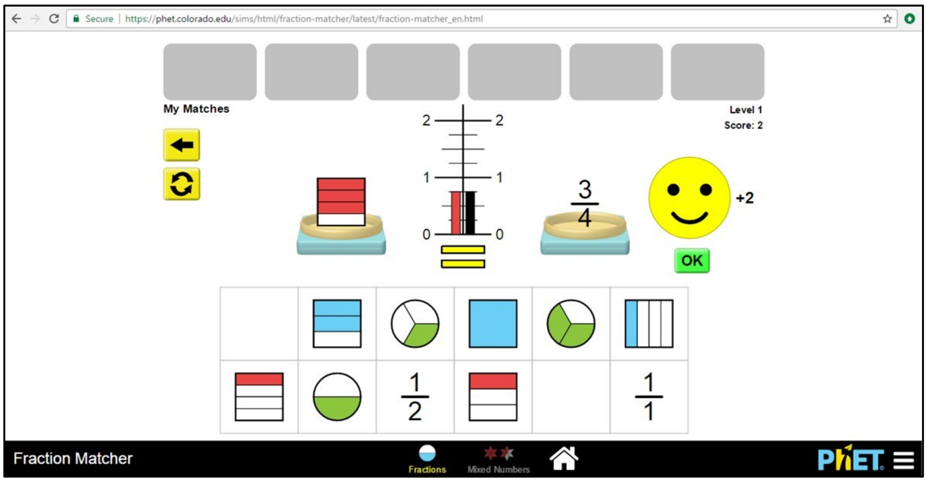

Figure 5: Screenshot of first level choosing level and shape in "Fraction Matcher"

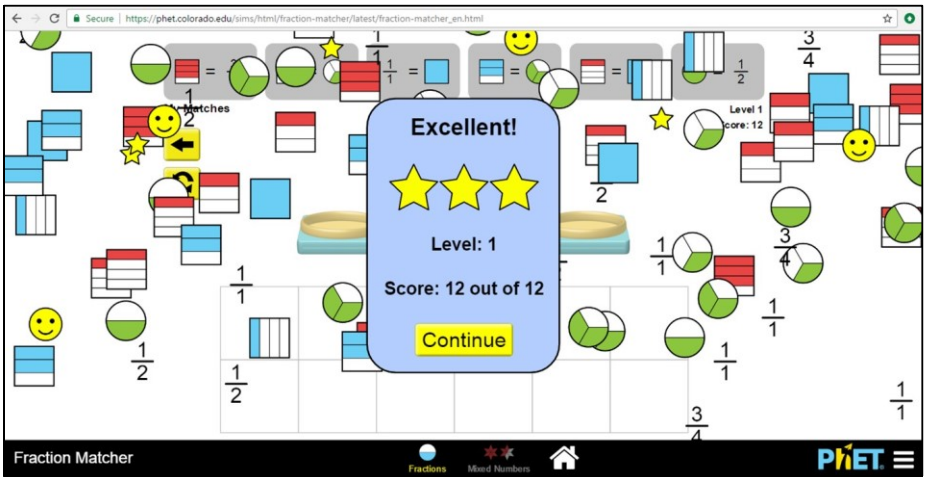

Figure 6: Screenshot when student completes the first level of "Fraction Matcher"

Although there are no instructions on what the student should do, making some improvisations helps an adult understand how the app works. Selecting the first level and making a selection of shapes it shows the following Figure 5. Completing the selection, the student can check if the choice he made was correct. If he was getting both grades corresponding to the specific exercise. The student can choose if he wants to go back and continue to another level or continue until he completes the first level. If he chooses to proceed to the next exercise of the first level, he selects two available shapes again. When student completes the six exercises, then the correct screen appears as in Figure 6. Sounds of success, and they fall by moving the shapes down. It generally creates a good climate for the student to continue with the next level.

In total the specific application does what it says its name, and it compares fractions. It is an interactive, enjoyable application but with limited capabilities and a specific purpose.

The Fraction Lab platform

Fraction Lab is an online platform for fractions developed within Project iTalk2Learn, funded by the European Union (November 2012 - October 2015). It is a system that allows students to understand the meaning of fractions. It is a multidisciplinary program that gathers expertise from machine learning, user modeling, intelligent teaching systems, natural language processing, educational psychology, and Mathematics education.

This application is online, does not require any installation, and runs on any web browser. The first screen displayed is the one shown in Figure 7.

The student can compare two fractions or add and subtract fractions. It can do so by selecting a number line, rectangular rectangles, object sets, and a jug content, as shown in Figure 8.

The student to compare two fractions in the Fractions Lab, he drags and drops any available representation and leaves it at the top of the application's frame. The comparison operator will then automatically come out and show the relationship between the two fractions (smaller, more significant, or equal).

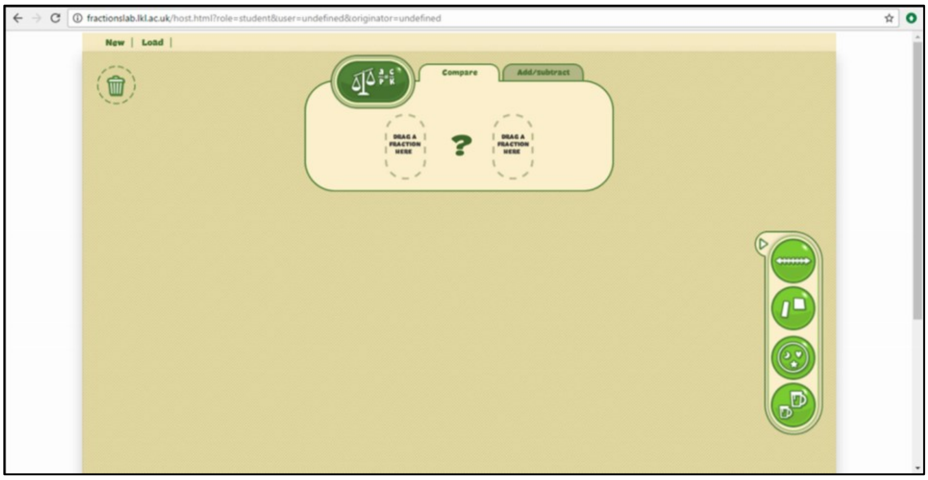

Figure 7: Screenshot of the home screen of the Fraction Lab platform

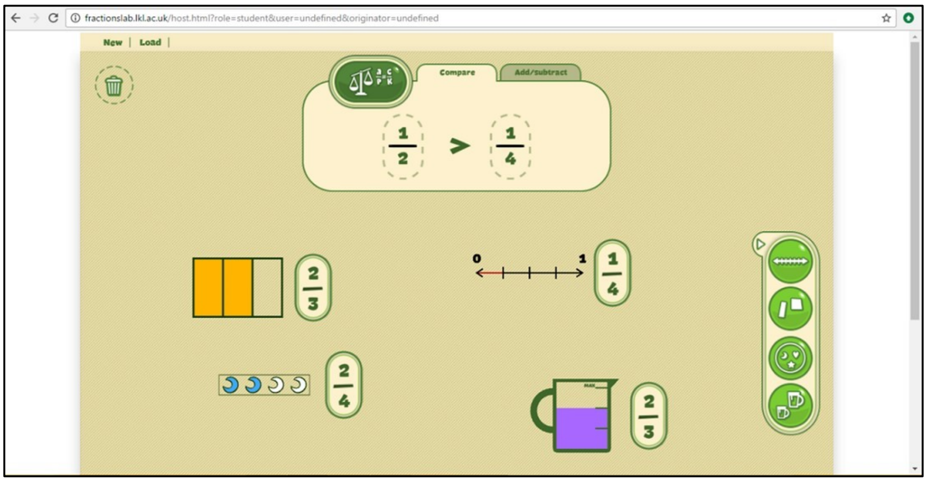

Figure 8: Screenshot of the representations that the application can display

The strong point of the application is that it is online and gives the learner the ability to have multiple representations. However, it does not have instructions, sound, and other operators beyond comparison, addition, and subtraction.

The MathTutor platform

The MathTutor platform is an online interactive system of Mathematics that enables students to engage in fractions, Figure 9 (access to the platform took place in December 2016). It enables students to interact with the system, make mistakes, ask for help from the system, and see the solution to their problems. It enables students to see a series of exercises in fractions.

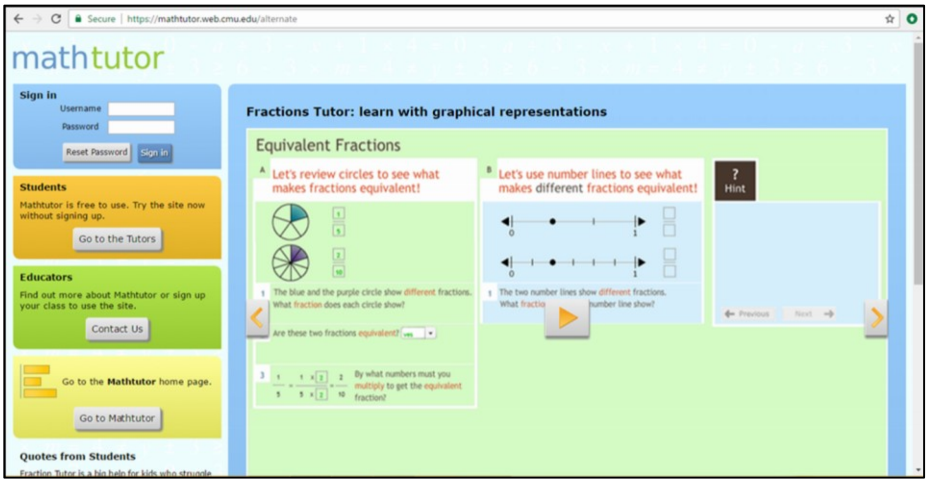

Figure 9: Screenshot of MathTutor platform for fractions

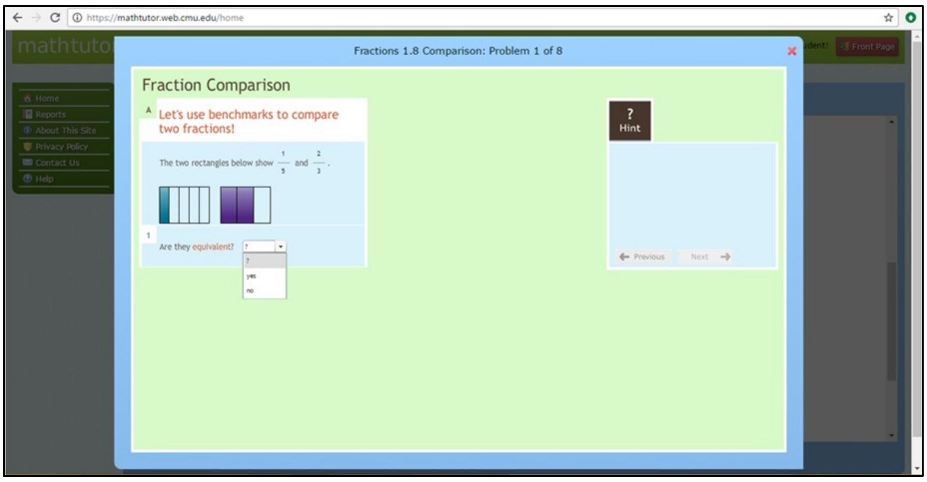

Figure 10: Screenshot of fraction's comparison in MathTutor platform

The student can use it either by acquiring codes so that the teacher can monitor his progress or enter without codes. There is a large selection of activities for fractions. A screenshot of the comparison of fractions in this platform is shown in Figure 10. There are instructions in the text in this application, and there is guidance on precisely what the student should do.

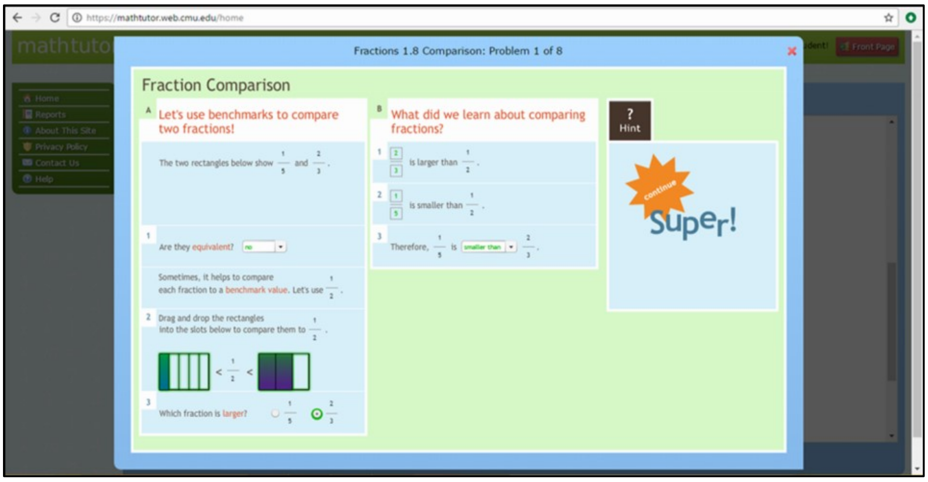

Figure 11: Screenshot when the first exercise in fraction comparison is completed in MathTutor platform

Students can select from pop-up lists his choice and answers the questions. In case he answers wrong a question, then it appears help (hint) that shows him exactly what he has done wrong. It allows him, if he has not understood the help text, to choose another hint or help. So he comes back to the question and answers again. In case his answer is correct, the program shows to the student the second exercise, which is based on the original. This continues until the entire exercise of comparing the two original fractions is completed. If the student responds correctly, then a pleasant message to encourage him appears in the right section that appears the help at a time, as shown in Figure 11. Completing this exercise, the student can proceed to the next exercise, a total of eight exercises to complete the section of the comparison of fractions.

The WolframAlpha platform

The WolframAlpha platform is an online interactive system for many sciences. In Mathematics and especially in fractions, it enables students to engage in all necessary operations (addition, subtraction, multiplication, division) both in the traditional representation of fractions and representation through controls (manipulatives) pies. In Figure 12 (access to the platform took place in September 2019), it appears the representation with pies adding two fractions.

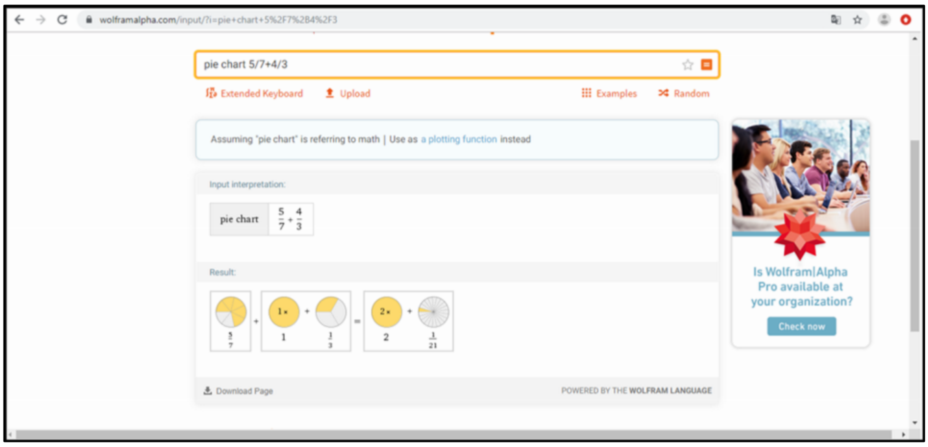

Figure 12: Screenshot of a pie representation of fractions in WolframAlpha platform

2.4.4 Simulations using ICT

Definition

Simulations are imitations of some situations or processes. Simulations mimic phenomena, allowing students to manipulate basic features or variables within a physical or abstract system. Because of their computing capabilities, computers are commonly used to construct real-life simulation situations [122].

Computer simulation is a program that contains a model from a system (physical or artificial) or a process [73]. According to [232] simulation is a method of studying a system and familiarization with its characteristics with the help of another system, which in most cases, is a computer.

Simulation - Emulation difference

At this point, a reference should be made to the distinction of simulation with emulation so that there is no confusion between the two terms. According to [232], emulation is the method of reproducing a system within or through another system similar to the first.

To understand the distinction, there is an example of a simulation of an aircraft. The simulation method creates a mathematical model on the computer that controls all the airplane flight parameters. This model can be entirely through a computer or using all the necessary controls on the airplane, but connected to a computer. On the other hand, there is an airplane cockpit with faithfully the instruments and the simulator cabin with the emulator's actual plane. The simulations are more economical concerning very costly simulations since they will have to construct a similar model with the real one that needs to be studied.

Types of simulations

Computer simulations are divided into two categories [73]: (a) simulations containing conceptual models, and (b) simulations based on operational models.

These two categories of simulations are used for training simulations [121]. The author says that the training simulations were created to facilitate the learning of students and trainees.

Training Simulations

The simulations are not particularly prevalent in education and are not applied as an auxiliary tool while teaching a lesson. The most important reasons that have not been disseminated according to [287] are: (a) they require a lot of time and are costly, (b) many educational simulations are commercial applications and a middle school cannot be purchased because of costs, and (c) many of free simulations that are free are not designed for educational purposes and do not run on web browsers or tablets.

To enable a simulation to become more prevalent in education according to [287] it should: (a) support the creation and reuse of models through open repositories, (b) convert into games, (c) exploit the multimedia capabilities of HTML5, and (d) to be able to run on tablets and other portable devices.

It also seems that when real problems are simulated through an interactive internet platform, they help elementary school students to influence cooperation and discussions with their peers positively and to improve their academic performance [94].

Where simulations used

The simulations are used in a wide range in the context of teaching and education (training). The simulations are also used in business, medicine, flights, and laboratories [122]. Also, they are used in the weather forecast [207], in the automobile industry [158] and in any area imaginable. Simulations are a way of studying a phenomenon, a situation, or presentation of a scientific model with low funding, easy construction usually about the actual situation, and not time-consuming to find a response in many science areas.

Profile of simulations users

The simulation users in order to learn how to solve problems using simulations are required to alter parameters, make decisions, and influence the simulation environment in such a way as to notice the changes they made to their solutions. The case of simulations allows students to interact with a representation of the problem. The form of the simulation will depend on the nature of the activity required to solve the problem. Learners should be directly involved in the problem they are investigating to experiment with the problem factors and immediately see the results of their experiments. Simulations of various kinds of [122] best support this commitment.

When users interact with the simulations by varying the input variables' values and observing the output variables' results, they control their understanding of the problem. These exploratory environments provide the trainees with opportunities to control the causal effects of the factors. Because the trainee (student) rarely has access to the underlying model, the trainees (students) must challenge the model rule through the handling of the environment [122].

Positive elements Problem-solving requires that stakeholders can somehow manage the elements of the problem, handle parameters, make decisions, and influence the environment in such a way as to examine the results of the solutions. Simulation cases provide students with the opportunity to interact with their problem representation [122].

Several people have survived air travel because the pilots had encountered these problems during the simulator training [122]. The apparent advantages of using the simulation are learners' ability to learn through mistakes without harming anyone [122].

One of the positive elements of simulations is that they are not very costly than other methods of studying phenomena or activities. What is needed in particular is the creation of the mathematical model and its use on the computer.

Also, the time to perform a simulation concerning a real phenomenon is much shorter, providing very close results to the actual phenomenon. It is no coincidence that in predicting the weather, the simulations give results many times as accurate as of the real one, helping the state take appropriate precautionary measures.

2.5 Artificial Intelligence

In [188], Mitchell defines the Machine Learning that "A computer program is said to learn from experience E concerning some class of tasks T and performance measure P, if its performance at tasks in T, as measured by P, improves with experience E.".

2.5.1 Designing a learning system

But the machines in order to learn should follow a learning system which is designed with principles and rules, so as not to fail the system. Mitchell in [188] considers that any effort to design a learning system should be followed by some basic steps, as shown below:

1. Which training system it will be selected. Trying to design the training system should determine the following:

   a) The choice of the type of education, whether it is direct or indirect. Mitchell in [188] accepts that "learning from the feedback of direct education is typically easier than learning from indirect feedback";

   b) The extent to which the trainee controls the sequence of training examples; and

   c) How well the distribution of the examples above which the final performance of the P. Mitchell should be measured [188] states that "generally learning is more credible when learning examples follow a breakdown Similar to those of future test examples";

2. Which target function it will be selected. At this stage, it is precisely determined which type of knowledge will be trained and how this will be used for the system's performance.

3. Which representation it will be selected for the target function. This phase selects a representation that the learning program will use to describe the V function learned.

4. Which algorithm of the approach of the function it will be selected. To train the objective V function, a set of training examples is required, each uniquely describing a condition and the value of the function for that state. In this phase, both the function's training values are calculated, and the weights for the trainee couples are determined. A well-known weight calculation algorithm is the LMS (Least Mean Squares) algorithm.

Summing up all the above steps, Mitchell in [188] proposes a final design of the learning system with four distinct modules of the program: (a) the Performance system, (b) the Critic, (c) the Generalized, and (d) The Experiment Generator.

2.5.2 Intelligent Tutoring Systems

An ITS or an experienced teaching system [123] is a software designed to emulate the behavior and guidance of the teacher [138]. These tutoring systems assist the real teacher since they provide personalized assistance for each student. However, in order to happen this there must be some evidence of the student's behavior, i.e., a profile has been created, as does the real teacher takes notes for the students to help them where hobbled. Finally, such a system aims to ascertain which point of the exercise the student made the wrong action. In this way, the system can help the learner providing him instructions, advice, solved examples, videos, etc. In education, the ITS with the tutoring model that adopts leads to individualized and interactive teaching [123].

Each ITS works uniquely, depending on the points that the designers want to give weight. What is certain is that such a system as it works whenever it intervenes. In whatever way it provides, the advice should have exercises designed in several steps to give answers at each step and later detect where the error was made to provide him with proper advice or assistance. Completing the exercises, the student's answers should be recorded in the student's profile whatever they may be (correct or wrong). In this way, depending on the course of understanding and solving the exercise, the intelligent system will be affected.

In any case, intelligent systems have not been created to replace the instructor, and the student can only interact with the machine. Instead, they are designed to provide multi-faceted and complex interaction in the learning process. Also, they can provide personalized assistance at the same time to many students, and even at any time, the student needs it, as long as they have access to the ITS. Finally, the shyness that some students have to raise their hands to ask for help or ask a question to their teacher does not exist in the ITS, since there is no concept of the natural order in other classmates' presence.

The current thesis will be studied ITSs that already exist and are available for free use and how they approach learning through the exercises and activities they provide.

2.5.3 Four critical elements of ITS

The Κόμης in [123] states that the expert teaching systems consist of four components: (a) the specialist, (b) the educator, (c) the interface, and (d) the student model.

Furthermore, the Intelligent Tutoring Systems consist of four elements [203] (as shown in Figure 13):

a) The Domain Model;

b) The Student Model;

c) The Tutoring Model;

d) The Interface.

The Domain Model includes all the material that the trainee should know. It can include theoretical knowledge, basic concepts, techniques, rules as well as problem-solving strategies to be adopted in the field that the learner must acquire. It includes elements of the curriculum of the field that deals with and can be organized in various ways, such as e.g. hierarchy, ontology.

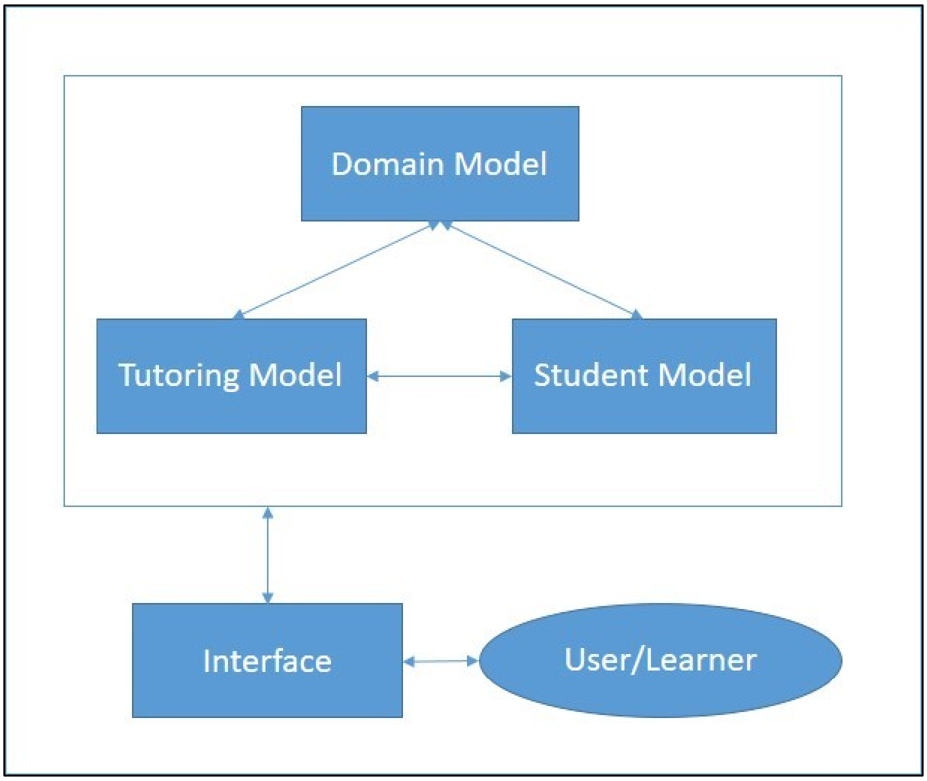

Figure 13: The four-component architecture in an ITS [203]

The Student Model may contain basic information about the student. Specifically, it can contain information about the student's cognitive level in the field of the Intelligent Tutoring System, about his academic progress and his emotional state. It is a dynamic model that helps in the educational process. In addition, the Student Model is considered a key component of ITS and is considered to be the means by which ITS can adapt the learning experience to suit the perceptual needs of the student [256]. Also in [203] is listed six key roles for the Student Model: corrective, processing, strategic, diagnostic, predictive, and evaluative.

The Tutoring Model is responsible for how it will help the student. Specifically, it collaborates with the other two models, Domain and Student, to decide which method it will apply and which strategy it will follow in order to help the student. One of its main functions is the content of the field it deals with as well as the way of presenting the content. It does not constantly intervene, but whenever it deems it necessary and in the most appropriate way.

The Interface element enables the interaction of the system and the learner. It is essentially the medium that connects the learner and the system. Provides the content of the learning environment in many ways (e.g. text, audio, and video) and through different representations (e.g. hyperlinks, simulations). Through this element the learner experiences the learning process and is important in the whole process of operation of ITS.

2.5.4 Affective Tutoring Systems

When asked to respond to teaching or to resolve an exercise, the student's emotional state is a critical factor in the session's outcome concerning teaching a teaching concept, whether it is successful. The teacher can often recognize the student's dominant emotion and use appropriate teaching strategies to achieve his learning and teaching goals. The teacher can recognize the student's emotional state is trying and science through technology to implement it. This is how the Affective Tutoring Systems arose.

Affective Tutoring Systems (ATSs) are the evolution of ITSs, which incorporate the relationship between emotion and learning [222]. They are systems that integrate ICT using sensors or recognize the student's emotions during the session to teach a concept or solve a problem. Then they use this information to achieve the objectives initially set out.

To detect the learner's emotions, questionnaires can be used electronically that the trainee usually completes at the beginning and at the ending of the session. Individual questions may also be displayed during the session to reflect the emotional state of the trainee. Knowing the system, the student's emotions and mood can intervene and help the student change emotional state and have a positive development in the teaching session. Such a system that attempts to change the student's mood to positive using various strategies is the MENTOR system [152].

In addition to the questionnaires and questions posed to the student during the execution of such systems to detect the student's emotional disposition, systems incorporate sensors of various kinds. Depending on the degree of discomfort that may develop in the trainee during the session of the teaching module, they are divided into three categories [222]:

a) Sensors that are not perceived by learners, such as a pressure sensor on the mouse;

b) Sensors that disturb the trainees less, such as the heart rate sensor;

c) In the sensors that cause discomfort to learners such as the camera that records every movement of the learner;

Various systems suggesting the use of sensors to detect the learner's emotional disposition are: AutoTutor [78], Guru Tutor [208], VALERIE [212] etc. Specifically for Mathematics have developed the FERMAT [308], PAT2Math [119], PrimeClimb [14] etc.

2.5.5 Convolutional Neural Networks

Simple Neural Networks have found great application in various technological fields. However, the increasing complexity of connections and neurons has led to their reduction and the development of other types of networks for recognizing image features that used Deep Learning techniques. These networks are called Convolutional Neural Networks.

Convolutional Neural Networks (CNN) are quite similar to the usual Neural Networks and are feed-forward networks. CNN is a special category of Deep Neural Networks (DNN) [251], and it has image data as input. They are mainly used for image recognition and in classification problems.

On a CNN, the input data is converted through the linked layers to a set of class scores at the output level [217]. The basic architecture for a CNN layer consists of three groups of layers (as shown in Figure 14):

a) The Input Layer;

b) Feature-extraction (learning) layers;

c) Classification layers;

The initial level, which is the Input Layer, usually accepts a color image as input. In fact, for each image, the width, height and number of three color channels are determined, one for each RGB color channel.

The second group of layers, which is essentially responsible for extracting features from the input image, consists mainly of a repetitive series of Convolutional and Pooling layers. A Convolutional layer receives the data and transforms it using the Rectified Linear Unit (ReLU) as the activation function.

The third group of layers, are the classification layers, where there is one or more fully connected layers and export class probabilities of scores. In this group the neurons are completely connected to the previous layer and a Fully Connected Neural Network emerges with a two-dimensional output about the number of classes we are interested in scoring.

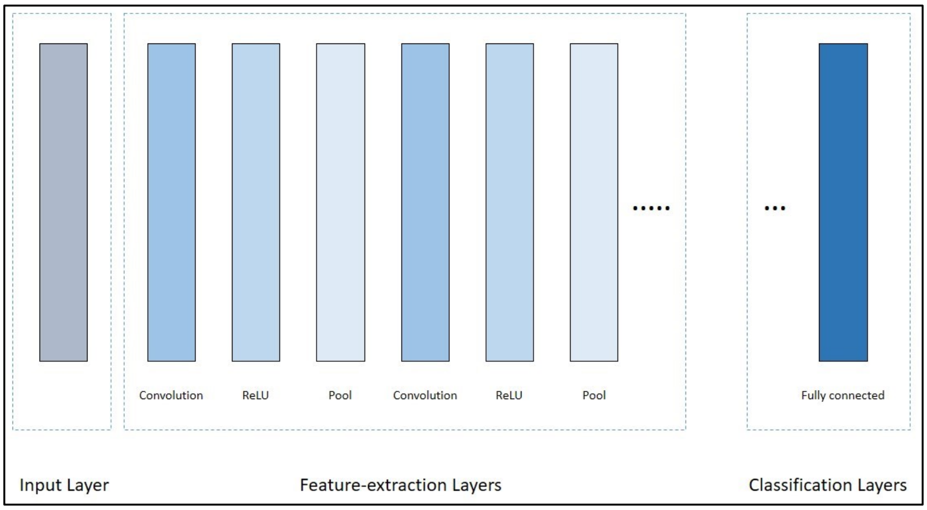

Figure 14: The architecture of a CNN [217]

2.6 Student's emotional state

The process of learning is a timeless question of people's search since ancient times. In every age of human history, there were always teachers who questioned the existing learning process and presented their learning (e.g., Socrates, Plato, Aristotle, etc.). There is no perfect learning system, nor can anyone mention that they found the panacea for learning. One thing is certain: learning involves two parts, the student and the teacher, to transfer knowledge from the teacher to the student. It is like the example of transmitter-receiver in communication.

This interaction between teacher and learner, to achieve the desired result incorporates many parameters, such as the parties' emotional state, the learning environment, their cognitive background, and other essential factors. The student's emotional state, i.e., the emotions that the student experiences during learning, were investigated by many educators of the century that passed us. Piaget is one of those who studied the involvement of emotion in the learning process.

As stated in his doctoral thesis [152], Leontidis for Piaget that in his "studies on students who had a weak cognitive background in mathematics, showed that emotional disturbances affect their performance". Also, Fennema in [87], states that at least two essential training outcomes related to emotional variables are: (a) beliefs, emotions, and emotions about Mathematics, and (b) learning about the Mathematics.

Also, surveys showed that in Mathematics, students are concerned, and they feel that they will not succeed [244], and this leads them to low-performance [306]. To avoid this, teachers should follow specific strategies that help particularly anxious students. Woodard in [294] proposed the following strategies:

a) Creating an environment that inspires trust and tranquility among students;

b) Creating collaborative groups;

c) Giving a second chance at tests;

d) Teaching slowly to understand the teaching;

e) Providing additional courses so that no one stays academically behind;

f) Teacher is always available for the student;

literature review

3

Chapter Three

3.1 Introduction

Students during the learning process of Mathematics have a set of negative emotions, and they have negative attitude for Mathematics [23], [254]. Thus, students are not interested in Mathematics and they try to avoid attending lessons in Mathematics [244]. As a result, students have lower performance in Mathematics, and they fail in mathematical examinations [47]. Also students tend to believe that they cannot succeed to Mathematics. So, having negative emotions to Mathematics students cannot develop the necessary skills to learn in depth Mathematics.

One area of Mathematics that is considered to be very difficult for students is fractions [300]. In addition, fractions are considered to be difficult both in learning and in teaching [153]. In a survey [300] the results carried out show that even simply comparing fractions such as 8/9 and 11/12 is considered a difficult exercise for students [39], [53], [72]. Studies that have been carried out have shown that students face various problems with fractions. Moreover, these difficulties follow them to the other levels of education. In a survey of 1/3 of students failed to add fractions with different denominators [27]. Additional problems with divisions are considered difficult for pupils in primary education [237], [55]. One solution to these problems that students faced with fractions it would be the multiple representations (e.g. visual or graphical) [236]. Also in [303] it is recognized that when knowledge is represented by teachers in many different ways, in contrast to the traditional symbolic notation, it could lead to a better understanding of mathematical concepts. Furthermore, the use of multiple representations for teaching fractions by teachers can help primary school students to understand fractions [276]. In [297] it is presented a study how using multiple representations in fractions multiplication can enhance students' learning.

In conclusion, teachers can use graphical representations as a global educational tool, such as flow-diagrams are used in learning programming concepts, schemas and tree diagrams. The graphical representations of fractions are applied by many software programs, whether they are standalone programs and web-based, or cloud applications located on repositories or e-learning platforms. Web-based applications that show fractions with graphical representations are: (a) the fractions simulations on Physics Education Technology (PhET), (b) Fractions Lab of iTalk2Learn project, and (c) Fractions Tutor platform of the online platform Mathtutor [238], and (d) the WolframAlpha platform.

3.2 Web Platforms with Fractions

3.2.1 PhET

The first web platform is PhET. PhET is a platform based on the exploration of interactive educational simulations of natural sciences and Mathematics. In this repository, there are 51 simulations (data as of March 2018) for primary education. Eight of these are for Mathematics, and only three specifically constructed for learning fractions. An advantage of the PhET repository is the ability to run applications both online and/or locally on a computer.

Programming language

PhET's simulations are written with a combination of programming languages, e.g., Java, Flash [221], and HyperText Marked Language v5 (HTML5). Since many web browsers have abandoned Flash technology, some applications could not be run through web browsers. A basic pre-condition for running the PhET platform simulations is to have the latest plug-ins installed in Flash and Java [221].

Interactivity

PhET's simulations introduce a different level of interactivity at each simulation. The user interaction is encouraged by the Graphical User Interface (GUI) and intuitive controls, using buttons, click-and-drag, slider, and direct response is provided to any user interaction [221]. Of course, the interaction in several simulations could be to a greater extent than already exists.

Open source code

Some simulations on the PhET platform are not open source applications, they are written in Flash that is not open-source software. However, applications are available for installation and running on a computer [221].

Executable on smart devices

The simulations provided by the PhET platform are available as an application for smart devices (tablets and smartphones) with both iOS and Android, and they are not free of charge. Furthermore, these applications cannot be downloaded for smart devices with different operating systems (e.g., Windows Phones).

Run as a standalone application on a computer The PhET simulations can be executed as standalone applications on a computer [221]. However, the latest plug-ins in Flash and Java [221] are required.

Run on well-known web browsers
PhET simulations specifically designed for fractions are mostly supported by any Web browser, because they have been created using HTML5.

Basic operations available
PhET fractions simulations provide limitations on the operations of fractions. In particular, there are few simulations available for fractions. These simulations provide fractions with different objects and the equivalence of the fractions through different representations. However, other basic operations (addition, subtraction, multiplication, division) are not available.

Multiple representations of fractions
The available simulations of the PhET platform allow for multiple representations of fractions. The system uses parallelograms (with horizontal and vertical partitions), pies, cylinders, number lines, and three-dimensional objects to represent fractions.

3.2.2 Fractions Lab

Fractions Lab is an online platform for fractions developed within Project iTalk2Learn, funded by the European Union (November 2012 - October 2015). It is a system that allows students to understand the meaning of fractions. It is an interdisciplinary platform that brings together expertise from mechanical learning, user modeling, intelligent teaching systems, natural language processing, educational psychology, and math education.

Programming language
Fractions Lab is part of the iTalk2Learn, which is an online platform [101], [102]. The decision to build an online Web platform has been made for the reasons described by Fernandez and Gutierrez-Santos [101] that it is more secure, requires no installation or further maintenance, and is maintained until the last version without any extra intervention. The user interface is based primarily on HTML5 and Flash. Additional components to create the user interface are using Javascript, Twitter Bootstrap, GWT, Apache Tiles, Thymelaf, Jquery Ajax, External Interface, Spring MVC [101], [102]. Other technologies such as Java Persistence API (JPA), Hibernate (ORM), MariaDB databases, etc. are used in other components of the platform [101], [102], [117].

Interactivity
In the Fractions Lab architecture, there is a presentation level that follows Model-View-Controller (MVC), as reported in [101], [102]. Specifically, each viewing (view) requests the model's information to producing an output representation to the user. In contrast, the model notifies the relevant sightings and controllers when a change in status [101], [102].

Open source code
The Fractions Lab is open source, as reported on its official website, and the platform's source code can be retrieved from http://www.italk2learn.eu/italk2learnplatform/.

Executable on smart devices
Fractions Lab has some limitations because it is mostly developed on Flash technology, such as tablet devices [101], [102]. Avoiding Flash technology could not be avoided because (a) many of the components of the iTalk2Learn platform are designed in Flash, and (b) HTML5 technology is not as developed as Flash technology for some specific applications [101], [102]. Attempting to use the Fractions Lab on devices with iOS, it has been determined that these limitations exist and the application cannot be viewed normally. Also, attempting to be used through mobile with android software, a diagnostic message appeared that Unity WebGL technology is not yet supported [5].

Run as a standalone application on a computer
Fractions Lab does not allow the application to run independently on a computer, but the iTalk2Learn code enables it to install this platform and use it online. The platform code can be retrieved from http://www.italk2learn.eu/italk2learn-platform/.

Run on well-known web browsers
Fractions Lab, as stated on its official website, is compatible with Chrome and Mozilla Web browsers.

Basic operations
Fractions Lab can perform two of the four basic operations of fractions (addition, subtraction) and comparison of fractions [101], [102], [117]. It can also perform the addition, subtraction, and comparison with all four representation shapes using the corresponding buttons available on the platform. Besides, there are the tools (a) equivalence, which provides feedback if two fractions are equivalent, (b) the partitioning tool, dividing a representation, and seeing the result of the numerator and the denominator, and (c) plotting; how two fractions are added together [106].

Multiple representations of fractions
The Fractions Lab uses four representations. Three of them (area, number line, and set of objects) are among the most essential [79]. Also, the liquid measurement is used as a fourth

representation (using a pot/jug) [264]. Overall, Fractions Lab uses various fractions representations that include continuous and distinct fractions, and fractions in one, two, or three dimensions (number line, area, and liquid measurement, respectively) [106].

3.2.3 Fractions Tutor

The Mathtutor platform is an interactive on-line math platform that encourages students to take part in various lessons involving fractions. It allows students to interact with the system through a series of exercises for fractions, make mistakes, use the help facility from the system, and find solutions to their problems. The fractions are provided through the Mathtutor platform, on a free, dedicated website named Fractions Tutor [238]. A password is provided by the teacher that facilitates the teaching process to students who use the system; this way, teachers can monitor and track each student's progress. There is a wide selection of activities for fractions. The Fractions Tutor mainly consists of three units: (a) Fractions Tutor - Unit 1, (b) Fractions Tutor - Unit 2, and (c) Fractions Tutor - Unit 3. There are many concepts in each section (create fractions, compare fractions, add fractions, subtract fractions) and a wide range of exercises.

Programming language

Mathtutor has been implemented using various programming tools. In particular, CTAT (Cognitive Tutor Authoring Tools) has been used to write the key components on both the client and server-side and Flash for the construction of the graphical user interface [5]. Fractions Tutor is designed with CTAT [238].

Interactivity

The Fractions Tutor has interactivity for graphical representations of fractions [238]. It is particularly given the ability to move objects with the known method drag & drop, the possibility of typing numbers, selecting from the drop-down list, and other options as they appear when performing exercises on the Fractions Tutor environment.

Open source code

Fractions Tutor contains key components written in Adobe Flash [5], which are not open-source code.

Executable on smart devices

There is not any available application of Fractions Tutor for smart devices (tablets and smartphones). It could run through a web-browser in a smart device but with some limitations since some of the key components are written in Adobe Flash.

Run as a standalone application on a computer
The Fractions Tutor's exercises and tasks are designed to run over the web and cannot be run as a standalone application on a computer.

Run on well-known web browsers
Fractions Tutor is compatible with Chrome and Mozilla Web browsers, and generally browsers that can run Flash applications. Of course, Adobe Flash Player is a requirement for the application to run successfully.

Basic operations
The Fractions Tutor covers a wide range of operations in the fractions and is reported in detail [238]. The most basic of the operations covered are the equivalent fractions, the comparison of fractions, and the addition of fractions with a sum greater than or equal to the unit [238]. After visiting the official website of Fractions Tutor, it was concluded that fractions abstraction exercises are also covered.

Multiple representations of fractions
Fractions visualized in Fractions Tutor are represented in various ways. In particular, they are represented as a pies, number lines, and as set of objects [238]. Fractions Tutor exercises and tasks show that the parallelogram can also be used as a representation.

3.2.4 The WolframAlpha platform

The WolframAlpha platform is an on-line interactive platform for many cognitive subjects. Specifically, it is a computer knowledge machine that generates real-time question and answers by performing calculations on its own vast internal knowledge base (https://www.wolframalpha.com/educators). In the field of Mathematics and especially in fractions, it enables students to deal with all the necessary operations (addition, subtraction, multiplication, division) both in the classical way of representation of fractions but also with the use of representations through manipulatives, such as pies.

Programming language
The WolframAlpha is written in Wolfram Language (https://en.wikipedia.org/wiki/WolframAlpha) which is a general multiparadigm computational language. It emphasizes symbolic computation, functional programming, rule-based programming, and can employ arbitrary structures and data (https://en.wikipedia.org/wiki/Wolfram_Language).

Interactivity
There is user control over the initial construction of the fractions by typing the corresponding fractions, but then it cannot intervene and modify the graph. There is feedback, enabling the

student to see step by step the execution of the actions and giving him instructions for solving the exercises.

Open source code
The WolframAlpha platform is not an open-source code.

Executable on smart devices
The WolframAlpha platform can run in smart devices. There is an application that someone can pay and download it from the appropriate web store, and then he can use it in a smart device.

Run as a standalone application on a computer
The WolframAlpha is designed to run in a web browser. Thus it cannot run as a standalone application on a computer.

Run on well-known web browsers
The WolframAlpha can run on well-known browsers without any particular problem. Furthermore, this platform is designed for the use of the Web.

Basic operations
The student can perform all four basic operations of fractions, i.e. addition, subtraction, multiplication and division. Also, the student can ask a question for the fractions' comparison by asking a real question, e.g. "Is 4/5 bigger than 3/5?".

Multiple representations of fractions
The fractions in WolframAlpha can be presented both with the traditional way of representing the fractions (e.g. 5/7), but also with the use of representations through manipulatives such as pies or number lines.

3.3 Affective Tutoring Systems dealing with Mathematics

3.3.1 Pat2Math

PAT2Math (Personal Affective Tutor to Math) is a web-based system that considers students' emotions by employing both system interface and facial expressions recognition [119].

PAT2Math consists of multiple agents, i.e., (a) Agent Interface, (b) Tutor Agent, (c) Agent Student Model, and (d) Agent Base Domain. PATequation tool is a Java applet, integrated to PAT2Math, an editor with a powerful expert system, assisting students on a step-by-step solution of 1^{st} and 2^{nd} degree equations. PATequation has been implemented using Drools 4.0, an ES shell implemented in Java. PAT2Math functionalities include instant response, hint, next step solution, and problem-solving [119].

To determine students' goals, Pat employs the MSLQ Questionnaire [118], which is presented during their first log in into the educational PAT2Math system. After completing the MSLQ, the next step is to determine the possible events that may occur within the educational environment to be aware of each one of students' desirability. Those events may be a student of agent action. For example, a student action could be "The student did not give the correct answer", while a system action "Pat did not give the appropriate assistance". In that manner, by knowing the event taken place and the students' goals, Pat can determine students' emotions. For a student who focuses on performance, the event "The student did not complete the task correctly or did not complete it at all" describes a negative fact, which causes the emotions of anger and distress. All the above are analytically described in [118].

The system can recognize seven emotions: joy, distress, satisfaction, disappointment, gratitude, anger, and shame, according to the Orthony, Clore, and Collins (OCC) model [118]. Those emotions arise from the students' actions on the system interface [119].

The tutor agent is a female cartoon named Pat. In Figure 15 there is Pat agent. It consists of two modules, i.e., the mind module and the body module, implemented in Java [118]. Being aware of each action in the tutoring environment and considering the goals set by each student, Pat can determine the emotional stage of a student. Each time a tutoring event occurs, the tutor agent concludes the student's emotional state and chooses the appropriate strategy to implement [118].

Pat has many available strategies, each of which is applied to each appropriate occasion. An example is described in [118], of a female student focused on performance, who feels incapable of completing a task if a task fails. Most of the time, she does not try harder when she faces difficulties, as she considers herself incapable. Pat responds with a message aiming at increasing the student's self-esteem and encourage her to perform the task with more effort.

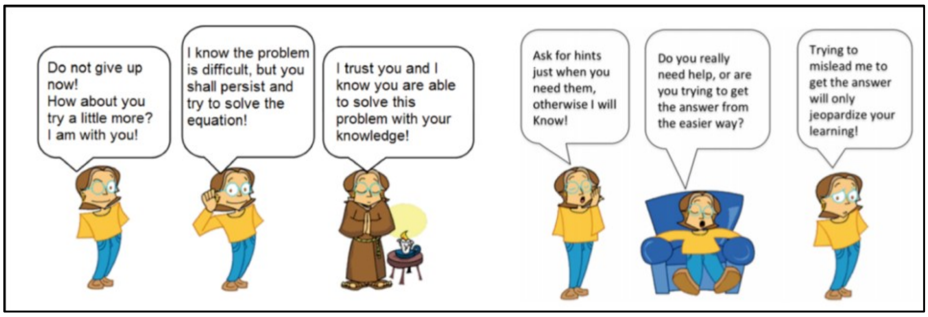

Figure 15: Pat agent showing that it is aware the student is gaming and learning (translated from original messages in Portuguese) [204]

The system's novelty is that it employs multiple agents, and each of its components can act independently, while at the same time actively cooperating with the rest of the components. The main reason behind the implementation strategy is the lack of any other open-source ITS on algebra. The authors aimed to: (a) to create and make available to every single school in Brazil, free of charge, and (b) develop an ITS which follows the formal curriculum of the Department of Education in Brazil [119]. Other studies have shown a lack of algebra for Brazilian students, which was the initial reason pushing towards implementing this system.

A vital feature of this system is that it focuses on algebra's cognitive field, especially on the field of linear and square equations, and that is specifically designed for elementary school students [182].

The tutor agent was evaluated in two ways [118] (a) with students, and (b) with specialists. Pat has integrated into the JADE system for the student evaluation, a simplistic tutoring environment, implemented in Java. There were three evaluation methods (a) JADE only, (b) JADE and Pat, and (c) JADE and Affective PAT. A total of 39 students between 12 and 19 years old participated in the experiment to evaluate Earth Time Zones. Eight tutors and psychologists participated in the evaluation of the students. The research came to the following conclusions: (a) although the tutoring environment with the assistance of Pat improved the students' performance, it is not evident whether the emotional agent works better compared to a tutoring environment with a non-emotional agent, and (b) regarding the specialists, the agent's existence and its tactics were very accurately designed.

In [120], two evaluations on the proposed emotional model on students are presented. The first evaluation (*direct* approach) focuses on measuring the precision of the emotional model. It was conducted on 24 students between the ages of 12 and 19 of the seventh grade in a Brazilian school. JADE, an on-line tutoring environment, was employed on the educational content section of Earth Time Zones. The results of this evaluation were not encouraging. Thus two proposals were suggested towards the improvement of the system: (a) the existence of a pop-up dialogue window, and (b) present the emotion of each student in a simple, understandable way. The second evaluation (*indirect* approach), which examines the effectiveness of an application employing the emotional model to adapt to the user, was performed on two groups of 13 students each. Results show that the model can provide some useful information on student's emotions. For the model evaluation, a self-referencing mechanism for each student was applied (using a simple pop-up dialogue window); students could record their emotions about the system in this window.

In [119], the PATequation tool is evaluated through research performed on 43 students (in a private Brazilian school) between the ages of 12 and 13. Students were divided into two groups; the first group consisted of 22 students who solved equations using PATequation, while the second consisted of 21 students, which solved equations without any assistance. Results showed that there is a strong indication that students' performance was higher when using PATequation. The observed improvement on students with the assistance of PATequation to solve equations was of high statistical importance.

Authors in [130] evaluated the effect of the meta-cognitive skill of monitoring knowledge through research among 107 7th grade students of 4 private schools in Brazil, aged between 12 and 14 years old. To perform the evaluation, researchers used a pre-test, a post-test and grades of meta-cognitive self-evaluation. Results confirmed the initial hypothesis that the education provided by the tutoring agent improved the students' ability to keep track of their knowledge. Also, there are several indications that the agent's meta-cognitive guidance improves their test results. Moreover, a few clues were found on a stronger correlation between the ability to monitor their knowledge and their learning capabilities, among the students assisted by the tutoring agent.

3.3.2 Fermat

Fermat [46] is an intelligent social network for teaching Mathematics, incorporating an ITS for teaching and learning. Fermat's architecture was originally presented in [46], and evidence of the implementation and development stages are presented in [308] and [25]. The Fermat system was developed to solve the main problems of Mathematics for primary education in Mexico since a substantial percentage of students who had inadequate or elementary level [23].

The system includes three basic units: (a) Expert, later renamed Domain [307], (b) Student, and (c) Tutor, which also has an affective sub-unit. A typical student profile contains: a) a static profile with personal and academic information, and b) a dynamic profile that contains information through the interaction with the system and recognition of the student's emotions [46]. The cognitive characteristics are derived from the student's background and the completed problems and tests. Finally, the emotional moods are detected using a camera, and a microphone [46].

To determine the student's cognitive level, as well as the proposed teaching method, a diagnostic test is initially performed [46]. The diagnostic test shows what student knows and also what to learn [23]. The emotional states which recognized by Fermat were initially considering seven emotion types (anger, disgust, fear, happiness, sadness, surprise and neutral) [46], and finally adapted to recognize five types of emotions (anger, happiness, sadness, surprise and neutral) [307]. A detailed description for extracting facial features and recognizing emotional status through efficient techniques is described in [308]. In [309], an analysis is made on how to recognize the learning style. It is worth noting that Kohonen neural networks and fuzzy logic are used to read the emotional state based on data that combine information from camera and microphone [46]. Reading the cognitive state was based mainly on ACT-R cognitive theory, creating rules and facts with XML rules [25], and fuzzy logic. The cognitive state also determines the next problem that the student will solve [46].

In [310], authors describe how neural networks can be implemented in a system to recognize learning styles and emotions. The proposed system combines a Self-Organizing Map (SOM) or Kohonen network to detect learning styles and a neural network for detecting the emotional state. Also, the authors state that fuzzy logic can be utilized to characterize the pedagogical state of a student. The suggested fuzzy system defines a set of variables to characterize errors, time, emotion, help, and learning style. These variables are loaded when the student solves an exercise. The output of the fuzzy system gives a variable that is the type and degree of difficulty of the next exercise that the student will solve. Besides, the type of exercise is determined according to the student's learning style.

Fermat also incorporates a pedagogical agent. In particular, the Tutor unit presents a pedagogical agent, the Microsoft Genie character of a character that makes some concrete actions (congratulate, confused, explain, and suggest) [46]. Additional actions were added, and finally, the agent reacts with the following eight actions (acknowledge, announce, congratulation, confused, get attention, explain, suggest, and think) [23]. When solving problems, there is a help button that sends tips or ideas through the agent [24]. The agent appears to the student and presenting different actions (pay attention, explain, suggest, or think actions). When a student successfully solves a problem, the agent reacts by acknowledge or congratulate action, or when the student asks for help (announce or confuse action).

An important feature of Fermat [46] is that it is a smart ATS social network for teaching Mathematics that presents the content of the course in a personalized way. Also, the cognitive field of the system is Mathematics, multiplication, division, and also fraction issues. However, the act of multiplication [310] and division [307] have been evaluated. Fermat has been applied to third grade elementary school students in multiplication [46] and natural numbers [22], and elementary school students in multiplication and division [307]. Additionally, it is addressed to elementary school students (second grade) for learning Mathematics [47]. The system was applied to some groups of students. It was initially applied to 25 students of the third grade class of elementary school examined in multiplication [46]. The process of assessing students was done by administering a pre-test, intervening using Fermat and providing a post-test. Also, another survey [23] in a sample of 72 students in the third grade elementary school was examined in multiplication. After some modifications and additions were made to Fermat system and almost ended in its final form, there was again intervention in 33 elementary students [307] in the acts of multiplication and division.

In both cases, the evaluation of the system showed that students with a lower cognitive level in Mathematics could see an improved performance [46], [307].

Additionally, the system develops various modules for second grade elementary school students learning Mathematics [47]. Lastly, several modules were developed in the field of natural numbers, specifically for third grade of elementary school taught in Mexico. Still, this version of Fermat has not yet been tested in students [22].

3.3.3 Easy with Eve

The Easy with Eve is an ATS for teaching Mathematics suitable for elementary school students [248]. Initially, the system was designed as an automated face expression analysis system [7], [249] and gesture analysis [7], but it ultimately uses real-time face expression analysis [250] for Mathematics.

In general, the architecture of Easy with Eve consists of the following components: (a) the facial expression analysis unit, (b) the student's model, (c) the teaching strategy unit, and (d) a set of teaching activities in the form of an animated pedagogical agent.

The Easy with Eve initially recognized seven emotions (surprise, happiness, sadness, puzzlement, disgust, anger and neutral) [249], [247], but eventually presented to recognize eight emotional states (neutral, smiling, laughing, surprised, angry, fearful, sad and disgusted) [250]. A web-camera [6], [7], [249], [247] and a system consisting of (a) an artificial neural network for face detection, (b) the extraction of facial features, and (c) the fuzzy face-expression classifier to detect student's emotional state [247].

The article [249] describes that an artificial neural network is being used to identify the person who is trained. The detection of facial expression of the student is achieved by using fuzzy rules for eyebrows, eyes and mouth. Facial expressions using a SVM method determine the student's emotional state. Emotions are recognized in real-time [250]. Also, the face expression analysis system runs in the background as the student interacts with the system.

There is a female pedagogical agent [6], [7], which called Eve [9], [247]. The way of operation, the actions, the expressions and what exactly Eve can do is described in [9], [8], [247], [250]. As described in [7], the pedagogical agent will be adapted in two ways: (a) by presenting the most appropriate material, and b) by empathizing with the student and providing oral and no-verbal encouragement to the student. Thus, the agent will be particularly beneficial in two ways: (a) if the most appropriate material for the student's emotional state is presented, then this will facilitate the learning process, and (b) if the agent is faithfully honest in her empathy, they could enhance the benefits of the motivation of personal impact. It seems in the article [250] that Eve can express a variety of emotions through facial expressions and can give a positive or neutral answer, ask questions, discuss problems or solutions, give advice or answer her questions. The strategies to be followed by the pedagogical agent are the result of coding of the study of videotaped teaching of real pedagogues to students aged 8-9 [7], [10], [247], [248]. These video recordings have shown that the neutral behavior of teachers and students is the most frequent, followed by the low-intensity students' smiling expression and then high intensity [9], [10], [247], [248]. The implementation of these strategies on Eve's part is fuzzy approach [9].

Article [250] introduces Easy with Eve, as a pioneering ATS that bases its teaching strategies on pedagogical actions of real-world pedagogues. There has been a study of observing human pedagogues, how they teach primary school students, and its results have been adopted in the pedagogical strategies of Easy with Eve. It is aimed at primary school students, mainly 8-9 years old [6], [248], [249].

The cognitive field that Easy with Eve deals is Mathematics. It was originally designed for basic operations also, subtraction, multiplication and division [249]. More specifically, it implements the seven first level of Mathematics strategy of New Zealand Numeracy Project and does not deal with fractions, percentages and proportions [8]. Easy with Eve mainly deals with addition problems [6], [7], [248].

For evaluating the Easy with Eve system, a survey was conducted [250] with 62 students aged 8-9 years. There was statistical evidence of a significant increase in pre-test and post-test results, and thus a positive effect on student's performance using the system.

3.3.4 Wallis

WaLLiS is an on-line interactive and intelligent learning environment specifically designed for Mathematics [176]. Its on-line environment offers material such as theoretical notes or examples presenting each appropriate math section implemented in the system. Its environment consists of a set of interactive web pages which offer easy interaction over various mathematical problems [178]. Moreover, the system combines content-based approaches with adaptive learning strategies excluded from the ITS [178]. The WaLLiS environment offers a dynamic content map, whereas a dynamic website provides access to theoretical notes or the exercise.

WaLLiS is designed based on a client-server model, consisting of the distinctive elements for the client and the server, which are responsible for presenting the content to the student, to adapt the content, and provide feedback [178]. Moreover, the feedback mechanism is designed in such a way as not to disturb the student from executing a task [178]. WaLLiS includes a student model, which keeps a history of the websites visited by each student. There is also a module that reports personal information of a student. Another distinct feature is that the content is adjusted to the student's specific features (color, size, etc.), as well as, on student's profile (e.g. grades). In contrast, it is not adjusted to the different educational styles or learning scenarios, as other adaptive systems do.

The goals determination mechanism is executed through the feedback mechanism, from the creator of the specific activity [176]. The author in [178] mentions that the emotional states of frustration, boredom, confusion are recognized, and all positive emotions such as satisfaction, joy, happiness etc. are categorized in a single group. In [222] evidence is presented that the WaLLiS system can recognize the following emotions: frustration, confusion, and mood (e.g., confident, happy, and enthusiastic). Also, emotions can be recorded by analyzing system log files [229]. Emotion detection is performed through a forecasting model based on the J4.8 algorithm; an induction tree based on an improved version of the decision tree algorithm C4.5 [229].

From the other hand, the system does not provide any tutoring agent, elaborate on the fact that the benefits of having such an agent are minimal [176]. On the contrary, the system records all information related to the activities performed by the student using a Javascript agent. The information recorded relates to student's actions, including mouse movements, mouse clicks, keyboard clicks etc. Those data are recorded to the server, as depicted in Figure 16. For information regarding the agent, the reader is referred to as [173].

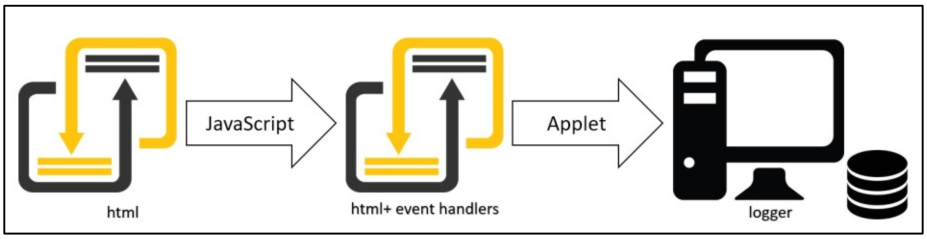

Figure 16: The architecture of the agent recording a student's actions

The system also records the goals set for each student, the feedback given to the student by the system and the pre and post results of the student's evaluation. Besides, the system provides scaffolding assistance and relevant suggestions regarding the student's misunderstandings on mathematical issues, according to predefined rules [175]. The strategy regarding the adaptation of the emotional state is implemented through the feedback mechanism provided by the system, along with the relative suggestions, updates and proposals it offers.

The essential characteristic of the WaLLiS system is the feedback mechanism, which provides feedback and suggestions to the student [178]. The feedback mechanism is inspired by cognitive acquisition theories (such as ACT-R), cognitive scaffolding theories, and follows similar approaches to those employed by the CMU cognitive tutors [178].

The feedback mechanism relies on different modules and their intelligence. The system's intelligence is based on JavaMath (a collection of graphical and mathematical objects, etc.) and the adaptation of the feedback mechanism developed for Dynamic Authoring aNd Tutoring Environment (DANTE) [176]. DANTE is a feedback mechanism which monitors students' demand process, whether they achieve their goals, mouse activity and provides a kind of emotional feedback which helps students to interpret their actions in a manner that prevents them from leaving their exercises [174]. A mechanism detects the goals set by the creator of the activity and must be fulfilled by the student. These goals form a tree structure, in which each goal consists of several objectives, each of which has several completion conditions and the related misconceptions, as shown in Figure 17 [178].

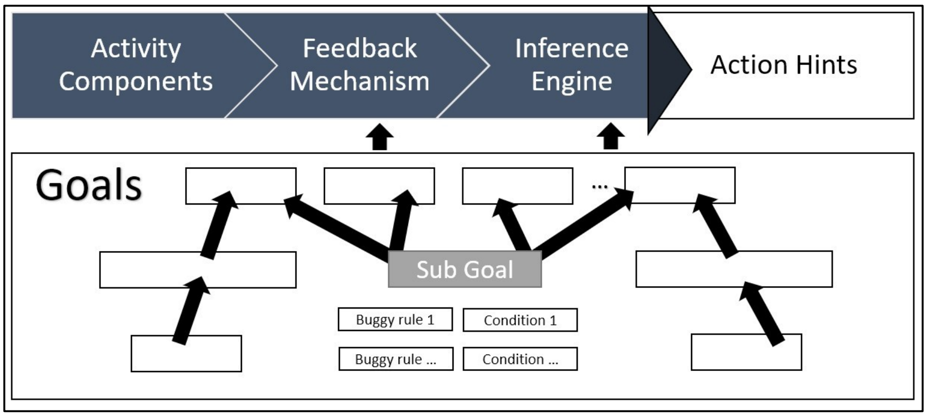

Figure 17: Goals presentation on the DANTE system

Apart from the static theoretical content and the examples, it provides interactive and exploratory activities [178]. It also employs decision trees to detect the emotional state of a student [229]. The creation of the proposed system was motivated by the lack of mathematical skills of science and engineering students [178]. The WaLLiS knowledge filed covers the spectrum of mathematical concepts such as functions, derivatives, integrals and vectors [176], [178].

The evaluation of the WaLLiS system was conducted with the participation of 20 students, divided into groups according to their qualifications and grades on a prerequisite course [177]. The goal of this research was to determine possible methods of predicting the emotional states, as well as the students' motives, while they work on interactive learning environments. By examining the log files per student, in which all interactions with the system were recorded, as well as, the students' feedback, it seems that it is possible to determine patterns which contribute to a conclusion regarding the emotional states of the students.

3.3.5 Prime Climb

Prime climb is an educational game designed by EGCMS team at the British Columbia University, to help students to learn factorization [60]. The educational game makes use of a Dynamic Decision Network (DDN) to models a student's emotional state following the OCC cognitive theory of emotions. Two players take part in the game, who cooperate to climb over mountains divided into numbered sectors. Each player can move only in one numbered sector who does not share any factor with the sector occupied by its associate. Points are deducted when the player makes a mistake. Before the game, the student uses the trial section of Prime climb, having an agent as an associate to get advice to think the reasons of fall, while at the same time getting specific directions on how to avoid falling or recovering after a fall [68].

Prime Climb employs DDN (an extension of Bayesian Networks) which are capable of monitoring environments that change over time. It also uses DDN to leverage information on the probable reasons, as well as, the observed results of a user's emotional reactions [65]. More specifically, Prime clime consists of two tools: (a) the magnifying glass, which presents the analysis of the factors of each number found on the mountain, and (b) the assistance frame which allows a player to communicate with the tutoring agent [312]. Prime Climb also has a special PDA tool, which provides a tree-analysis of a number's factorization. Figure 18 depicts the presentation layer of the DDN, on modelling of a user's emotional state.

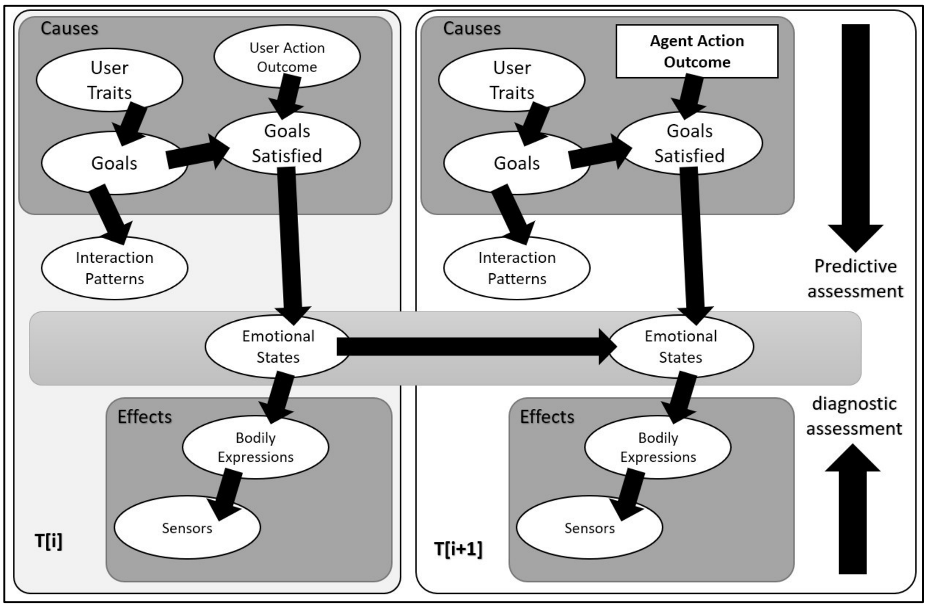

Figure 18: A high presentation layer of DDN on modelling a user's emotional state

The nodes and the structure of the evaluation model are presented in Figure 19. The elements are shown as nodes in Figure 20, occurred after two studies [68], of 23 and 10 students respectively, who interacted with Prime Climb. Both studies evaluate system's log files [68]. Nodes and connections occurred are based on subjective observations of real interactions. Then the mode improves dynamically using data from an official user study [312]. Zhou et al. [312] present a method through which a student's goals are connected with the student's personality, as well as the connection between these goals and their interaction patterns.

For the reason that Prime Climb is an educational game, it does not seem to implement procedures regarding a student's initial knowledge-base on natural numbers factorization. It only provides the opportunity to explore various mathematical structures (e.g., multiplication, division etc.), through practical climbing, while a tutoring agent which assists the player. User's goals seem to be very specific, as shown in Figure 20 [68]. To determine the personality of a student, the game was evaluated using Goldberg's 100 standard markers test over the five types of the personality of the Five Factors Theory [312].

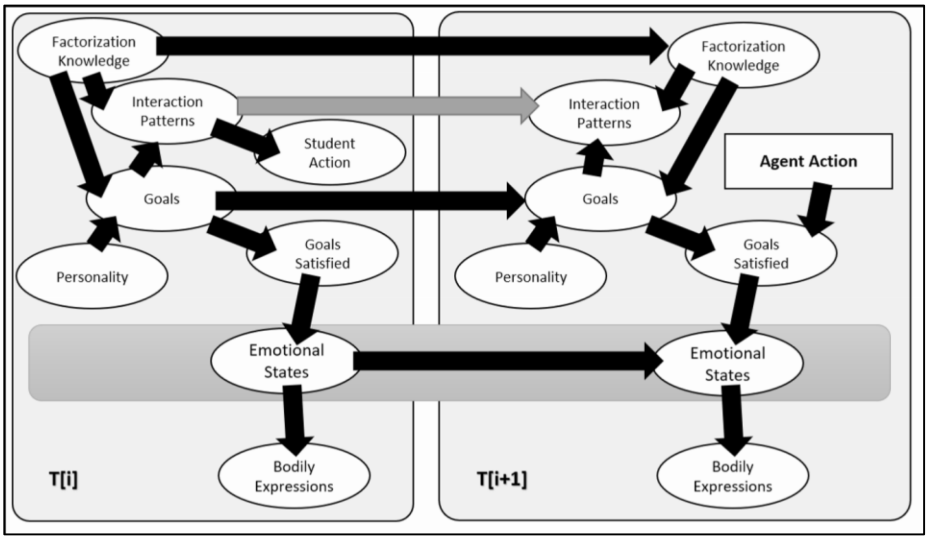

Figure 19: Two DDN's time intervals of the student's emotional status

The prime climb can recognize six out of the 22 OCC emotions (joy, distress, admiration, reproach, pride, and shame) [62]. Regarding each one of them [60]:

a) Joy for the incident faced by the user;

b) Distress for the incident faced by the used;

c) Reproach to the entity caused the incident;

d) Admiration for the entity caused the incident;

e) Pride for the entity which caused the incident, when it is the same entity; and

f) Shame for the entity which caused the incident, when it is the same entity;

Those six emotions are modelled by three nodes which can take values: emotion for the game, emotion for self and emotion for the agent [64].

The emotional state of each student was recorded through biometric records combined with game event records which could trigger an emotional reaction, such as the climber's fall [14]. It is mentioned in [62] that biometric signals were recorded, regarding skin conductance, muscle electromyography for those (muscles) involved in frowning and eyebrows, blood pressure and respiration. Four different sensor sets were used [14] to record those biometric characteristics: a skin conductance sensor, a heart rate sensor, and a set of electromyogram sensors attached to the student's forehead. Each biometric record was synchronized with game log files which could result in an emotional reaction. An electromyogram sensor was attached to each student's forehead [65], while all actions were logged in a log file, used later on to evaluate the emotional state of each student.

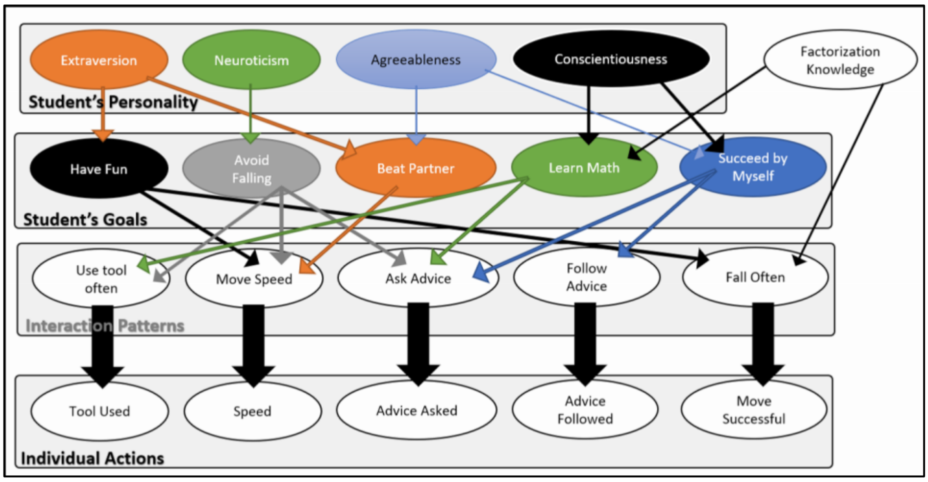

Figure 20: Two time-lines of the DDN model on a student's emotional state [68]

Emotion detection is performed through the records of each biometric sensor, which are synchronized with the game's log files that show possible emotional reactions (e.g., a fall or a climb) [14]. Emotional states are categorized using a clustering method based on the data collected from the biosensors [14].

A user's emotional state is modelled based on the OCC model and is the result of a user's assessment between the current interactions events compared to the user's goals. An interaction event may be an action caused by the student or the agent which creates a new time entry in DDN [60].

One of the tutoring agents, as mentioned in [60], is a climbing instructor, who can provide personalized assistance by request. In particular, the instructor can: (a) assist the student in case of a mistake by encouraging him regarding the reasons which led to the mistake, and (b) remain neutral. The goal of a tutoring agent is to preserve student involvement on high levels, while at the same time providing assistance to learn number factorization [14]. Microsoft Agent Package has been used to develop the tutoring agent, who is animated as a wizard named Merlin [67]. To decide on how and when to intervene or not, the agent relies on a probabilistic model (a Dynamic Bayesian Network) that makes judgments on student's knowledge on factorization. Its main goal is to provide personalized support to the students for them to learn number factorization during climbing while maintaining a high level of involvement in the game [312]. In the second version of Prime Climb, an extra level was added, which included definitions and examples [66].

The system observes all actions performed by a student and through the agents provides an appropriate level of advice. The aim is to change the student's emotional state to reach a specific goal. Figure 21 illustrates an action of an agent represented as a decision node in DDN, such as the agent action. A student's emotional state is categorized into three groups (joy/distress, pride/shame, and admiration/reproach). It is presented in three nodes (emotion for-event, emotion-for-self, emotion-for-agent) which can take of two values. An example [68] is the case where a student appears to have a high probability of feeling shame. In such an occasion, the agent decides to provide minimum advice to make the student feel better regarding his performance. If there is a high probability for the student to feel shame, the agent must act appropriately to regain its reliability.

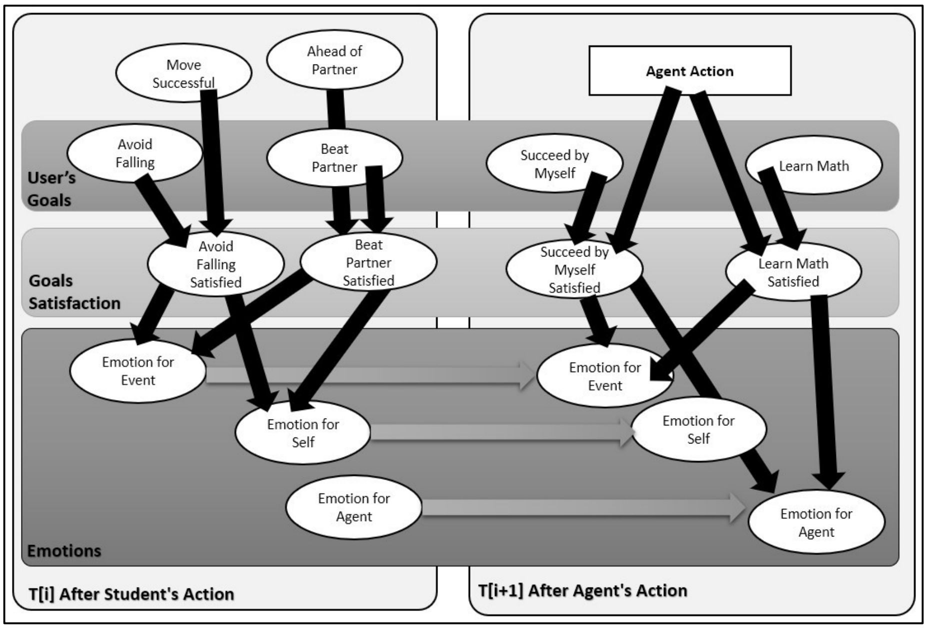

Figure 21: A sample of the nodes involved in an agent's action evaluation and emotion analysis [68]

The result of an agent's or student's action is subject to the evaluation of the student [65]. Each goal Satisfied node affects the emotion for a game node in each step. If a student action has created such a step, then each goal Satisfied node affects the emotion for the self node, while if the agent has created it, the emotion for the agent node is affected.

The novelty of the system is that the Prime Climb agent is based on a probabilistic mathematical model to provide personalized assistance which aims at helping students to learn number factorization through an educational game [67]. The presentation of the relations of the probabilistic nature between a student's emotional state, their causes and their effects, is performed through the DDN [68]. An extra characteristic is that it provides the Magnifying glass tool, which makes use of a tree form to provide several analysis into factors.

The same technique is discussed in [312] where it provides: (a) a model to recognize, in real-time, a variety of specific emotions, (b) evaluates student's emotions in the context of an interactive learning procedure, and (c) is the first model which performs a probabilistic presentation of the OCC cognitive theory of emotions. Moreover, information regarding a student's current emotional state, as well as, of those emotions in favor of a student's learning ability, will hold the tutoring agent to choose the correct interventions [14]. The novelty of the proposed model, as stated in [65], is that it is a first attempt to create a model which exploits data coming from sources such as: (a) what causes the emotions (predictive model), and (b) emotions results (diagnostic model). The predictive model is based on student actions and the agent's interventions, while the diagnostic model is based on recording data from the electromyographic sensors. Another novelty is that the use of information relative to student goals is used as a source of evidence.

The Prime Climb learning field is mainly the factorization of natural numbers. It is designed for sixth and seventh grade students [14] of the American education system, which corresponds to the sixth grade of elementary school and the first grade of secondary school, respectively, of the Greek education system.

Before the model was completed, it was evaluated using a direct technique [61] where 10 students participated in this experiment. The results showed that pop-up windows did not affect students and that students were not afraid to express their emotions. A second study was conducted in 20 students and showed that they were not annoyed by the pop-up window and that the questions were entirely understandable. To evaluate the effectiveness of the Prime Climb learning agent, a study was performed over 20 of the seventh grade. Students were divided into two groups, a control and an experimental group. Results were promising and showed that providing personalized assistance through an animated tutoring agent can improve student learning using educational games [67].

In another evaluation performed on 20 students of the seventh grade, reports the precision of the probabilistic models' elements, which contribute to the emotional assessment of the students [63]. The evaluation of student goals can be combined with information regarding the reasons and the consequences of their emotional reaction. Results showed that if the student's goals can be accurately defined, then the emotional model can preserve a precise estimation of the student's current emotional state.

Another study reported in [64] was conducted with an autonomous agent (not with the Wizard of Oz technique as in previous studies), on 66 students of the sixth and seventh grades from three local schools. A pre-test on number factorization was conducted, then a post-questionnaire was given to define the goals each student had during the game. Finally, students were asked to fill a personality test. Data analysis showed that a new goal, called Want Help, had to be introduced and that students tend to give different priorities to their goals.

To evaluate the effect of adaptive feedback provided by a tutoring agent during a learning game, a new study emerged [66] on sixth grade students that were divided into three groups: (a) without an agent (13 students), (b) initial version with the tutoring agent (14 students), and (c) new version with the tutoring agent (17 students). After that, the intervention occurred as planned, and then students were evaluated with a post-test questionnaire. Results showed that there was not any difference in learning levels between the three groups of students. By further examining the log files, authors concluded that the reason of the result above was that the tutoring agent offered advice related to knowledge and skills already known to the students, meaning that most probably students did not pay any attention to this kind of advice. A possible solution would be to improve the model's accuracy such that the agent interferes only when there is an actual reason to do so.

3.3.6 Conclusions for the five ATS

PAT2Math can be considered as an ATS with several advantages and novelties, as it has multiple agents that operate independently and also have the ability to work together. Another observation is the PATequation tool can be used to provide step-by-step guidance on solving first and second-degree equations, setting goals through the MSLQ questionnaire and moving a pedagogical agent. As a limitation, it can be assumed that the strategies applied by the agent have to do mainly by actions of the students' interaction with the system and there is no recording of the emotional state of the student with any artificial means (e.g., camera). Also, the student's personality is not taken into account. Moreover, no representations are used for the representation of the equations with the interaction between student and system to be kept at the minimum. Also, there is only one form of static exercises and no multiple representations of exercises or concepts to be taught. Furthermore, the tool cannot estimate or record at the beginning the level of knowledge of a student. Finally, it appears that this system is not open source code.

Name	Architecture	Pedagogical Agent			Emotions		Emotional state of student	
		Name	Characteristics	Can do	Number	Type	Mean	Method
PAT2Math	Has pedagogical agent PAT and 4 other agents: Agent Interface, Tutor Agent, Agent Student Model, Agent Base Domain	Pat	Animated agent (woman), can move and change face expressions.	Every time an tutoring event occurs, the tutor agent comes to a conclusion regarding the student's emotional state and chooses the appropriate tactic to implement	7	joy/ distress, satisfaction/ disappointment, gratitude/anger, shame (according to OCC model)	log files	Takes into consideration students' emotions by employing both system interface and facial expressions recognition
Fermat	3 basic units: Expert (later renamed to Domain), Student, Tutor that has a sub-unit called Affective	Genie	Animated agent (man)	Give advices, ideas and generally can make 8 actions	5	anger, happiness, sadness, surprise, neutral	web camera	Neural Networks Kohonen
Easy with Eve	Consists of 4 components: Student's model, Teaching strategy unit, Facial expression analysis unit, Set of teaching activities in the form of an animated pedagogical agent	Eve	Humanoid animated agent (woman), with body, can move and change expression	Gives positive or neutral answer, make questions, discuss problems or solutions, do suggestions, answers to questions	7	surprise, happiness, sadness, puzzlement, disgust, anger, neutral	web camera	It consists of: a) artificial neural network for face detection, b) extraction of facial features, c) fuzzy face-expression classifier. It uses Support Vector Machines (SVMs).
WaLLiS	Based on Client-Server model. There is student's model, Studex component and feedback mechanism	-	There is no animated agent, but a text box with advices, information etc.	Gives information, advices and help to student.	3+positive emotions	Frustration, boredom, confusion, and a category with positive emotions (satisfaction, joy, happiness etc.)	log files	Based on forecasting model based on an algorithm called J4.8.
Prime Climb	Has Dynamic Decision Network, using nodes and connections between its components	Merlin	Animated agent (man)	Give advices and guidance	6	joy/ distress, admiration/ reproach, pride/ shame (according to OCC model)	Biometric signal sensors and log files	Dynamic Bayesian Networks and Clustering (Unsupervised Machine Learning techniques)

Table 1: List of basic features (Architecture, Pedagogical Agent, Emotions, Emotional State) of five ATSs

Name	Student's Level	Cognitive Field	Originality	Recognize emotional State of student using camera	Detect Student's Cognitive Field	Detect Student's Personality	Multiple Representations
PAT2Math	Elementary	Mathematics: linear and square equations	Multiple agents that can act independently, while at the same time actively cooperating with the rest of the components	No	No	No	No
Fermat	Elementary	Mathematics: multiplication and division	Smart social network ATS for teaching mathematics, that presents the content of the course in a personalized style	Yes	Yes	No	No
Easy with Eve	Elementary	Mathematics: addition	It bases its teaching strategies on pedagogical actions of real-world pedagogues.	Yes	No	No	No
WaLLiS	University (science and engineering students that use mathematics)	Mathematics: functions, derivatives, integrals and vectors	Feedback mechanism	No	No	No	No
Prime Climb	Elementary	Mathematics: factorization natural numbers	Uses probabilistic student model and provide personalized support	No	Yes	Yes	No

Table 2: List of features of five ATSs

Fermat is regarded as an ATS with many positive aspects, such as being a smart social network for presenting course context in a personalized manner. It also has a moving tutoring agent able to perform specific tasks. A student's learning level is evaluated through a diagnostic test on the examined mathematical concepts. Emotion detection is performed using Fuzzy Expert Systems. Some limitations of the proposed system reported are the limited number of actions the tutoring agent can perform, the one-dimensional presentation of mathematical operations, the absence of multiple representations of the exercises, and the fact that there is no way to reflect a student's personality. Moreover, the detection of a student's emotional state is performed only once, before the start of a proposed exercise. In contrast, the emotional state can be altered in multiple ways during the exercise, due to many factors. Also, the system's ability to recognize five emotions may lead the system to choose the wrong strategy. Finally, there is no evidence that the system is open-source code.

Easy with Eve can be regarded as an ATS with several positive aspects, such as the fact that the teaching strategies applied are based on teaching techniques employed by real teachers. Another advantage is that emotion recognition is performed using cameras in real-time. It is also a positive fact that there is a moving tutoring agent with a human voice, able to express multiple emotions. A drawback of this system is that there is no way to reflect a student's emotional personality through a test, nor the knowledge level of mathematical terms which are used by the proposed system. Also, there are no dynamic exercises, which are presented through videos, while there is no interaction between the student and the exercise. Also, the system has not been tested on a large scale and is not reported of being open-source code.

Wallis can be considered as an ATS with many positive elements. The components that make it stand out are the dynamic map of content, the feedback mechanism that does not interrupt the student from his work, customized educational strategies, scaffolding, and interactive web pages with the content of each module. A limitation is considered to be the absence of a pedagogic agent moving to attract more attention to the learner; since feedback through the text frame may not be perceived by the student. Additionally, no initial screening with a diagnostic test is the student's cognitive level as well as the personality of the student. There are also no multiple representations of the proposed exercises. Also, no sense of sensation can be detected through a device (e.g., a camera) but only by considering information from the log files; which cannot fully reflect the student's emotions since they do not directly ask questions about the student's emotional state. Also, openness to more massive student potential and its assessment from a larger sample may be a component of improving the model. Finally, it is not reported as an open-source system.

Prime Climb can be an ATS with substantial novelty. It has many advantages, such as the animated cartoon-like agent, the intervention, when needed, of the agent, its ability to show number factorization in a tree-like structure and the attempt of detection students' emotions using a probabilistic cause-causality model. However, Prime Climb lacks on that is has not been tested on a large scale, since a significant number of electromyography sensors would be required, while at the same time duals of students with similar skills would be needed to be capable to respond better to the game. In such a way, the two students would simultaneously perform the same task, according to the predefined educational curriculum on numbers factorization. Another drawback is that signal recording does not ensure that collected data are solely relevant to the procedure of the educational game, since there may be several external factors which may affect the data. For example, it has not been taken into account whether those signals are due to the anxiety a student may feel while wearing the device, due to the lighting of the classroom, or noises created by others, or even other factors relevant to the school and the courses attended. Also, this system is not open source.

All the features of the five systems under review, which are presented in detail in this paper, are summarized in Table 1 and Table 2. Table 1 presents the basic features of the five ATSs: their architecture, the characteristics of the pedagogical agent (name, characteristics, actions), and the way (mean, method) of recognizing the student's emotional state. Table 2 presents additional features of the five ATSs: student's level of education, the cognitive field of system, originality of the system, using a camera to detect the emotional state of the student, detect student's cognitive field, detect student's personality and using multiple representations.

3.3.7 Discussion

There is a high interest from both the academic and educational community in the development of systems with integrated techniques of determining an individual's emotional state, and using that state to help students with understanding mathematical concepts or even change their perspective against Mathematics. Moreover, this trend appears to be worldwide, since there are many systems developed around the globe, such as in Canada (Prime Climb), Mexico (Fermat), Brazil (PAT2Math), New Zealand (Easy with Eve) and United Kingdom (WaLLiS). Those applications cover all levels of the educational system, beginning from the first level (Fermat, Easy with Eve, PAT2Math, Prime Climb), to the second (PAT2Math, Prime Climb), and the third (Wallis). By examining the systems mentioned above, it seems that whether it is an educational game (Prime Climb), or a social network (Fermat), or teaching mathematical concepts (Easy with EVE), or a Web-based system (PAT2Math, WaLLiS), there exist opportunities for detecting the emotions of a student while interacting with these systems. The emotional detection through appropriate strategies as already discussed provides evidence for improving students to understand mathematical concepts better and change their perspective for Mathematics [20]. Moreover, recent research showed that positive psychological attitudes and predisposition to Mathematics is an essential factor (along with others) that influences how a student is performing in Mathematics [54].

We note that further research can be conducted on ATS specifically designed and developed for other mathematical fields, such as, in trigonometry, matrices, fractions, etc. There is also the opportunity to examine further the factors that may be causing various kinds of emotions during the use of an educational system, and then examine these factors to determine whether these are caused due to internal (e.g., exercise difficulty) or external (e.g. difficulties in the classroom environment) situations. Being aware of the emotion's origin, it may be easier to convert a negative emotion to a positive one through the appropriate intervention.

Research design

4

Chapter Four

4.1 Introduction

In this chapter, it is presented the design of the current research. Specifically, they are described as the phases and the methodology to achieve the aims and the objectives that were stated early at the beginning of the current thesis. In the first phase of the research, called *Survey on fifth and sixth graders*, a survey was conducted in fifth and sixth-grade students to recognize the problem. In the second phase of the research, it is described in *Survey on e-learning systems for fractions*, the survey that took place for web-based e-learning systems with fractions. In the third phase, called *Survey on Affective Tutoring Systems*, it is described as a study for Affective Tutoring Systems for mathematical concepts. In the fourth phase, called *Teachers' focus group*, it is stated an interview implemented with the focus group technique to Greek in-service teachers. In the fifth phase, called *Design and implement a proposed system*, it is presented the design and the implementation of the proposed web-based system. In the sixth phase, called *Applying the proposed system to students*, it is presented the evaluation of the proposed system into students.

4.2 Survey on fifth and sixth graders

The first step of the research was to identify the problem and to provide a solution. Thus, we conduct a survey in fifth and sixth grades of an elementary school in comparing fractions. We gave them a booklet with 12 exercises comparing pairs of fractions. All exercises had proper fractions comparisons, which means that the numerator was less than the denominator. For example, they had to compare 3/9 with 5/9. The intervention lasted a teaching hour. All students answered, and then the results of their answers were studied.

The results showed that elementary school students have a problem with fractions, especially when comparing pairs of proper fractions. The survey that was conducted, the results and all the information are presented analytically in Chapter 5.

This research essentially confirmed the problem that was mentioned at length at the beginning of the work, that is, that elementary school students have problems with fractions.

4.3 Survey on e-learning systems for fractions

The second step was to identify learning systems on the web that are used to help students in the fractions. Thus it was surveyed web-based systems for Mathematics, and especially in the area of fractions. After searching in the literature, we have concluded into three web-based systems. The survey and the results are presented in [172]. Later another system was examined; it was the WolframAlpha.

This survey presents in detail the critical elements of these systems. They studied in depth their architecture, the type of representations of fractions they use, the operations they can perform in fractions as well as other vital factors. The survey and its results are presented in Chapter 6. Some of these features were also taken into account when designing our proposed system for fractions.

4.4 Survey on Affective Tutoring Systems

After studying web-based systems for fractions, then we wanted to explore systems that use Artificial Intelligence that has been used in general in Mathematics.

Thus, the third step of the research was to identify Affective Tutoring Systems for mathematical concepts and to study them in-depth using the literature review. After searching in electronic databases (i.e. Google Scholar, Scopus, and EBSCO), we have concluded into five Affective Tutoring Systems.

The results of the survey highlighted significant findings which were taken into account during the design of the proposed system for fractions. The survey and the results were published in paper [171], and it is presented in Chapter 7.

4.5 Teachers' focus group

The next step, after studying systems for fractions as well as ATS, was to explore the views, strategies, and methods used by active primary school teachers to teach fractions in the classroom.

The research was to interview Greek Teachers about their strategies, methods and attitudes towards fractions. The method that was used was the focus group. Thus, it was interviewed two separate groups of teachers, in total 12 teachers, using the focus group method.

The results of the research highlighted significant findings which were taken into account during the design of the proposed system. The results, the findings, as well as the reflections and the discussion about them, are presented in Chapter 8.

4.6 Design and implement a proposed system

The fifth step of the research was to design and to implement a proposed system for fractions. Taking into account (a) students' research in fractions, (b) teachers' interviews about their strategies for teaching fractions, (c) research on online systems for fractions, and (d) ATS on Mathematics, we proceeded to design an online system for fractions. This system recognizes the emotional state of students during the solution of fractional exercises with different representations. Thus, the teacher, knowing the emotions brought about by each representation, can suggest exercises to the student with a specific representation of the fractions and thus help him to improve his performance in the fractions. This system is analyzed in Chapter 9.

4.7 Applying the proposed system to students

The sixth step of the research was to apply the proposed system for fractions to students. An intervention was made in primary school students using the proposed system we had developed. Students were asked to compare pairs of proper fractions. When the students gave their answer to the exercises, then the system recognized the emotional state that the students had. From the analysis of the results, important findings emerged regarding the relationship between the representations and the performance of the students during the exercises of comparison of fractions, but also of the emotions that evoke them. The results of the research, the findings, as well as other important details of the research, are detailed in Chapter 10.

Survey on students

5

Chapter Five

5.1 Introduction

Primary school students encounter difficulties in various fields of Mathematics. Research has shown that primary school students have difficulties with rational numbers [202], [280], [283], and especially with fractions [39], [53], [57], [72], [103], [181], [184], [233], [278]. Students encounter difficulties in fraction comparisons [97], [300], in addition of fractions [103], in subtraction of fractions [128], in multiplication of fractions [258], in division of fractions [55], and in ordering a pair of fraction [76]. Some of the factors that affect students' performance in fractions may be the use prior knowledge of whole numbers into fractions, the different gender, the grade etc.

Firstly, these difficulties often stem from the fact that students' prior knowledge of whole numbers affects them [88]. Also, students try to apply properties of integer numbers to fractions, for example when comparing 4/7 and 2/3 consider that since 4 is greater than 2, and 7 is greater than 3, then 4/7 is greater than 2/3, which is wrong. Furthermore, as mentioned in the [271] "But the only alarming ailment is the following one, namely, the temptation to deal with fractions in the same manner as with natural numbers. The student may get off on the wrong foot here when compiling and shortening descriptions of division processes in symbolic form.", and when students add 1/4+1/4, then they write 2/8 as an answer, instead of 1/2 which is the correct answer. This problem is known as Natural Number Bias [283]. The four main areas where mistakes are made systematically due to Natural Number Bias are size, operations, representations and density [283].

Secondly, some of these difficulties in Mathematics probably have their roots in student's gender. The gender gap in Mathematics is an important and divisive issue of academic debate [89]. Boy students are better at Mathematics than girl students in primary school [89]. This is also confirmed from the research of the Organization for Economic Co-operation and Development (OECD), which shows that in most countries, girls underperform boys in Mathematics [205]. But recently, according to [206], many countries have been prosperous in closing gender gaps in learning outcomes, including Mathematics. Also, in [206], mentioned that "15-year-old boys are more likely than girls, on average, to fail to attain a baseline level of proficiency in reading, mathematics and science". But, a survey [3] showed that it no significant relationship between gender and performance in fractions.

Thirdly, the difficulties are different in each grade of elementary school. From one hand, fifth graders have misconceptions and make mistakes in operations of fractions, especially in ordering, addition, subtraction and multiplication [267], [31], and they have difficulties in understanding the problems of fractions [137], [305]. Also, fifth graders consider fractions to be two independent natural numbers, and they use the phenomenon of Natural Numbers Bias [268]. Additionally, fifth graders use the Natural Number Bias when they are doing operations in fractions [108]. From the other hand students in sixth grade had misconceptions in fractions [299], but different than fifth graders. In a survey, [74] to investigate the performance of children from grade 3 to 6 comparing pairs of decimal fractions has shown that sixth graders have better scores than fifth graders. Probably this happens because at this stage of the education, i.e. in sixth grade, the understanding of decimal fractions become more precise [74]. Also in [3] and [96] the sixth graders had better performance than fifth graders in fractions tasks. Furthermore, in a survey [98] there were changes in the students' performance in comparing fractions, and analytically there was no difference comparing congruent items between fifth and sixth graders. Still, there was a significant difference to incongruent items between the fifth and sixth graders.

In this research, we studied the difficulties that students encounter in fractions, which are essentially rational numbers, especially in the field of number size. Specifically, in the study, we focus on comparing two rational numbers with fractional representation, just as [97] did in their study. The study aims to identify any difficulties of primary school students in the process of comparing fractions. Also, we try to identify if there is any relation between performance and gender in fractions. Finally, we investigate if the students' grade affects students' performance in fractions. Four research questions are being asked:

a) Do elementary school students perform better when comparing two fractions with common components (same denominator or numerator)?

b) Do elementary school students perform better when comparing congruent fractions pairs;

c) Is there any relationship between gender and performance in fractions; and

d) Is there any relationship between the level of grade and performance in fractions;

5.2 Related work

In study [97], conducted a study with 438 participants from fifth to tenth grade examining the effect of congruency and gap thinking in students' responses and reasoning. It was used as a test of four fraction comparison items. Firstly the results of the study show that both primary and secondary students encounter difficulties with fraction comparison items. Also, it reveals that Natural Number Bias is the main reason for students' failure in fraction comparison. The results of the study show that gap thinking has influenced student's responses. Also, it revealed that the gap effects could explain the differences between congruent and incongruent items.

In another research [57] conducted a study with 323 children of sixth grade in Victoria, Australia. The students were asked to find the larger of two fractions for eight pairs. Also, students have to give reasons for their answers. The students did not give their answer using pen and paper, but they had to do all tasks mentally - the results have shown that the difficulty of the pairs was found to be close to that predicted. But the fractions with the same numerator and different denominator were surprisingly tricky for students. The researchers also suggested the strategies of benchmarking (or transitive thinking) and residual thinking, which is used by a student who was generally successful in these kinds of tasks.

In a different research [95] conducted a study in 502 students from fifth, sixth and seventh-grade classes from schools in Santiago, Chile. It was used a computerized task with pairs of fractions, and each student could see the pairs in the screen for a limited time of 10 seconds. The study aimed to investigate specifically individual differences in fraction comparison within a quantitative framework, with a particular focus on congruency and the presence of standard components. It was used clustering analysis based on four item types (common component or not, congruent items or not). Half of the students follow a congruency-based implementation of the Natural Number Bias (larger component-larger fractions strategy), but several clusters showed opposite patterns.

In the study of Aksu [3], a study was conducted with 155 participants from sixth grade. It was conducted a test with three types of context, namely: (a) concept test, (b) operation test, and (c) problem-solving test. One of the findings is that students perform better scores, also preferably in multiplication when these problems are in word problems. The results showed that there is no significant relationship between gender and performance in fractions. Also, one of the findings was that there were significant differences in performance on the three tests according to previous mathematics grade.

Also, in a cross-sectional study [98] was conducted on 438 students, from fifth to tenth grade in comparing fractions. The instrument was a test of four fraction comparison items. These items were designed based on congruency and gap conditions. The results showed that overall, that sixth-graders answered better than fifth graders, and there was a statistical significance when they compared incongruent pairs of fractions.

5.3 Method

Participants were 115 primary school students (fifth and sixth grade). Students were from mixed socio-economic backgrounds. The instrument was a test consisting of 12 fraction comparison items, see Table 3. All fractions in pairs were proper fractions, which means the numerator was smaller than the denominator. In each of the 12 questions, students had to answer which one of the items of the pair was greater. Also, there was no time limit for their answers. All students had normal vision, and a parent of each child signed an informed consent form to participate in the study.

Number	**Pair of Fractions**	**Congruent or Incongruent (C/I)**	**Same or Different denominator od numerator (S/D)**
Pair 1	3/9 vs. 5/9	C	S
Pair 2	6/8 vs. 7/8	C	S
Pair 3	4/6 vs. 5/6	C	S
Pair 4	1/7 vs. 1/5	I	S
Pair 5	3/4 vs. 6/8	I	D
Pair 6	1/3 vs. 4/9	C	D
Pair 7	3/5 vs. 2/3	I	D
Pair 8	7/9 vs. 7/8	I	S
Pair 9	7/8 vs. 8/9	C	D
Pair 10	3/8 vs. 2/4	I	D
Pair 11	2/3 vs. 5/6	C	D
Pair 12	4/5 vs. 4/7	I	S

Table 3: Characteristics of the items

Every students' answer classified as correct or incorrect. If the student wrote the correct answer, then the answer was classified as correct. But if the student wrote the wrong answer, then the answer was classified as incorrect. Also, unanswered items were classified as incorrect.

5.4 Results

The mean score was 8.25 (SD=2.571). The mean accuracy, computed of correct answers, was 68.8% (SD=21.4). Differences between the two grades were significant (fifth grade: 60.5%, sixth grade: 77.5%). Also, in the study of [96] students of sixth grade had better scores than students of fifth grade (60.0 % and 58.7% respectively) in a fraction comparison questionnaire with 24 pairs of fractions.

Table 4 shows the percentage of students who answered correctly the appropriate fraction for the pair. The fraction pairs are presented in ascending order of success.

Pair of fractions		**%Correct**
Fraction 1	**Fraction 2**	
7/8	8/9	41.7
3/4	6/8	53.9
3/5	2/3	54.8
3/8	2/4	54.8
4/5	4/7	56.5
2/3	5/6	61.7
1/3	4/9	64.4
7/9	7/8	72.2
1/7	1/5	81.7
3/9	5/9	91.3
6/8	7/8	95.7
4/6	5/6	96.5

Table 4: Percentage of grades fifth & sixth students answering correctly

Students performed better in pairs of fractions which had a common component (numerator or denominator). Specific students in 1st, 2nd, 3rd, 4th and 8th pair of fractions had a score of 91.3%, 95.7%, 96.5%, 81.7%, and 72.2% of correct answers respectively.

Unlike in pairs of fractions that there were no standard components (specific in 5th, 7th, 9th, and 10th item) students had lower achievement from 41.7% up to 54.8%.

Students had a score of 72.2% of correct answers in item 8 when they have to compare 7/9 vs 7/8. According to Natural Number Bias, we expected the student to have a lower score, as in a similar study [57] student in pair comparison 4/7 vs 4/5 had 37.2% correct answers. Probably, this happens because students of our study have the ability to use a pen and paper. Also, they did not have a limitation of time, unlike the students of the other survey [57] have to undertake all the tasks mentally.

Furthermore, the student had a low score (41.7%) in item 9, when they have to compare 7/8 vs 8/9. This means that students who tried to compare numerator and denominator separately, and not as a fraction, they failed. The Natural Number Bias is the main reason for this failure.

Additionally, the total score in comparing pair fractions was lower (60.2%) if we remove the first three items that are generally easy tasks since students have to compare only the numerator. This reveals that students face difficulties in fractions and especially when they make comparisons of fractions with different components in numerator and denominator.

	With common components	**Without common components**	**Mean**
Congruent	94.5%	55.9%	75.2 %
Incongruent	70.1%	54.5%	62.3 %
Mean	82.3%	55.2%	68.8 %

Table 5: Average scores in the fraction comparison questionnaire

Table 5 presents accuracy rates per item types. It is obvious that in fractions with common components, students do it better than in fractions without common components (82.3% and 55.2% respectively). Also, students have better performance in fractions with congruent (75.2%) than in fractions with incongruent (62.3%).

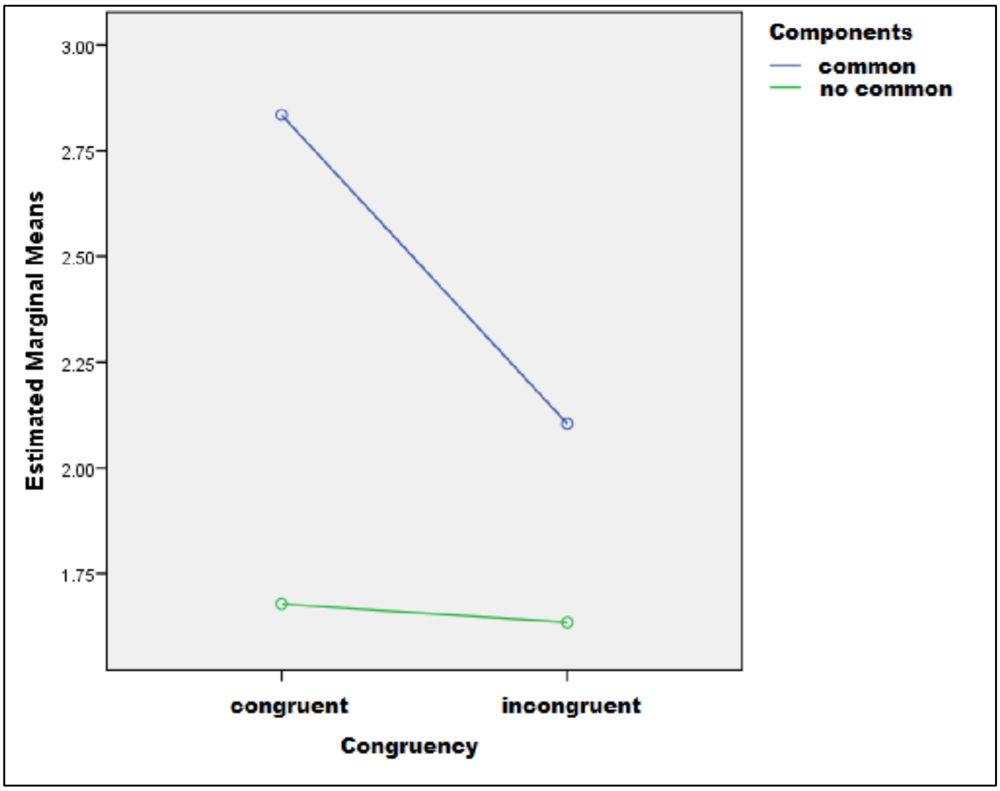

Figure 22: Relationship between congruency and common components.

Using a 2-way repeated measure, ANOVA showed a statistical effect of the presence or absence of standard components. Fraction pairs with common components were answered in average 27.1% better than pairs without standard components ($F(1,114)=116.7$, $p<0.001$). Fractions pairs with congruency were answered in average 12.9% better than incongruent items ($F(1,114)=26.2$, $p<.001$). Overall there was a significant statistical relationship between congruency and standard components ($F(1,114)=28.3$, $p<.001$). Also, Figure 22 shows the relationship between these two factors (congruency and standard components) in fractions. This means that students have better scores in congruent fractions with standard components than in incongruent fractions with no standard components. All these findings agree with the research of [96]. Furthermore, in [98], there was a statistically significant difference ($p=0.01$) between fifth and sixth grade students when they compare incongruent items. Still, there was no statistically significant difference in comparing congruent items, with the sixth graders to have a more significant percentage score than fifth graders.

The Independent-Samples T-Test showed that there was no significant difference in score between boys (8.52, $SD=2.335$) and girls (8.00, $SD=2.773$), $t(113)=1.085$, $p=0.280$. This is in line with a survey of 155 sixth graders [3] where the relationship between gender and performance on three types of tests (concept, operations, and problem-solving) in fractions was not significant. Also, these results are inconsistent with Lindberg, Hyde and

Petersen [162] meta-analysis and Hyde et al. [113] analysis of data, which showed no gender difference in performance at elementary grades. Also, the slightly better performance of boys compared to girls in the present study in terms of fractions, is in the same direction as the results of the OECD [205] which show that boys in Greece are slightly better in performance in Mathematics than girls in Greece.

Also, the Independent-Samples T-Test showed that there was a significant difference in score between fifth and sixth graders ($t(113)=4.641$, $p<.001$) with the mean score for fifth graders 7.25 ($SD=2.623$) and sixth graders 9.30 ($SD=2.062$). Also, the average score of sixth graders was 1.608 points better than fifth graders. This is in line with the survey of [3] that mentioned: "As the previous mathematics grade increased, the performance of the students on the tests also increased". Also, in [96] students of sixth grade doing better in fractions than students of fifth grade. Furthermore, in [98], the sixth graders overall have better percentage score than fifth graders.

5.5 Discussion

We survey fifth and sixth grade students to investigate their performance in fraction comparisons. Students had to compare pairs of fractions with standard components (same numerator or same denominator), and pairs of congruent or incongruent fractions. The results of the study shown that students have difficulties with fractions tasks. These difficulties, for students to compare pairs of fractions correctly, are due to some factors. One of the main factors is the phenomenon of Natural Number Bias, and it is in line with [97] and [245]. Of course, misconceptions and lower achievement in Mathematics are some other factors.

Nevertheless, Students had better scores in fractions pairs with standard components than in fraction pairs without common components. This is in line with the study of [96]. It is easier for students to compare fractions with standard components (numerator or denominator) since they have to compare only one number. If this number is the numerator, then the students achieve better scores in the comparison of fractions.

Also, as expected students have higher scores to fractions with congruent items that fractions with incongruent items, and it is in line with the study of [96]. This reveals that Natural Number Bias has a strong influence on fifth and sixth graders, as also mentioned in [96]. Also, our findings are in line with the study [98] that reveals that the difference between congruent and incongruent items was significant to students for fifth and sixth grade.

Furthermore, one of the findings of the survey is that gender is not playing a significant role in the performance. The difference in score between boys and girls is negligible. Moreover, in the same results, that there is no significant difference in gender and performance in fractions test, conclude the study of [3].

Another finding of the study is that sixth graders they are doing better in fractions than the fifth graders. This is line with [96], [3], [98] and probably this happens because the students of sixth grade have spent more time and exercises with fractions as they are one year older than students of fifth grade.

Of course, the study had some limitations, such as the number of pairs in fractions comparison and the reasoning of students why they came up with that answer. In our plans is a research in comparison to fractions with more pairs and a more significant sample of elementary students. Also, it will be asked by students to justify their answers through an interview or writing down their answers in a paper.

Survey on e-learning systems 6

Chapter Six

6.1 Introduction

According to students, learning mathematical concepts and processes often creates a set of negative emotions and builds up a negative attitude towards learning Mathematics [23], [254]. As a result, students are not interested in lessons of Mathematics and do not show interest in attending them [244], and as a consequence students often fail in Mathematics examinations [47]. Under this negative situation, students are not being able to acquire the necessary skills or develop the necessary mindset to learn Mathematics as they are often assume that they will not succeed in such courses. Fractions is considered to be a difficult area of Mathematics for students [300]. It is believed that fractions are among the most difficult areas of Mathematics for both teaching and learning [153]. Indeed, in a survey conducted in [300] suggests that even the comparison of simple fractions, such as, 8/9 and 11/12 is considered as a difficult exercise for most elementary school students [39], [53], [72]. The concepts behind the various operations of fractions are often difficult to accumulate, because students are required to use their imagination several times to describe, visualize and understand fractions. Students should be able to understand the problem and eventually come up with a strategy as a solution to the given problem. Recent studies showed that students that face various difficulties with fractions inherit these difficulties to the next levels as they progress. As a result the 1/3 of students fail to add fractions with different denominators [27]. Also, division problems are considered as difficult for students at the primary even at the elementary level [237], [55]. Although the understanding of fractions is grounded on solid mathematical theory it is argued that [236] multiple representations of fractions (e.g., visual or graphical representations) could be a solution to promote learning of fractions. Yeh et al. [303] recognized that when knowledge is represented in many different ways, in contrast to the traditional symbolic notation, can lead to a better understanding of mathematical concepts. More specifically, multiple representations of fractions can help primary school students to understand fractions [276]. Wyberg et al. [297] present a study on how applying multiple representations in fractions multiplication can enhance students' learning.

In general, graphical representations can be seen as a universal educational tool: flow-diagrams are used in learning programming concepts, schemas and tree diagrams in biology - to mention just a few examples. Graphical fractions representations are used in various software, both as stand-alone programs, as well as, web-based or cloud applications located on repositories or e-learning platforms. Such web-based applications are the: (a) fractions simulations on Physics Education Technology (PhET), (b) Fractions Lab of iTalk2Learn project; and (c) Fractions Tutor platform of the online platform Mathtutor [238].

This study explores applications that are part of the above aforementioned platforms/repositories and deal with fractions and their representation. The aim of this exploratory research is to present the basic components of the above systems dealing with the teaching of fractions, their positive elements, any limitations, and the originality of each system. Also, a comparative analysis is performed between the applications of fractions studied in this article to highlight their positive elements and any missing points that could be incorporated into a new proposed software or cloud application for fractions.

6.2 Analyzing web-based platforms/repositories

This section presents the key features encountered in the following platforms:

a) The PhETs simulations dealing with fractions;

b) the Fractions Lab application; and

c) the Fractions Tutor;

PhET is a platform based on the exploration of interactive educational simulations of natural sciences and Mathematics. In this repository, there are a total of 51 simulations (data as of March 2018) for primary education. Eight of these are for Mathematics and only three specifically constructed for learning fractions. An advantage of the PhET repository is the ability provided to run applications both online and/or locally on a computer.

Fractions Lab is an online platform for fractions developed within Project iTalk2Learn, funded by the European Union (November 2012 - October 2015). It is a system that allows students to understand the meaning of fractions. It is an interdisciplinary platform that brings together expertise from mechanical learning, user modeling, intelligent teaching systems, natural language processing, educational psychology, and math education.

The Mathtutor platform is an interactive online platform for Mathematics that encourages students to take part in various lessons involving fractions. It allows students to interact with the system through a series exercises for fractions, make mistakes, use the help facility from the system, and find solutions to their problems. The fractions are provided through the Mathtutor platform, on a free, dedicated website named Fractions Tutor [238]. A password is provided by the teacher that facilitates the teaching process to students that use the system, this way teachers can monitor and track the progress of each student. There is a wide selection of activities for fractions. The Fractions Tutor mainly consists of three units:

a) Fractions Tutor - Unit 1;

b) Fractions Tutor - Unit 2; and

c) Fractions Tutor - Unit 3;

There are many concepts in each section (create fractions, compare fractions, add fractions, subtract fractions) and a wide range of exercises. The rest of this section outlines the basic features of the three systems considered in this study.

6.2.1 Programming language

PhETs simulations are written with a combination of programming languages e.g., Java, Flash [221] and Hyper Text Marked Language v5 (HTML5). Since many web browsers have abandoned Flash technology, many of the applications cannot be run through web browsers. A basic pre-condition for running the PhET platform simulations is to have the latest plug-ins installed in Flash and Java [221]. Fractions Lab is part of the iTalk2Learn, which is an online platform [101], [102]. The decision to build an online Web platform has been made for the reasons described by Fernandez and Gutierrez-Santos [101] that it is more secure, requires no installation or further maintenance, and is maintained until the last version without any extra intervention. The user interface is based primarily on HTML5 and Flash. Additional components to create the user interface are using Javascript, Twitter Bootstrap, GWT, Apache Tiles, Thymelaf, Jquery Ajax, External Interface, Spring MVC [101], [102]. Other technologies such as Java Persistence API (JPA), Hibernate (ORM), MariaDB databases, etc. are used in other components of the platform [101], [102], [117]. Mathtutor has been implemented using various programming tools. In particular, CTAT (Cognitive Tutor Authoring Tools) has been used to write the key components on both the client and server side, as well as, Flash for the construction of the graphical user interface [5]. Fractions Tutor is designed with CTAT [238].

6.2.2 Interactivity

PhET's simulations introduce a different level of interactivity at each simulation. The user interaction is encouraged by the Graphical User Interface and intuitive controls, using buttons, click-and-drag, slider and a direct response is provided to any user interaction [221]. Of course, the interaction in several simulations could be to a greater extent than already exists. In the Fractions Lab architecture, there is a level of presentation that follows Model-View-Controller as reported in [101], [102]. Specifically, each viewing (view) requests information required by the model to produce an output representation to the user, while the model notifies the relevant sightings and controllers when a change in status [101], [102]. The Fractions Tutor has interactivity for graphical representations of fractions [238]. In particular, it is given the ability to move objects with the known method drag & drop, the possibility of typing numbers, selecting from drop-down list, and other options as they appear when performing exercises in Fractions Tutor environment.

6.2.3 Open Source Code

Simulations on the PhET platform are not open source applications, as many of them are written in Flash that is not open source software. However, applications are available for installation and running on a computer [221]. The Fractions Lab is open source, as reported on its official website, and the platform's source code can be retrieved from http://www.italk2learn.eu/italk2learnplatform/. Fractions Tutor contains key components written in Adobe Flash [5], which are not open source code.

6.2.4 Executable on smart devices

The simulations provided by the PhET platform are available as an application for smart devices (tablets and smart phones) with both IOS and Android and they are not free of charge. Furthermore, these applications cannot be downloaded for smart devices with different operating system (e.g., Windows Phones). Fractions Lab has some limitations, because is mostly developed on Flash technology, such as use in tablet devices [101], [102]. Avoiding Flash technology could not be avoided because: (a) many of the components of the iTalk2Learn platform are designed in Flash, and (b) HTML5 technology is not as developed as Flash technology for some specific applications [101], [102]. Attempting to use the Fractions Lab on devices with iOS, it has been determined that these limitations exist and the application cannot be viewed normally. In addition, attempting to be used through a mobile with android software, a diagnostic message appeared that Unity WebGL technology is not yet supported [5].

6.2.5 Run as a standalone application on a computer

The PhET simulations can be executed as stand-alone applications on a computer [221]. However, the latest plug-ins in Flash and Java [221] are required. Fractions Lab does not allow the application to run independently on a computer, but the iTalk2Learn code is provided to enable it to install this platform and use it online. The platform code can be retrieved from the following link (http://www.italk2learn.eu/italk2learn-platform/). The Fractions Tutor's exercises and problems are designed to run over the Internet and cannot be run as a standalone application on a computer.

6.2.6 Run on well-known web browsers

PhET simulations specifically designed for fractions are mostly not supported by any Web browser with an exception one of the three simulations with fractions, because it has been created using HTML5. Fractions Lab, as stated on its official website, is compatible with Chrome and Mozilla Web browsers. While running the Fractions Lab on Microsoft Edge, the mouse pointer seems to disappear. Fractions Tutor is compatible with Chrome and Mozilla Web browsers, and generally browsers that can run Flash applications. Of course, Adobe Flash Player is a requirement for the application to run successfully.

6.2.7 Basic operations

Available PhET fractions simulations provide limitations on operations of fractions. In particular, there are only three simulations available for fractions. These simulations provide the concept of the formation of fractions with different objects, and the equivalence of the fractions through different representations. However, other basic operations (addition, subtraction, multiplication, division) are not available. Fractions Lab can perform two of the four basic operations of fractions (addition, subtraction), and comparison of fractions [101], [102], [117]. It can also perform the addition, subtraction, and comparison with all four representation shapes using the corresponding buttons available on the platform. In addition, there are the tools [106]:

a) Equivalence, which provides feedback if two fractions are equivalent;

b) The partitioning tool, dividing a representation and seeing the result of the numerator and the denominator; and

c) Plotting; How two fractions are added together;

The Fractions Tutor covers a wide range of operations in the fractions and is reported in detail [238]. The most basic of the operations covered are the equivalent fractions, the comparison of fractions and the addition of fractions with a sum greater than or equal to the unit [238]. After visiting the official website of Fractions Tutor, it was concluded that fractions abstraction exercises are also covered.

6.2.8 Multiple representations of fractions

The available simulations of the PhET platform allow for multiple representations of fractions. In order to represent fractions the system uses parallelograms (with horizontal and vertical partitions), pies, cylinders, number lines, and three dimensional objects. The Fractions Lab uses four representations. Three of them (area, number line, sets of objects) are among the most important [79]. In addition, the liquid measurement is used as a fourth representation (using a pot/jug) [264]. Overall, PhET uses a variety of fractions representations that includes continuous and distinct fractions, and fractions in one, two or three dimensions (number line, area, and liquid measurement respectively) [106]. Fractions visualized in Fractions Tutor are represented in various ways. In particular, they are represented as pies, number lines and as set of objects [238]. The exercises and problems of the Fractions Tutor show that the parallelogram can also be used as a representation.

6.3 Results

Examining the features of each of the three systems considered in our study, it seems that the PhET simulations for fractions, Fractions Tutor and Fractions Lab are mostly dependent on Flash technology; which is no longer recommended for web applications. Many of these apps are not suited to run on smart devices (phones and tablets), at least free of charge. Only PhET simulations are available for smart devices but it is not free of charge and not available for all operating systems (e.g., not available for Windows Phones). It turns out that there is an objective difficulty if these applications need to run directly in a computer lab in schools, since a proper web browser needs to be selected in order for the application to run properly and will need updating to the latest versions of Java and Flash plug-ins.

There is a good level on interactivity, for all of them. Specifically, there is interactivity of PhET fractions' simulations different for each application, with several selector controls, and response to user interaction is immediate. Similarly, Fractions Tutor except that the student follows a serial sequence to solve the exercises. In order for the next step to appear, the previous one must be correctly completed. The same process applies to all exercises available, i.e., to go to the last exercise, first you have to answer correctly the previous ones. Fractions Lab is highly interactive, the learner can even move the representational fraction objects at any location, the learner can add several different objects, and make changes to shapes, colors, etc. However, one of the limitations is that it does not have ready-made exercises to solve, as Fractions Tutor and PhET simulations have.

Studying the three platforms, only Fractions Lab has open source code and can therefore be expanded further. The other two platforms, PhET and Fractions Tutor, are not open source. In addition, only PhET simulations enable them to be embedded within another web-page, and they can also run as standalone applications on a computer.

The fractions applications of the three platforms can run on Web browsers, but require Adobe Flash Player installations. The PhET's fractions simulations are programmed in HTML5, and therefore can be run on any web browser. The three above systems appear to provide the concept of fractions, allow comparability of fractions and equivalence of fractions. Those that provide the ability for addition and subtraction are Fractions Tutor and Fractions Lab. In fact, the Fractions Tutor provides complete exercises in the basic operations of addition and subtraction, many exercises, and counseling is provided to the learner during the exercises.

Finally, all three systems provide the possibility of multiple fractions representations. All three platforms use the pie, the number line, and the rectangle. In addition, PhET simulations use cylinder and 3D objects, while Fractions Lab uses a liquid meter representation and a set of objects. Fractions Tutor also uses the representation of set of objects.

6.4 Discussion

Observing both the applications along with the simulations provided by the three platforms, it is clear that each of the approaches attempt to represent fractions in a different way. There are both advantages and elements that can be improved in all three systems. In particular, the Fractions Tutor seems to be the most complete system of teaching fractions if compared with the other systems studied in this article. Fractions Tutor enables the student to have a great deal of interaction, to present fractions with multiple representations, to provide help and advice in performing the basic operations of addition and subtraction, and a comprehensive set of exercises. Another positive factor is that it can be considered praise and reward after every complete exercise or work. It is also positive that it is possible to record the student's actions and results in the application, having entered with his own credentials. The results can then be used by the teacher to identify weaknesses and thus support the teaching methodology to help improve student's cognitive progress in fractions. From the other hand, the system is considered limited in terms not providing multiplication and division capabilities for fractions. In addition, the progression system requires that each exercise must be completed correctly for being able to look at the next exercise. Furthermore, the data used for the construction of the exercises are static and do not change dynamically each time the application is running. Another limitation is that it requires dependencies to exist and manual installation of supporting plug-ins to be able to execute successfully on some web browsers. A final disadvantage can be considered the fact that it is not an open source application, is not extensible, and does not support execution on smart devices.

The PhETs simulations are limited, thus it cannot be considered as a complete system for teaching fractions, except for specific purposes. In particular, the simulations of PhET fractions don't provide scaffolding assistance or a set of specific targets. In particular, the simulations developed and provided on PhET assume that teachers use scaffolding and the objectives of each simulation [1]. Moreover, because of the variety of students with different goals when using the simulations, no specific questions or targets have been incorporated into the simulations [1]. They also don't provide exercises for the basic operations of fractions. Finally, the system provides no evidence of being open source code or scalable, and is not provided free of charge for smart devices. Of course, the simulations available for PhET fractions have high interactivity, they have movement and sound, they cheer the student when an exercises is completed successfully, and has the capability to present fractions in multiple visual representations. In addition, the software applications provided are also available as standalone applications to run directly from a computer.

Finally, Fractions Lab is a system that can be used as a supportive tool used by teachers during fractions teaching. It provides multiple representations for fractions, has great interactivity and it is compatible with all the well-known web browsers. It is also open source and it can be expanded. Finally, it allows the execution of two basic operations, addition and subtraction. As a limitation it can be assumed that it does not itself provide completed exercises to solve by the students, cannot be executed on smart devices and does not provide support for two basic operations (multiplication and division). Finally, it is not provided as a stand-alone application to run directly on a computer.

6.5 Conclusions

The academic community seems to be interested in the development of systems that support the teaching of mathematical concepts that dealing with lessons in fractions. Two of the platforms studied in this study (i.e., PhET, Mathtutor) have been developed as University projects. From the other hand, the development of Fractions Lab was supported by state funding for creating a web application that can be used for supporting and promoting the understanding of fractions. Studying the basic characteristics of the fractions simulations, it appeared that the existing systems present several advantages but also have various difficulties both on representing the concepts, but also with regards to design and implementation. By attempting to overcome the difficulties encountered in these systems, but at the same time adopting many of the advantages of the aforementioned systems, we suggest a set of features to be considered during the design and implementation phases for any application to be developed for the teaching of fractions. It is essential that a newly created platform for fractions should be developed as a web-platform (using HTML5 or Javascript). Web or cloud applications are becoming very popular for educational purposes for the following reasons: (a) they are more secure, (b) do not require installation or compliance, and (c) updates to the latest version require no additional intervention from end users [101], [102], [117]. Considering the above, we envision that a newly developed system should be able to run seamlessly on all well-known web browsers and smart devices. Finally, the system should be able to utilize multiple representations for fractions, be highly interactive, should provide full support to all basic operations of fractions (addition, subtraction, multiplication, and division), and should provide dynamic content through a repository of exercises and a tutoring experience during exercise solving.

Survey on ATSs

7

Chapter Seven

7.1 Introduction

Facilitating middle-level and high-level school students on learning mathematical concepts has often characterized as a rigorous and a challenging task for educators [23]. Indeed, communicating mathematical theories, in an understandable form to students, requires educators to develop effective problem-solving strategies and communicate them to students. Briefly, these strategies refer to the theory, guidelines, and steps used by an individual to understand the conditions for a specific mathematical problem and finally determine the right steps for solving it [302]. Wilkerson-Jerde and Wilensky [291] argue that educators are expected to help students to build a cognitive mathematical structure for solving mathematical problems. However, how educators can effectively communicate and trigger the construction or improvement of mathematical literacy remains a challenge.

For students, the process of accumulating and acquiring mathematical skills and techniques during their mathematical training is also not trivial. Different studies showed that even academically capable students have a strong tendency to avoid Mathematics [19], [179]. Mathematics' anxiety has a negative impact on students which often results into restricting their educational mathematical training and forecloses potential career options [19]. Furthermore, fear of failure, self-efficacy beliefs, and achievement goals are factors that influence students' performance in Mathematics [214].

To address students' negative attitude toward learning mathematical concepts, various alternative methodologies have been proposed based on the physical presence of an educator or not. In personal tutoring, educators tend to adapt the educational material to student needs; however, such approaches are costly [23]. With the recent technological trends and especially with the increasing availability of the Internet, there has been a significant increase in the development of web-based learning systems.

Often, such computer pedagogical agents adopt a series of learning-by-doing exercises to direct the learning processes and drive knowledge construction. However, as pointed by Arroyo et al. [18], the intelligent dimension introduced by a computerized learning system should be granted by its ability to understand the physiological activity of students (i.e., by recognizing various emotions) and being able to react to them [18]. In the literature, such systems are well known as Intelligent Tutoring Systems (ITSs) and have been an active field of research in recent years. Alkhatlan and Kalita [11] conduct a comprehensive historical survey for ITSs.

In particular, an ITS implements Artificial Intelligence (AI) techniques in computer-assisted teaching by allowing a full Socratic Method process to develop with an individualized teaching. A classic ITS consists of four interdependent components: (a) a student model, (b) a pedagogical unit, (c) a cognitive unit, and (d) a communication unit [248]. These four elements of a traditional architecture of ITSs could be found with different names, but they used for the same purposes. Specifically, Alkhatlan and Kalita [11] refer that an ITS consists of four basic components: (a) a domain model, (b) a student model, (c) a tutor model, and (d) a user interface model. The student model contains basic information for the student, and this is the reason that the student model is a basic component of an ITS [248]. The domain model stores the domain knowledge, and the tutor model stores pedagogical knowledge and is responsible to decide when and how to intervene [11].

In the last years, a lot of effective ITSs have been developed, and the key areas of research that ITSs were developed are affective tutoring system (ATS), cultural awareness in education, game-based tutoring systems, adaptive intelligent web-based educational system, collaborative learning, data mining in ITSs, and authoring tools [11]. In education also, a number of ITSs have been developed on various subject areas including specific fields of Mathematics. It has been noted that computerized educational systems may be more effective if they can trigger the appropriate emotional states of the students [65]. To enable ITSs and educational games to detect the current emotional state of students, such systems analyze sensed data from various biometric devices and record students' actions (e.g., bodily expressions influenced by emotional reactions). This is essential for such systems in order to develop the constructive reasoning necessary for stimulating the learning processes. It is also important that educational games are able to recognize and react to various kinds of emotions [60]. ITSs that have the capability of understanding or even reacting to students' emotions are known as ATSs.

ATS, according to Ammar, Neji, Alimi, and Gouarderes [15], is an ITS that can recognize human emotions in different ways. The system can embed some devices (camera, microphone, special mouse, etc.) to recognize student's emotions [11]. Once student's emotions are recognized, the system processed it and tries to change the emotional state of student. Furthermore, one of the main aims of an ATS is to change the student's emotional state from negative to positive [307]. Emotions are very important in learning, and thus positive emotions enhance learning while negative emotions could weaken the process of learning [115], [142].

The progress of modern ATSs is huge in the past decade, and it is important to keep updated on the changes that are made in this field. For this reason, this document includes a review of ATS developed for mathematical purposes through relatively relevant recent documents. The main contribution of this study is that it presents detailed information for some ATS as well as the main emotional states that a student experiences when solving mathematical problems. In particular, we present the number of emotional states as well as the type of emotions that the ATSs recognize and the type of architecture and techniques used by these ATS. This study attempts to find common elements that may exist in the five examined ATSs and help future researchers and developers to use them in order to develop ATSs with modern methods and architectures, such as the use of deep learning. In this article, we study five known ATSs that deal with teaching various mathematical concepts in the three levels of education. Each one of these five systems has its own way to detect students' emotional states in order to help them understand the various mathematical concepts. In particular, our discussion emphasizes on how these systems recognize the kind and number of emotions and the way that they do it in order to find the emotional state of the student. The research questions that are asked are:

a) What kind of students' emotions ATSs for mathematical concepts recognize?;

b) Whether there is a minimum or maximum number of emotions that ATSs for mathematical concepts could recognize; and

c) Whether ATSs for mathematical concepts follow common strategies for recognition students' emotional state;

This study is structured as follows: The next section includes the literature review, in which the basics terms for ATS are presented. Then, research methodology is explained. This is followed by a section in which the analysis of the five ATSs (their architecture, their way of functioning, the way and means of detecting the students' emotional state, the strategies of integrating emotions in the systems, the cognitive field of mathematics in each one, and any evaluation for them) is detailed. Then, the research results and three research questions are also presented. After that, a discussion to the summarized results is presented.

Finally, some conclusions with regard to the systems studied along with a critical discussion and suggestions for any future work exploiting the present research are summarized.

7.2 Related Work

An ITS, a computer teaching system, is based on AI and provides individualized teaching [139], [161], [189], [284]. It can also provide instructions and feedback in a personalized way, that is, in a way that it can replace the real teachers [161], every time it interacts with the learners. In addition, if this system can detect emotions of learners and gives feedback to improve learning effectiveness, then it is called ATS [161].

ATS is a type of ITS which detects nonverbal behavior in the real time (e.g., using a camera) and makes use of this information for personalized interaction with the student [249]. It also adopts the emotional state of the student, emulating the behavior that teachers have [6]. In general, ATSs encounter two main issues: (a) the reliability of recognition of the student's emotions and (b) the knowledge of how best to tailor this information when the student's emotions have been identified [9]. Sarrafzadeh et al. [248] state that there are four main components in an ATS: (a) a student model, (b) a set of strategies, (c) a cognitive field, and (d) a pedagogical unit that interacts with the learner. In addition, the recognition of the user's emotional state is divided into two main categories: (a) methods designed to detect emotions based on their physical effects, and (b) methods designed to predict emotions based on the understanding of their causes. ATS is a useful tool and could improve learning outcomes [288].

ATSs are used in many areas of education. Gordon et al. [100] presented the results of a social robot tutor during a second-language learning game on a tablet. In addition, a study that used an ATS for learning a foreign language showed that this ATS improves students' learning interest and provide adequate feedback by recognizing students' emotional state during learning processes [161]. Thompson and McGill [274] presented an ATS called Genetics with Jean, which teaches the subject of genetics, and the evaluation of the system showed measurable improvements in perceived learning. ATEN is an ATS for built environment management [124]. Another ATS, called JavaTutor, is for introductory computer science, and it supports students through both cognitive and affective feedbacks [289]. Furthermore, ATSs could be used in medicine education [4]. An ATS in special education was also developed, and the results showed that there is a significant improvement in learning [191].

Emotions are important for the field of education. Especially, for educational games, recognition of emotions is important [60]. The recording of emotions is mainly done in three ways: (a) with web cameras, (b) using sensory devices, and (c) using log files. The main sensors that can be used are skin conductance sensors, cardiac pulses sensors, electromyography sensors on the forehead [14], as well as pressure sensors on the mouse and the keyboard. The students' emotional state detected by ATS is usually described by theory. A theory that has been used in such AI systems is Ortony, Clore, and Collins (OCC) cognitive theory. OCC cognitive theory [211] describes the emotions of a person as the result of this individual's assessment of how the current state fits with the individual's goals and preferences [68].

One of the ATSs objectives is to change the emotional state of student [307]. ATSs have implemented different approaches to change the emotional state of students. A strategy that implements ATSs for changing the emotional state of students is through animated pedagogical agents. This technique is widespread and a lot of ATSs use it (i.e., PAT2Math, Fermat). Research has shown that animated pedagogical agents seem to tend to increase student involvement and motivation [189], [282]. Also, it appears that the provision of personalized teaching with the use of a pedagogical agent improves student learning [67]. On the other hand, the benefits of having animated pedagogical agent are minimal [176]. Thus, there is a different approach for changing student's emotional state is through dialogue. A pop-up window or a dialogue box can replace the pedagogical agent. Mao and Li [170] conducted a survey in order to examine critical factors affective learner's satisfactions in ATSs. Ten variables most important to ATSs were discussed and divided into three dimensions: (a) learner, (b) agent tutor, and (c) technology. The hypothesis of the number of emotions recognized by ATSs is also examined in this study. The results of the study showed that the learner's attitude toward ATSs depends on some factors, including the number of emotions that has a significant influence on learner's satisfaction.

Ivanova [115] conducted a research in the areas of facial expression and gesture recognition in the context of the role of emotions in improvement of ITSs and enhancing the learning quality. That study presented the categories of methods in facial expression recognition techniques and the existing approaches for facial expression recognition using ITSs. Furthermore, Ivanova's study presented a summarized table with some techniques regarding affect recognition and what kind of emotions are in the scope of research. That paper concludes that facial expression recognition approaches are not universal among different cultures.

Petrovica and Ekenel [223] presented an analysis of emotion recognition methods used in recent ATSs. They study six ATSs for mathematical purposes including four ATSs (Prime Climb, Easy with Eve, Fermat, and PAT2Math) which are also examined in the current study. Also, they examined sensors, emotional data detection, and emotional classification. That study also examined whether these ATSs use facial expression or log files for emotion identification.

In an empirical study, Alqahtani and Ramzan [12] examined the physiological signals in automated tutoring systems. That study compared ITS and a detailed discussion about them is also presented. The classification methods, the methods that are used, and the recognized emotions are some of the highlighted fields that are examined. The study concluded that the use of physiological signals in ITSs is an interesting and promising field. Furthermore, this study examined in detail four ATSs (Easy with Eve, PAT2Math, Prime Climb, and Wallis).

7.3 Research Methodology

For the purpose of this study, we searched electronic databases relevant to information Technology, Mathematics, education, and psychology, that is, Google Scholar, Scopus, and EBSCO. Because there were many articles, we had to restrict our search. The research was restricted first to articles written in English language. Second, the articles have to be peer-reviewed, and finally, the articles have to be published the last decade or last 15 years, based on the last evaluated papers about these systems. Furthermore, in a recent research [12], the studied papers for ITSs were between 2004 and 2018. Because there is a lack of complete ATSs for Mathematics in general, the time period of survey was extended near to 15 years in order to be found ATSs dealing with mathematical concepts for the current comparison in this survey. Also, we search for ATS around the world in order to examine what methods and strategies are used in different continents and countries. Furthermore, we focus to ATSs that deal with mathematical concepts in elementary school in particular, and of course, these ATSs must have been evaluated and tested with students.

Furthermore, the lack of the available ATSs for the purposes of this study was one big obstacle. Thompson and McGill [274] mentioned that while there are a lot of prototype ATSs, there is limited number of prototypes that have been evaluated of their impact on learning outcomes. Furthermore, in the same study, there is not a standard implementation architecture of ATSs. In addition, Alqahtani and Ramzan [12] examined 19 modern ITSs that were published between 2004 and 2018, and only seven of them deal with mathematical concepts. All these seven modern ITSs were initially developed before 2013, and this proves that the examined field has lack of ATSs for mathematical purposes. Furthermore, in Mao and Li [170], it is mentioned that there is no specified research identified important variables dealing with agent-based ATSs. Finally, we only included articles that mention ATSs in details, presenting their architecture, methodologies, strategies, and further information in order someone to understand the way that this system can recognize the emotional state of student and can help him with advice and suggestions. Of course, we tried to find articles from all over the world, that they referred to ATSs that dealing with various mathematical concepts.

We chose to discuss these five ATSs because we found papers that described their architecture in detail, they had been previously evaluated, and they presented no obstacles to using our survey. Of course, in the future, we intended to extend our survey to more ATSs dealing with mathematical concepts. Prime Climb, one of the ATSs, the under consideration, have also been shown to contribute to significant learning outcomes [274]. Easy with Eve, Pat, Prime Climb, and WaLLiS are also examined in a study [12].

7.4 Analysis of five ATSs

We have reviewed literature for completed ATSs that dealing with mathematical concepts. Taking into consideration, the criteria that stated in research methodology of this article, we present five ATSs. We have driven to these five ATSs because we found a lot of data for the whole system, from the initial design of architecture to the final evaluation.

7.4.1 PAT2Math

PAT2Math (personal affective tutor to math) is a web-based system which takes into consideration students' emotions by employing both system interface and facial expressions recognition [119]. PAT2Math consists of multiple agents: (a) agent interface, (b) tutor agent, (c) agent student model, and (d) agent base domain. PATequation tool is a Java applet, integrated to PAT2Math, which is an editor with a powerful expert system, assisting students on a step-by-step solution of first- and second-degree equations. PATequation has been implemented using Drools 4.0, an expert system (ES) shell implemented in Java. PAT2Math functionalities include instant response, hint, next step solution, and problem solving [119]. To determine students' goals, PAT2Math employs the MSLQ [118] which is presented during student's first login into the educational PAT2Math system. After completing the motivated strategies for learning questionnaire (MSLQ), the next step is to determine the possible events that may take place within the educational environment in order to be aware on the desirability of each one of them. Those events may be one action of student or agent.

For example, a student action could be "The student did not give the correct answer", while a system action "Pat did not give the appropriate assistance". In that manner, by knowing the event taken place and the students' goals, PAT2Math is able to determine students' emotions. For a student who focuses on performance, the event "The student did not complete the task correctly or did not complete it at all" describes a negative fact, which causes the emotions of danger and distress. All the above are analytically described in [118].

The system is able to recognize seven emotions: joy, distress, satisfaction, disappointment, gratitude, anger, and shame, according to the OCC model [118]. These events may result from either the student's actions or the software itself [119].

The tutor agent is a female cartoon, named Pat. It consists of two modules, that is, the mind module and the body module, implemented in Java [118]. Being aware of each action in the tutoring environment as well as taking into consideration the goals set by each student, Pat can determine the emotional stage of a student. Each time a tutoring event occurs, the tutor agent comes to a conclusion regarding the student's emotional state and chooses the appropriate tactic to implement [118]. Pat has many available strategies, each of which is applied in each appropriate occasion. An example is described in [118] of a female student focused on performance, who feels incapable of completing a task, in case a task fails. In most of the times, she does not try harder when she faces difficulties, as she considers herself incapable. Pat responds with a message aiming at increasing the student's self-esteem and encourage student to perform the task with more effort.

The system's novelty is that it employs multiple agents, and each of its components is able to act independently while at the same time actively cooperating with the rest of the components. The main reason behind the implementation strategy is mainly the lack of any other open-source ITS on algebra. The aim of the authors was: a) to create and make available to every single school in Brazil, free of charge, and b) to develop an ITS which follows the formal curriculum of the Department of Education in Brazil [119]. Other studies have shown a lack in algebra for Brazilian students, which was the initial reason pushing toward the implementation of this system. A key feature of this system is that it focuses on the cognitive field of algebra, especially on the field of linear and square equations, and that is specifically designed for elementary school students [182].

The tutor agent was evaluated in two ways [118]: (a) with students, and (b) with specialists. For the student evaluation, Pat was integrated in the java agents for distance education framework (JADE) system (a simplistic tutoring environment implemented in Java). There were three evaluation methods: a) JADE only; b) JADE and Pat; and c) JADE and affective PAT. A total of 39 students between 12 and 19 years old participated in the experiment to be evaluated on Earth Time Zones. Eight tutors and psychologists participated in the evaluation of the students. The research came to the following conclusions: (a) Although the tutoring environment with the assistance of Pat improved the students' performance, it is not obvious whether the emotional agent works better compared with a tutoring environment with a non-emotional agent, and (b) regarding the specialists, the agent's existence and its tactics were very accurately designed.

Jaques, Vicari, Pesty, and Martin [120] presented the two evaluations on the proposed emotional model on students. The first evaluation direct approach focuses on measuring the precision of the emotional model, and it was conducted on 24 seventh-grade students between the ages of 12 and 19 years, in a Brazilian school. JADE, an online tutoring environment, was employed on the educational content section of Earth Time Zones. The results of this evaluation were not encouraging, and thus two proposals were suggested toward the improvement of the system: (a) the existence of a pop-up dialogue window, and (b) present the emotion of each student in a simplistic understandable way. The second evaluation indirect approach, which examines the effectiveness of an application employing the emotional model in order to adapt to the user, was performed on two groups of 13 students each. Results show that the model can provide some useful information on student's emotions. For the model evaluation, a self-referencing mechanism for each student was applied (using a simple pop-up dialogue window), in this window students could record their emotions about the system. Jaques et al. [118] evaluated the PATequation tool through a research performed on 43 students between the ages of 12 and 13 years. Students were divided into two groups. The first group consisted of 22 students who solved equations using PATequation, while the second consisted of 21 students who solved equations without any software assistance. Results showed that there is strong indication that students' performance was higher when using PATequation. That is, the observed improvement on students with the assistance of PATequation to solve equations was of high statistical importance.

Authors in [130] evaluated the effect of the metacognitive skill of monitoring knowledge through a research among 107 seventh-grade students of four private schools in Brazil aged between 12 and 14 years. To perform the evaluation, researchers used pre-test, post-test, and grades of metacognitive self-evaluation. Results confirmed the initial hypothesis that the education provided by the tutoring agent improved the students' ability to keep track of their knowledge. Also, there are a number of indications that the agent's metacognitive guidance improves their test results. Moreover, a few clues were found on a stronger correlation between the ability of monitoring their knowledge and their learning capabilities, among the students assisted by the tutoring agent.

7.4.2 Fermat

Fermat is an intelligent social network for teaching Mathematics, incorporating an ITS for teaching and learning [46]. Fermat's architecture was originally presented in [46], and evidence of the implementation and development stages is presented in [308] and [25]. The Fermat system was developed to solve the main problems in Mathematics for secondary education in Mexico, where a very large percentage of the students had an inadequate Mathematics background [23].

The system includes three basic units: (a) expert, later renamed domain [307], (b) student, and (c) tutor, which also has an affective sub-unit. A typical student profile contains: (a) a static profile with personal and academic information, and (b) a dynamic profile that contains information through the interaction with the system and recognition of the student's emotions [46]. The cognitive characteristics are derived from the student's background and from the completed problems and tests. Finally, the emotional moods are detected using a camera and a microphone [46].

To determine the student's cognitive level as well as the proposed teaching method, a diagnostic test is initially performed [46]. The diagnostic test shows the student's knowledge and what needs to learn [23]. The emotional state which recognized by Fermat were originally considering seven kind of emotions (i.e. anger, disgust, fear, happiness, sadness, surprise, and neutral; [46]) and finally adapted to recognize five kind of emotions (i.e. anger, happiness, sadness, surprise, and neutral) [307]. A detailed description for extracting facial features and recognizing emotional status through efficient techniques is described in [308]. It is presented in [309] an analysis on how to recognize the learning style. It is worth noting that Kohonen neural networks and fuzzy logic are used to recognize the emotional state based on data that combine information from camera and microphone [46]. Reading the cognitive state was based mainly on adaptive control of thought-rational (ACT-R) cognitive theory, creating rules and facts with XML rules [24] and Fuzzy Logic. The cognitive state determines the next problem that the student will solve [46].

Zatarain-Cabada, Barron-Estrada, and Zatarain-Cabada [310] described how neural networks can be implemented in a system to recognize learning styles and emotions. The proposed system combines a self-organizing map or Kohonen network to detect learning styles and a neural network for detecting the emotional state. In addition, the authors state that fuzzy logic can be utilized to characterize the pedagogical state of a student. The suggested fuzzy system defines a set of variables to characterize errors, time, emotion, help, and learning style. These variables are loaded when the student solves an exercise. The output of the fuzzy system gives a variable that is the type and degree of difficulty of the next exercise that the student will solve. In addition, the kind of exercise is determined according to the student's learning style.

Fermat also incorporates a pedagogical agent. In particular, the tutor unit presents a pedagogical agent, the Microsoft Genie character, of a character that makes some concrete actions (congratulate, confused, explain, and suggest; [46]). Additional actions were added, and finally, the agent reacts with the following eight actions (acknowledge, announce, congratulation, confused, get attention, explain, suggest, and think; [23]). When solving problems, there is a help button that sends tips or ideas through the agent [24]. The agent appears when the student makes by presenting different actions (i.e. pay attention, explain, suggest, or think actions). The agent reacts (with acknowledge action or congratulate action), when a student successfully solves a problem.

An important feature of Fermat [46] is that it is a smart ATS social network for teaching Mathematics that presents the content of the course in a personalized way. In addition, the cognitive field of the system is Mathematics, multiplication, division, and also fraction issues. However, the operation of multiplication [310] and division [307] has been evaluated. Fermat has been applied to third-grade primary school students in multiplication [46] and natural numbers [22] and primary school students in multiplication and division [307]. In addition, it is addressed to elementary school students (second grade) for learning Mathematics [47].

The system was applied to some groups of students. It was originally applied to 25 students of third-grade class of elementary school and examined in multiplication [46]. The process of assessing students was done by administering a pretest, intervening using Fermat and providing a post-test. Also, another survey [23] in a sample of 72 students in third-grade class of elementary school was examined in multiplication. After some modification and additions were made to the Fermat system, it was almost in its final form. There was again intervention in 33 elementary students [307] in the acts of multiplication and division. In both cases, the evaluation of the system showed that students with a lower cognitive level in Mathematics could see an improved performance [46], [307]. In addition, the system develops various modules for second grade of elementary school students learning Mathematics [47]. Finally, several modules were developed in the field of natural numbers specifically for third grade of elementary school taught in Mexico, but this version of Fermat has not yet been tested in students [22].

7.4.3 Easy with Eve

The Easy with Eve is an ATS for teaching Mathematics suitable for primary school students [248]. It is mentioned as the first fully functioning ATS [115]. Initially, the system was designed as an automated face expression analysis system [7], [249], and gesture analysis [7], but it ultimately uses real-time face expression analysis [250] for Mathematics. In general, the architecture of Easy with Eve consists of the following components: (a) the facial expression analysis unit, (b) the student's model, (c) the teaching strategy unit, and (d) a set of teaching activities in the form of an animated pedagogical agent. The Easy with Eve initially recognized seven emotions (surprise, happiness, sadness, puzzlement, disgust, anger, and neutral; [249], [247]) but eventually presented to recognize eight emotional states (neutral, smiling, laughing, surprised, angry, fearful, sad, and disgusted; [250]). A web camera [6], [7], [249], [247] and a system consisting of: (a) an artificial neural network for face detection, (b) the extraction of facial features, and (c) the fuzzy face expression classifier to detect student's emotional state [247].

Sarrafzadeh et al. [249] described that an artificial neural network is being used to identify the person who is trained. The detection of facial expression of the student is achieved by using fuzzy rules for eyebrows, eyes, and mouth.

Facial expressions using a support version machines method determine the student's emotional state. Emotions are recognized in real time [250]. In addition, the face expression analysis system runs in the background as the student interacts with the system.

There is a female pedagogical agent [6], [7], which called Eve [9], [247]. The way of operation, actions, expressions, and what exactly Eve can do is described in the literature [8], [9], [247], [250]. As described in [7], the pedagogical agent will be adapted in two ways: (a) by presenting the most appropriate material, and (b) by empathizing with the student and providing oral and no-verbal encouragement to the student. Thus, the agent will be particularly beneficial in two ways: (a) if the most appropriate material for the student's emotional state is presented, then this will facilitate the learning process, and (b) if the agent is faithfully honest in her empathy, then could enhance the benefits of motivation of personal impact. It seems in [250] that Eve can express a variety of emotions through facial expressions and can give positive or neutral answers, ask questions, discuss problems or solutions, and give advice or answer her own questions. The strategies to be followed by the pedagogical agent are the result from coding of the study of videotaped teaching of real pedagogues to students aged 8 to 9 years [7], [10], [247], [248]. These video recordings have shown that the neutral behavior of teachers and students is the most frequent, followed by the low intensity students' smiling expression and then high intensity [9], [10], [247], [248]. The implementation of these strategies on the Eve's part is fuzzy approach [9].

Sarrafzadeh et al. [250] introduced Easy with Eve, as a pioneering ATS that bases its teaching strategies on pedagogical actions of real-world pedagogues. There has been a study of observing human pedagogues, how they teach primary school students, and its results have been adopted in the pedagogical strategies of Easy with Eve. It is aimed at primary school students, mainly 8 to 9 years old [6], [248], [249]. The cognitive field that Easy with Eve deals is Mathematics. It was originally designed for basic operations such as addition, subtraction, multiplication, and division [249]. More specifically, it implements the first seven levels of mathematics strategy of New Zealand Numeracy Project and does not deal with fractions, percentages, and proportions [8]. Easy with Eve mainly deals with addition problems [6], [7], [248].

For evaluating the Easy with Eve system, a survey was conducted [250] with 62 students aged 8 to 9 years. There was a statistical evidence of a significant increase from pre-test to post-test results and thus a positive effect on student's performance using the system.

7.4.4 Wallis

WaLLiS is an online interactive and intelligent learning environment specifically designed for Mathematics [176]. Its online environment offers material such as theoretical notes or examples presenting each appropriate mathematical section implemented in the system. Also, its environment consists of a set of interactive web pages which offer easy interaction over various mathematical problems [178]. Moreover, the system combines content-based approaches with adaptive learning strategies excluded from the ITS [178]. The WaLLiS environment offers a dynamic content map, whereas a dynamic website provides access to theoretical notes or the exercise.

WaLLiS is designed based on a client-server model, consisting of the distinctive elements for the client and the server, which are responsible to present the content to the student, to adapt the content, and to provide feedback [178]. Moreover, the feedback mechanism is designed in such a way not to disturb the student from executing a task [178]. WaLLiS includes a student model, which keeps a history of the websites visited by each student. There is also a module that reports personal information of a student. Another distinct feature is that the content is adjusted to the student's specific features (color, size, etc.) as well as on student's profile (e.g., grades), while it is not adjusted to the different educational styles or learning scenarios, as other adaptive systems do.

The goals determination mechanism is executed through the feedback mechanism from the creator of the specific activity [176]. Mavrikis [178] mentions that the emotional states of frustration, boredom, and confusion are recognized, and all positive emotions (such as satisfaction, joy, happiness, etc.) are categorized in a single group. Petrovica, Anohina-Naumeca, and Ekenel [222] presented an evidence that the WaLLiS system can recognize the following emotions: frustration, confusion, and mood (e.g., confident, happy, and enthusiastic). Furthermore, Alqahtani and Ramzan [12] refer that WaLLiS can recognize the following emotions: frustrated, enthusiastic, confused, confident, bored, and happy. Also, emotions can be recorded by analyzing system log files [229]. Emotion detection is performed through a forecasting model based on the J4.8 algorithm, an induction tree based on an improved version of the decision tree algorithm C4.5 [229].

On the other hand, the system does not provide any tutoring agent and elaborate on the fact that the benefits of having such an agent are minimal [176]. On the contrary, the system records all information related to the activities performed by the student using a JavaScript agent. The recorded information relate to student's actions, including mouse movements, mouse clicks, keyboard clicks, and so on. These data are recorded to the server. Information regarding the agent are referred to [174].

The system also records the goals set for each student, the feedback given to the student by the system, and the pre- and post-results of student's evaluation. In addition, the system provides scaffolding assistance and relevant suggestions regarding the student's misunderstandings on mathematical issues, according to predefined rules [175]. The strategy regarding the adaptation of the emotional state is implemented though the feedback mechanism provided by the system, along with the relative suggestions, updates, and proposals it offers. The key characteristic of the WaLLiS system is the feedback mechanism, which provides feedback and suggestions to the student [178]. The feedback mechanism is inspired by cognitive acquisition theories (such as ACT-R) and cognitive scaffolding theories and follows similar approaches to those employed by the Carnegie Mellon University (CMU) cognitive tutors [178].

The feedback mechanism relies on different modules and their intelligence. The system's intelligence is based on JavaMath (a collection of graphical and mathematical objects, etc.) and on the adaptation of the feedback mechanism developed for DANTE (Dynamic Authoring aNd Tutoring Environment) [176]. DANTE is a feedback mechanism which monitors students' demand process, whether they achieve their goals, mouse activity, and provides a kind of emotional feedback which helps students to interpret their actions in a manner that prevents them from leaving their exercises [174]. A mechanism detects the goals set by the creator of the activity and must be fulfilled by the student. These goals form a tree structure in which each goal consists of a number of objectives, and each one has a number of completion conditions and the related misconceptions [178].

Apart from the static theoretical content and the examples, it provides interactive and exploratory activities [178]. It also employs decision trees to detect the emotional state of a student [229]. The creation of the proposed system was motivated from the lack of mathematical skills of science and engineering students [178]. The WaLLiS knowledge field covers the spectrum of mathematical concepts such as functions, derivatives, integrals, and vectors [176], [178].

The evaluation of the WaLLiS system was conducted with the participation of 20 students, divided in groups according to their qualifications and grades on a prerequisite course [177]. The goal of this research was to determine possible methods of predicting the emotional state as well as the students' motives, while they work on interactive learning environments. By examining the log files per student, in which all interactions with the system were recorded as well as the students' feedback, it is possible to determine patterns which contribute to a final conclusion regarding the emotional states of the students.

7.4.5 Prime Climb

Prime Climb is an educational game designed by electronic games for education in Mathematics and Science (EGEMS) team at British Columbia University, with the purpose of helping students to learn factorization [60]. The educational game makes use of a Dynamic Decision Network (DDN) to models a student's emotional state following the OCC cognitive theory of emotions. Two players take part in the game who cooperate in order to climb over mountains divided into numbered sectors. Each player is able to move only in one numbered sector who does not share any factor with the sector occupied by its associate. Points are deducted when the player makes a mistake. Before the game, the student uses the trial section of Prime Climb, having an agent as an associate to get advice in order to think the reasons of fall while at the same time getting specific directions on how to avoid falling or recovering after a fall [68].

Prime Climb employs DDN (an extension of Bayesian networks) which are capable of monitoring environments that change over time. It also uses DDN to leverage information on the probable reasons as well as the observed results of a user's emotional reactions [65]. More specifically, Prime Climb consists of two tools: (a) the magnifying glass which presents the analysis of the factors of each number found on the mountain, and (b) the assistance frame which allows a player to communicate with the tutoring agent [312]. Prime Climb also has a special PDA tool, which provides a tree analysis of a number's factorization.

The nodes and the structure of the evaluation model are presented in [68]. The elements are shown as nodes, they are presented in [68], occurred after two studies [68], of 23 and 10 students respectively, who interacted with Prime Climb. Both studies evaluate system's log files [68]). Nodes and connections occurred are based on subjective observations of real interactions. Then, the mode improves dynamically using data from an official user study [312]. Also, Zhou and Conati [312] present a method through which student's goals are connected with student's personality as well as the connection between these goals and their interaction patterns.

For the reason that Prime Climb is an educational game, it does not seem to implement procedures regarding student's initial knowledge base on natural numbers factorization. It only provides the opportunity to explore various mathematical structures (e.g., multiplication, division, etc.), through practical climbing, while a tutoring agent which assists the player. User's goals seem to be very specific [68]. To determine the personality of a student, the game was evaluated using the Goldberg's 100 standard markers test over the five types of personality of the Five Factors Theory [312].

Prime Climb can recognize 6 of the 22 OCC emotions (i.e. joy, distress, admiration, reproach, pride, and shame; [62]). Regarding each one of them: (a) joy for the incident faced by the user, (b) distress for the incident faced by the user, (c) reproach for the entity caused the incident, (d) admiration for the entity caused the incident, (e) pride for the entity which caused the incident, when it is the same entity, and (f) shame for the entity which caused the incident, when it is the same entity [60]. Those six emotions are modeled by three nodes which can take values: emotion for game, emotion for self, and emotion for the agent [64].

The emotional state of each student was recorded through biometric records combined with game event records which could trigger an emotional reaction, such as the climber's fall [14]. It is mentioned in [62] that biometric signals were recorded, regarding skin conductance, muscle electromyography for those (muscles) involved in frowning and eyebrows, blood pressure, and respiration. Four different sensor sets were used [14] to record those biometric characteristics: a skin conductance sensor, a heart rate sensor, and a set of electromyogram sensors attached to the student's forehead. Each biometric record was synchronized with game log files which could result in an emotional reaction. An electromyogram sensor was attached to each student's forehead [65], while all actions were logged in a log file, used later on to evaluate the emotional state of each student.

Emotion detection is performed through the records of each biometric sensor, which are synchronized with the game's log files that show possible emotional reactions (e.g., a fall or a climb [14]). Emotional states are categorized using a clustering method based on the data collected from the biosensors [14]. A user's emotional state is modeled based on the OCC model, and it is the result of a user's assessment between the current interactions events compared with user's goals. An interaction event may be an action caused by the student or the agent which creates a new time entry in DDN [60].

One of the tutoring agents, as mentioned in Conati [60], is a climbing instructor, who can provide personalized assistance by request. In particular, the instructor can: (a) assist the student in case of a mistake by encouraging him or her regarding the reasons which led to the mistake, and (b) remain neutral. The goal of a tutoring agent is to preserve student involvement on high levels while at the same time providing assistance to learn number factorization [14]. Microsoft Agent Package has been used to develop the tutoring agent, who is animated as a wizard named Merlin [67]. To decide on how and when to intervene or not, the agent relies on a probabilistic model (a dynamic Bayesian network) that makes judgments on student's knowledge on factorization. Its main goal is to provide personalized support to the students in order for them to learn number factorization during climbing while maintaining a high level of involvement in the game [312]. In the second version of Prime Climb, an extra level was added which included of definitions and examples [66].

The system observes all actions performed by a student and through the agents provides an appropriate level advice. The aim is to change the student's emotional state to reach a specific goal. A sample of the nodes involved in an agent's action evaluation and emotion analysis is presented in [68]. A student's emotional state is categorized into three groups (i.e. joy and distress, pride and shame, admiration and reproach) and is presented in three nodes (emotion for event, emotion for self, and emotion for agent) which can take of two values. An example [68] is the case where a student appears to have high probability to feel shame. In such an occasion, the agent decides to provide minimum advice in order to make the student feel better regarding his or her performance. If there is a high probability that the student will feel shame, the agent must be programmed appropriately to maintain its rapport with the student.

The result of an agent's or student's action is subject to the evaluation of the student [65]. Each goal satisfied node affects the emotion for game node in each step. If such a step has been created by a student's action, then each goal satisfied node affects the emotion for self-node, while if it has been created by the agent, the emotion for agent node is affected.

The novelty of the system is that the Prime Climb agent is based on a probabilistic mathematical model in order to provide personalized assistance, which helps students to learn number factorization through an educational game [67]. The presentation of the relations of the probabilistic nature between student's emotional state, their causes, and their effects is performed through the DDN [68]. An extra characteristic is that it provides the magnifying glass tool, which uses a tree form to provide a number analysis into factors.

The same technique is discussed in Zhou and Conati [312] where it provides: (a) a model to recognize in the real time a variety of specific emotions, (b) it evaluates student's emotions in the context of an interactive learning procedure, and (c) it is the first model which performs a probabilistic presentation of the OCC cognitive theory of emotions. Moreover, information regarding a student's current emotional state as well as of those emotions in favor of a student's learning ability will held the tutoring agent to choose the correct interventions [14]. The novelty of the proposed model, as stated in [65], is that it is a first attempt to create a model which exploits data coming from sources such as: (a) what causes the emotions (prognostic model), and (b) emotion results (diagnostic model). The prognostic model is based on student's actions and the agent's interventions, while the diagnostic model is based on recording data from the electromyographic sensors. Another novelty is that the use of information relative to student goals is used as a source of evidence.

The Prime Climb learning field is mainly the factorization of natural numbers. It is designed for sixth grade and seventh grade students [14] of the American education system.

Before the model was completed, it was evaluated using a direct technique [61] where 10 students participated in this experiment. The results showed that pop-up windows did not affect students and that students were not afraid to express their emotions. A second study was conducted in 20 students and showed that they were not annoyed by the pop-up window and that the questions were fully understandable. A study was performed over 20 of the seventh grade in order to evaluate the effectiveness of the Prime club learning agent. Students were divided into two groups: a control and an experimental group. Results were promising and showed that providing personalized assistance through an animated tutoring agent can improve student learning using educational games [67].

In another evaluation performed on 20 seventh grade students, it is reported the precision of the probabilistic models elements, which contribute to the emotional assessment of the students [63]. The evaluation of student goals can be combined with information regarding the reasons and the consequences of their emotional reaction. Results showed that if student's goals can be accurately defined, then the emotional model can preserve a precise estimation of the student's current emotional state.

In another study [64], a survey was conducted with an autonomous agent (not with the Wizard of Oz technique as in previous studies), on 66 students of the sixth and seventh grades from three local schools. A pre-test on number factorization was conducted, and then a post-questionnaire was given to define the goals each student had during the game. Finally, students were asked to fill a personality test. Data analysis showed that a new goal, called Want Help, had to be introduced, and that students tend to give different priorities to their goals.

To evaluate the effect of adaptive feedback provided by a tutoring agent during a learning game, a new study emerged [66] on sixth grade students that were divided into three groups: (a) without an agent (13 students), (b) initial version with the tutoring agent (14 students), and (c) new version with the tutoring agent (17 students). After that, the intervention occurred as planned, and then students were evaluated with a post-test questionnaire. The results showed that there was no difference in learning levels among the three groups of students. Based on the log files, the authors concluded that this lack of difference was because tutoring agent suggested knowledge and skills that were already familiar to the students and so was ignored. A possible solution would be to improve the model's accuracy such that the agent interferes only when there is an actual reason to do so.

7.5 Results

PAT2Math can be considered as an ATS with several advantages and novelties, as it has multiple agents that operate independently and also have the ability to work together. Another observation is that the PATequation tool can be used to provide step-by-step guidance on solving first-degree and second-degree equations, setting goals through the MSLQ, and moving a pedagogical agent. As a disadvantage, it can be assumed that the strategies applied by the agent have to do mainly by actions of the students' interaction with the system, and there is no recording of the emotional state of the student with any artificial mean (i.e., camera). Also, the student's personality is not taken into account. Moreover, no multiple representations are used for the representation of the equations with the interaction between student and system to be kept at the minimum. In addition, there is only one form of static exercises and no multiple representations of exercises or concepts to be taught. Furthermore, the tool lacks the ability to estimate or record at the beginning the level of knowledge of a student.

Fermat is regarded as an ATS with many positive aspects, such as being a smart social network for presenting course context in a personalized manner. It also has a moving tutoring agent able to perform specific tasks. A student's learning level is evaluated through a diagnostic test on the examined mathematical concepts. Emotion detection is performed using Fuzzy Expert Systems. Some limitations of the proposed system reported are the limited number of actions the tutoring agent can perform, the one-dimensional presentation of mathematical operations, the absence of multiple representations of the exercises, and the fact that there is no way to reflect student's personality. Moreover, the detection of a student's emotional state is performed only once, before the start of a proposed exercise, while the emotional state can be altered in multiple ways during the exercise, due to many factors.

Easy with Eve can be regarded as an ATS with several positive aspects, such as, the fact that the teaching strategies applied are based on teaching techniques employed by real teachers. Another advantage is that emotion recognition is performed using cameras in the real time. It is also a positive fact that there is a moving tutoring agent with a human voice, able to express multiple emotions. A limitation of this system is that there in no way to reflect student's emotional personality through a test, nor the knowledge level of mathematical terms which are used by the proposed system. Also, there are no dynamic exercises, which are presented through videos, while there is no interaction between the student and the exercise.

Wallis can be considered as an ATS with many positive elements. The components that make it stand out are the dynamic map of content, the feedback mechanism that does not interrupt the student from his work, customized educational strategies, scaffolding, and interactive web pages with the content of each module. A limitation is considered to be the absence of a pedagogic agent moving to attract more attention to the learner; as feedback through the text frame may not be perceived by the student. In addition, the program provides no screening with a diagnostic test to evaluate the student's cognitive level as well as his personality. It also lacks multiple representations of the exercises and as there are only log files rather a camera, the program cannot determine the student's emotional state. In addition, the model could be improved if more students could assess it.

Prime Climb can be an ATS with substantial novelty. It has many advantages, such as the animated cartoon-like agent, the intervention, when needed, of the agent, its ability to show number factorization in a tree-like structure, and the attempt of detection students' emotions using a probabilistic cause-causality model. However, Prime Climb lacks on that it has not been tested on a large scale, as a great number of electromyography sensors would be required, while at the same time two students with similar skills would be needed in order to be capable to respond better to the game. In such a way, the two students would simultaneously perform the same task, according to the predefined educational curriculum on numbers factorization. Another drawback is that signal recording does not ensure that collected data are solely relevant to the procedure of the educational game, as there may be a number of external factors which may affect the data. For example, it has not been taken into account whether those signals are due to the anxiety a student may feel while wearing the device, due to the lighting of the classroom, or noises created by others, or even other factors relevant to the school and the courses attended.

7.5.1 First Research Question

The first research question of this study referred to student's emotions that could be recognized by an ATS for mathematical concepts. It is important to recognize the emotional state of a student, when solving exercises and problems with ATS, because if the ATS knows the current emotional state then it could try to change it to positive emotion which is a main objective of an ATS [307]. Furthermore, the number of emotions that a system could recognize was studied in [170] and it seems to be a basic factor in ATSs.

The five ATSs recognize positive and negative emotions. The PAT2Math can recognize joy, distress, satisfaction, disappointment, gratitude, anger, and shame. The Fermat can recognize anger, happiness, sadness, surprise, and neutral. The Easy with Eve can recognize surprise, happiness, sadness, puzzlement, disgust, anger, and neutral. The WaLLiS can recognize frustration, boredom, confusion, enthusiasm, confidence, and happy. The Prime Climb can recognize joy, distress, admiration, reproach, pride, and shame. The emotions that can be recognized from ATSs are presented in Table 6.

We notice that the emotions come with its pair (negative/positive). This method is used by PAT2Math and Prime Climb. Both of them are based on OCC model. Also, there are systems that use balance into their strategies to recognize the student's emotions. Specifically, there is a neutral emotion and the same number of positive and negative emotions, for example, one neutral, two positive, and two negative emotions. This method is used by Fermat and Easy with Eve. In [84], the ITS that used for facial recognition expression recognizes emotions in pair (happiness, anger, surprise, sadness, puzzled, and disgusted). Also, in Ratliff and Patterson [234], the recognized emotions are fear, joy, surprise, anger, disgust, sadness, and neutral. Furthermore, there are two ATSs (PAT2Math and Prime Climb) that recognize emotions using the OCC model. Thus, they use pair of emotions (an emotion and its opposite, i.e., joy and distress). Also, there are ATS (Fermat, Easy with Eve, and WaLLiS) that use two groups of emotions, positive and negative.

Name	Recognized emotions	Number of emotions
PAT2Math	Joy/distress, satisfaction/disappointment, gratitude/anger, and shame (according to OCC model)	7
Fermat	Anger, happiness, sadness, surprise, and neutral	5
Easy with Eve	Surprise, happiness, sadness, puzzlement, disgust, anger, and neutral	7
WaLLiS	Frustration, enthusiasm, confusion, confidence, boredom, and happiness	6
Prime Climb	Joy/distress, admiration/reproach, and pride/shame (according to OCC model)	6

Table 6: Emotions that can be recognized from ATSs

Having these observations into account, we could conclude that an ATS for mathematical concepts could recognize a combination of negative, neutral, and positive emotional states. Positive emotions could be joy, satisfaction, gratitude, happiness, surprise, admiration, and pride. Negative emotions could be anger, disappointment, distress, shame, sadness, puzzlement, disgust, frustration, boredom, confusion, and reproach. Some of the above emotions (fear, anger, happiness, sadness, disgust, and surprise) are mentioned as basic emotions in affective computing systems [170]. Also, in a survey of Alqahtani and Ramzan [12] where 19 modern ITSs were examined, the above emotions of this study and some more (interested, concentrated, flow, enthusiastic, anxious, curious, focused, confidence, excitement, smiling, disinterested, worried, inactive, scared, and amused) are mentioned as emotions that recognized by ITSs.

7.5.2 Second Research Question

The second research question of this study referred to a number of emotions that an ATS for mathematical concepts could recognize. Of course, the number of emotions that an ATS can recognize is one of the key factors of students' satisfaction with ATSs [288]. Furthermore, the number of emotions that a system could recognize was studied in Mao and Li [170], and it seems to be a basic factor in ATSs and is counted as second important factor that affecting learner's satisfaction.

The five ATSs recognize both positive and negative emotional states. The Fermat can recognize five emotions and its evaluation shows that this number of emotions could lead in a way to recognize the student's emotions during mathematical learning. On the other hand, there are two systems that recognize seven emotions, PAT2Math and Easy with Eve, and do it with a successful way. The other two systems, Prime Climb and WaLLiS, recognize six emotions. These results are presented in Table 6. Ivanova's study [115] presented the systems for facial expression recognition and recognition of gesture, and the recognized emotions for the scope of research are between six and seven.

Furthermore, we notice that some ATSs (PAT2Math and Prime Climb) recognize emotions with their pair (negative/positive). Also, there are ATSs (Fermat and Easy with Eve) that use a neutral emotional state and the same number of positive and negative emotions, for example, one neutral, two positive, and two negative emotions. In Ratliff and Patterson [234], the recognized emotions are fear, joy, surprise, anger, disgust, sadness, and neutral. Furthermore, in an experimental comparison of 19 modern ITSs, the number of recognized emotions was between two and eight, with average number of recognized emotion to be 5.2 [12].

Having these findings into account, we could accept that a minimum number of five kind of emotions (two positive, one neutral, and two negative), six kind of emotions (three positive and three negative), or seven kind of emotions (three positive, one neutral, and three negative) could lead to a save way to recognize the emotional state of student during learning mathematical concepts. In addition, Mao and Li's study [170] mentioned that six basic emotions (fear, anger, happiness, sadness, disgust, and surprise) are the majority in affective computing systems. Once recognizing the students' emotions, the ATS can help students to improve their performance. Of course, this is not a panacea but a finding, but it depends from the model that uses the ATS in order to recognize the student's emotional state. Also, further research is needed to approve this finding.

7.5.3 Third Research Question

The third research question of this study tries to find whether there is a common way or strategy that ATSs follow in order to recognize student's emotional state. After reading these five ATSs in depth, there are some common elements. Specifically, all of them use a pedagogical agent. Most of them use an animated agent except WaLLiS, which uses a text box with advice, information, and so on. The pedagogical agents could give a plethora of information and advice. These results are presented in Table 7.

Also, these ATSs record the student's emotional state using web camera or log files. Most of them use neural networks in order to recognize student's emotional state. There is a list of basic features of these five ATSs in Table 7.

Having these observations into account, we could provide that different strategies and techniques used by different systems in order to accomplish their targets. This also agrees with a survey [11], which mentions that it is very difficult to find ITSs with the same architecture. In addition Thompson and McGill [274] referred that these systems are complex and they have different architectures. The most of the examined systems in this study use animated pedagogical agent, use media (web camera or log files or combination of them) to recognize student's emotional state, and they accomplish their goals through AI using different techniques.

Name	Agent Name	Type	Actions
PAT2Math	Pat	Animated agent (woman),can move and change face expressions	Chooses the appropriate tactic to implement
Fermat	Genie	Animated agent (man)	Gives advices and ideas
Easy with Eve	Eve	Humanoid animated agent (woman), with body, can move, and change expression	Answers to questions, make questions, discuss problems or solutions, and do suggestions
WaLLiS	-	Text box with advice, information, and so on	Gives information, advice, and help
Prime Climb	Merlin	Animated agent (man)	Gives advice and guidance

Table 7: List of basic features of pedagogical agents of five ATSs

7.6 Discussion

All the features of the five systems under review, which are presented in detail in this article, are summarized in Tables 6 to 9. Table 6 presents kind and number of emotions that can be recognized from ATSs. Table 7 presents a list of basic features (name, type, and actions) of pedagogical agents of five ATSs. Table 8 presents the basic features of the architecture of the five ATSs: their architecture and the way (mean and method) of recognizing the student's emotional state. Table 9 presents further features of the five ATSs: student's level of education, cognitive field of system, originality of the system, using web camera to detect emotional state of student, detect student's cognitive field, and detect student's personality.

The five ATSs that examined in details in this study are used for mathematical training in different fields (equations, multiplication and division, addition, functions, derivatives, integrals, vectors, and factorization natural numbers). These ATSs are used mainly for the elementary school except Wallis, which used in university because of its content. Some of them use web camera, and others use log files in order to recognize the emotional state of student. Both of them show that they can recognize with success students' emotions and can help students for mathematical training.

Name	Architecture	Mean	Method
PAT2Math	Agent interface, tutor agent, agent student model, agent base domain, and pedagogical agent	Log files	Takes into consideration students' emotions by employing both system interface and facial expressions recognition
Fermat	Three basic units: expert, student, tutor that has a subunit called affective	Web camera	Kohonen neural network
Easy with Eve	Four components: student's model, teaching strategy unit, facial expression analysis unit, and set of teaching activities	Web camera	Use of SVM. Consists of: a) artificial neural network for face detection; b) extraction of facial features; and c) fuzzy face expression classifier
WaLLiS	Client-server model. There is: a) student's model; b) Studex component; and c) feedback mechanism	Log files	Forecasting model based on J4.8 algorithm
Prime Climb	Dynamic decision network using nodes and connections between its components	Biometric signal sensors and log files	Dynamic Bayesian networks and clustering

Table 8: List of basic features of the Architecture of the five ATSs

One finding of this research is that a minimum number of emotions could be used in order to motivate students for Mathematics. This minimum number is between five and seven. We cannot for sure say if an ATS that can recognize five emotions is better than a different ATS that uses seven emotions. But we can say that an ATS that could recognize five to seven emotions could give the appropriate information in order to help student for mathematical training. PAT2Math that recognizes seven kind of emotions helps student to improve their performance in Mathematics [119], [130]. Also, Fermat that recognizes five kind of emotions helps students improve their performance [46], [307]. Furthermore, Easy with Eve, which recognizes seven kinds of emotions, has positive effect on student's performance [250]. Furthermore, there are other systems that could recognize six kind of emotions [84] or seven kind of emotions [234].

This research concludes that ATSs have to recognize both positive and negative emotions, as they tend to occur in pairs. This is also observed in other survey, where a system that used for facial recognition expression, recognizes kind of emotion in pairs [84]. Furthermore, another finding is that ATSs which use web cameras for recognition of facial expression (Fermat and Easy with Eve) could recognize a neutral state. This is also observed in other survey, where a system uses a web camera and among to recognize kind of emotions is the neutral [234]. It is obvious that a lot of times the face expression cannot be categorized, and it is important to have the neutral state. Also, another finding is that the gender or the type of pedagogical agent seems not to affect the whole process, as in this study, there were woman (Eve and Pat), man (Genie and Merlin), cartoon (Pat, Genie, and Merlin), and humanoid (Eve). It appeared that generally the pedagogical agent helps students in the educational process and motivate students to Mathematics, independently of the gender. This is in contrast to research that found an effect of female pedagogical agent on motivation and learning than a male pedagogical agent [228].

Name	PAT2Math	Fermat	Easy with Eve	WaLLiS	Prime Climb
Student's level	Elementary	Elementary	Elementary	University	Elementary
Cognitive field	Linear and square equations	Multiplication and division	Addition	Functions, derivatives, integrals, and vectors	Factorization natural numbers
Originality	Multiple agents	Smart social network, personalized style	Teaching strategies based on pedagogical actions of real-world pedagogues	Feedback mechanism	Probabilistic student model, personalized support
Web camera	No	Yes	Yes	No	No
Student's cognitive field	No	Yes	No	No	Yes
Student's personality	No	No	No	No	Yes

Table 9: List of features of the five ATSs

In addition, the ATSs that have animated pedagogical agent are shown, through the evaluations of each system that presented in details earlier in this research, to help finally students. This agree with other studies that the presence of animated pedagogical agent has small but significant effect on learning [252] or has encouraging evidence of learning [192].

Finally, it seems that when the mean of student's emotions recognition is a web camera, then the ATS has more chances to be accurate in prediction of student's emotions. This helps the ATS to propose the appropriate suggestions and exercises in order the student to gain its goals. We conclude to this finding for the four ATSs that use web camera from the evaluations through surveys in students that were mentioned earlier in this study in "Analysis of five ATS" section.

7.7 Conclusions

There is a high interest from both the academic community and educational community in the development of systems with integrated techniques of determining an individual's emotional state, and using that state to help students with understanding mathematical concepts or even change their perspective against Mathematics. Moreover, this trend appears to be worldwide, as there are many systems developed around the globe, such as in Canada (Prime Climb), Mexico (Fermat), Brazil (PAT2Math), New Zealand (Easy with Eve), and the United Kingdom (WaLLiS). Those applications cover all levels of the educational system, beginning from the first level (Fermat, Easy with Eve, PAT2Math, and Prime Climb), to the second (PAT2Math and Prime Climb), and to the third (Wallis).

By examining the aforementioned systems, it seems that whether it is an educational game (Prime Climb), or a social network (Fermat), or teaching mathematical concepts (Easy with Eve), or a web-based system (PAT2Math, WaLLiS), there exist opportunities for detecting the emotions of a student while interacting with these systems. The emotional detection through appropriate strategies, as already discussed, provide evidence for improving students to better understand the mathematical concepts, and change their perspective for Mathematics [20]. Moreover, recent research showed that positive psychological attitudes and predisposition to Mathematics is an important factor (along with others) that influences how a student is performing in Mathematics [54].

This study shows that there are systems that could recognize a minimum number of emotions (five to seven) that can represent the emotional state of student during learning mathematical concepts and they can motivate students. This minimum number of emotions could be taken into consideration when it could be designed a new ATS for Mathematics. In a survey for affective computing techniques for ITS, there are systems that recognize between six and seven emotions [115]. Also, there is a group of emotions, which follows

OCC method, which could be into consideration when it is designed an ATS for mathematical concepts.

Furthermore, this study shows that there are a lot of strategies and architectures that could be proposed for the design of an ATS for Mathematics. Also, a study [115] concludes that facial expression recognition approaches are not universal among different cultures. Thus, we cannot propose one specific architecture or strategy, but for sure, the techniques and methods that they will be used have to adopt AI. AI may be considered as a key factor and taking into consideration, when it is designed an ATS. Also, if this proposed system has animated pedagogical agent, independently of the gender, then according to the findings of the current survey, it will have a lot of chances to motivate students in Mathematics. Of course, the findings of this study indicate that using a web camera to recognize student's emotional state is very helpful.

We note that further research can be conducted on ATS specifically designed and developed for other mathematical fields, such as, in trigonometry, matrices, fractions, and so on. There is also the opportunity to further examine the factors that may cause various kinds of emotions during the use of an educational system, and then examine these factors to determine whether these are caused due to internal (e.g., exercise difficulty) or external (e.g., difficulties in the classroom environment) situations. Being aware of the emotions origin, it may be easier to convert a negative emotion to a positive one through the appropriate intervention.

Teachers' Focus group

8

Chapter Eight

8.1 Introduction

In the current chapter, there are presented the results of the interview with Greek teachers of elementary school using the focus group method. There are presented the questions that were asked and the teacher's answers.

Fractions are a difficult learning subject for primary school students [39], [53], [72]. Thus, many students face difficulties in fractions [103], [233], [290], [181], [57]. The results of a survey showed that students have difficulty comparing the 8/9 and 11/12 fractions [300]. Except from students, adults also face difficulties with fractions [262], [180], [279]. Furthermore, students have difficulties in understanding the basic operations of fractions, such as the addition of fractions [103]. A research study showed that 1/3 of students fail in the addition of fractions with different denominators [27]. However, another research study found that students have higher achievements when adding than when subtracting fractions [128]. Division is another difficult aspect of Mathematics [107] and many students face difficulties with division [55]. Thus, since a fraction is essentially a division, many students face difficulties, and there are misconceptions regarding fractions.

Additionally, students face difficulties in understanding the multiplication of fractions [233], [258]. Overall, learning fractions is one of the significant difficulties in the development of mathematical thinking among students [230]. Moreover, students have problems when asked to create different representations of fractions [128]; and they also have misconceptions about fractions [128] and difficulties in understanding the concept of fractions [281]. In conclusion, students face many different challenges when dealing with fractions.

To help students overcome these difficulties, teachers try to apply different strategies and approaches concerning fractions. They use geometrical shapes to introduce fractions, such as circles and parallelograms [276]. Additionally, they often refer to pizza as an example of fractions [269], and some teachers use the example of apples and watermelons when teaching fractions [26]. Furthermore, to teach the part-whole relationship, it is suggested the use of bread, apple, cardboard to present the half-quarter concepts [2]. Of course, teachers also use other real-life examples, such as chocolate or cake, to introduce fractions to students.

Additionally, an example from sports, namely hurdles, has been used in research studies for students to understand the multiplication of fractions with a natural number [233]. Another example from sports, using the swimming context, was found to stimulate the students' informal understanding of the meaning of fractions and it can be used in addition to other teaching material for fractions [103]. Furthermore, in a study presented that storytelling is another technique that may help students understand fractions and incredibly low-performance students [156]. In this context, every teacher is trying to use techniques, materials, and real-life objects to teach fractions to students.

Generally, teachers can represent fractions in various ways and use various objects (e.g. circle, parallelogram), different from whole numbers (e.g. number line), but this may create further problems to students because they need to learn new algorithms. The fractions' approach should be different and multifaceted to achieve its purpose, which is the learning of fractions. The multifaceted construct of fractions (e.g. part-whole, ratio, quotient, operator, and measure) further increases the difficulties students are faced with [28], [153]. But, teachers sometimes use the multifaceted construct of fractions because students learn in different ways and have different learning styles. This means that the most effective mode of teaching or studying may be different for each student [216]. There are different learning modalities (visual, auditory, tactile, and kinesthetic), and teachers have to teach fractions in different ways if they do not want to leave any student behind. Also, teachers need to know the students' cognitive level to select the most appropriate materials, lessons, and exercises for their students.

To identify the cognitive level of students regarding fractions, some teachers use assessment tests. The traditional assessment provides grades based on test scores [150]. As mentioned in [131], teachers use data from diagnostic assessments to adjust their instructions, i.e. by identifying the areas students have mastered and failed. Also, teachers use test data to provide feedback to their students; however, such feedback may not offer the diagnostic data required to support later learning [150]. Thus, students may still fail to understand mathematic concepts and could encounter difficulties with fractions. Finally, teachers sometimes do not use appropriate strategies and techniques, and thus, their students may fail to understand fractions. More specifically, teachers sometimes do not use appropriate examples to engage students, and also do not use manipulatives in the classroom as often as they should.

In this context, the teachers trying to implement the Analytical Curricula use various supervisory means and technologies to pass on knowledge to their students. Curricula sometimes indicate such means, but sometimes, teachers select the tools that they consider meaningful and reliable, based on their experience. The use of visual representations can help students understand the sense of numbers, decimals, and fractions [69]. Through multiple representations, the educator tries to help all students and to communicate knowledge. The use of fingers, abacus, chopsticks, cubes, and other manipulatives, help teachers present a mathematical operation and also serve as a tool for problem solving.

Apart from the traditional way of learning, the teaching of Mathematics can also be done with the use of new technologies. Specialized software is available to help students understand mathematical concepts. Since general strategies regarding the use of technology in the classroom cannot be uniformly applied for all subjects, a specialized strategy is required for the technology to be efficient and meaningful in the classroom [126]. The teaching time, the classroom, the quality and the size of the classroom, and the level of the students' computer knowledge are some obstacles that need to be overcome, for the teacher to implement the teaching module using new technologies in the classroom. The software's multiple representations include pies, area models, and other objects that the students are familiar with in their daily routines. For example, the circle is linked with pizza or with the traditional pie, because it is easier for the student to think that he has to distribute a pizza to some friends rather than having to break a circular disc into some pieces. Such platforms, using various representations of fractions are Fractions Lab, Fractions Tutor, and PhET [172].

In conclusion, students encounter many difficulties when asked to solve problems with fractions. Even with the assistance of teachers, the problem persists, and many students are still facing difficulties with fractions. Thus, there is an urgent need to design a platform for the teaching of fractions, which should be designed based on the examples, strategies, and methods that the teachers use in the classroom when teaching fractions.

The purpose of this research is to identify the strategies, techniques, and methods that in-service teachers of primary schools in Greece utilize in the classroom to teach fractions and assist their students. Also, this research has attempted to map the difficulties that in-service teachers have identified in their students. The current research was conducted via the focus groups methodology, through which we attempted to explore how in-service teachers teach some concepts of fractions. Areas explored include teaching strategies for fractions, representations of fractions, and tools/manipulatives used for fractions. The findings of this study will be taken into consideration in the design of a web based system for fractions. The research questions in this study were:

a) What are the difficulties that students encounter with fractions? and

b) Which strategies, methods, and tools do primary teachers use when teaching fractions?

8.2 Related work

The literature review identified past studies conducted to explore the strategies teachers use when teaching fractions. These studies have also examined the methods that the teachers use to engage their students as well as the manipulatives being used for this purpose.

More specifically, one study [195] attempted to identify effective strategies used by teachers to improve the teaching of fractions. Several students (178 sixth graders and 92 seventh graders) and eight female teachers participated in this study. The researchers observed 12 lessons for fractions over four months, collected data from videotaping teaching sessions, observation notes, and also from interviews with the teachers. The interviews included four questions focusing on engagement strategies, teaching styles, classroom materials, and promotion of thinking and reflection. The findings of this study suggest that there are areas which can be improved in the teaching of fractions. One such area of improvement relates to how the teachers could engage students, i.e. using real-life examples and explaining the applications of fractions in the context of the students' lives. Another area of improvement is identified in the use of manipulatives. Moreover, teachers could help students more efficiently when they are aware of the students' prior knowledge.

In another study [26], the main goal was to explore the teachers' perspectives regarding the teaching of fractions. This study included 126 third grade primary teachers (91 females and 35 males). The study was a descriptive survey, and the participants were administered a questionnaire, consisting of 14 open-ended and pre-coded questions. The findings of this study show that teachers focus on the part-whole and the division meanings of fractions. Also, the majority of teachers introduce fractions using examples and materials from daily life, but some teachers prefer geometric shapes. Finally, the teachers believe that the significant difficulties and misconceptions regarding fractions among their students relate to the ordering of fractions, operations of fractions, proper and improper fractions, and the meaning of numerator and denominator.

8.3 Research method

8.3.1 Participants

Twelve (12) primary school teachers participated in our survey; on average, the participants had a teaching experience of 27.5 years, their classroom experience was mid-career and later career, and all participants were certified computer users.

The participants were from two different groups. The first group included six teachers, four males and two females, while the second group included six teachers, three males and three females. Each group of participants was interviewed at a different venue on a different date, and the interviews were held in an empty dining area, in non-working hours, away from school to avoid the participants being influenced by school-related emotions. The main subject of the interviews was the teaching of fractions, and the same moderator conducted all interviews. Each interview lasted approximately two hours, was conducted in a relaxed mood, and was focused on specific questions. The discussion guide included seven questions that were raised and discussed in more detail below.

8.3.2 Focus group method

Focus groups are a research method used to identify the perceptions, attitudes of respondents, and actually, the use of this method has increased in recent years [293]. Focus groups are essentially group discussions, in which a group of respondents discuss a specific subject under the coordination of a moderator.

According to [293], a focus group consists of 4-12 respondents, one moderator and has a duration of 1-2 hours. The respondents, with the moderator, discuss a specific topic in a non-frightening environment that will encourage team interaction. The aim is to explore the participants' opinions, attitudes, emotions, and ideas.

The respondents participating in focus groups are in a relaxed mood, share their views, express comments on what is said, and urge each other to speak up [38]. Self-disclosure among participants is encouraged for the researchers to gather and analyze qualitative data [293]. Indeed, this methodology produces findings which would not be captured through the use of a standardized questionnaire, because there are no limitations on the respondents' answers.

8.3.3 Procedure

The moderator explained the main goals of the interviews to the participants and informed them that they have the right not to answer or not to participate in the discussion of a subject they are not comfortable with. All participants were free to leave at any time. The basic axis of the semi-structured interviews included the following questions:

a) Do you use any diagnostic test to establish the student's level of knowledge regarding fractions?

b) Do you use any questionnaire or any test to identify the student's learning style?

c) When you teach fractions to students for the first time, how do you teach them?

d) What representations of fractions do you believe are more helpful to the students?

e) Which sections of fractions do students experience the most difficulties with?

f) What strategy do you use to compare fractions, and which representations? and

g) What strategy do you apply regarding unlike fractions?

All the teachers participated in focus groups discussion, which was audio recorded while the moderator was also taking notes. The audio file was then transcribed and translated from Greek into English, based on the thematic analysis. This process was followed in order not to lose any of the details discussed during the focus groups [140]. Furthermore, the participants were asked to elaborate on the answers provided for each question asked. A qualitative analysis of the data and the key points from the teachers' data was carried out. The transcription of the focus group, performed by the moderator, was anonymized to ensure the participants' confidentiality. The main focus of the discussion included the following areas:

a) Strategies implemented by the teachers to identify the students' learning profile;

b) Methods used by teachers in teaching fractions;

c) Difficulties that the students encounter regarding fractions;

d) Representations used by teachers when teaching fractions;

The qualitative data collected through focus groups were processed through thematic analysis for the researchers to identify the systematic recognitions, to organize and understand the conceptual schemes in a large set of qualitative data [41]. The thematic analysis was based on the following six-phase approach [41]:

a) Familiarizing yourself with the data;

b) Generating initial codes;

c) Searching for themes;

d) Reviewing potential themes;

e) Defining and naming themes;

f) Producing the report;

8.4 Results

8.4.1 Teachers' strategies in identifying the student's profile

The first area explored was how teachers could identify the prior knowledge of students regarding fractions to establish connections with the new knowledge. The data suggest that the respondents do not use any questionnaires or other tests to identify the students' level of knowledge regarding fractions. Instead, they mostly employ the repetition of previous exercises on the whiteboard, because they prefer to repeat the basic concepts. One teacher, Angelo (pseudonym), stated that recognizing the students' level of knowledge regarding fractions is not necessary, since "the book gives you this opportunity". Another teacher, Kostas (pseudonym), said that he reminds the previously taught knowledge regarding fractions in one lesson before he begins teaching new concepts". Only one teacher, Sofia (pseudonym), is using a test of fractions that she has prepared herself, to identify her students' existing knowledge regarding fractions.

The participants were asked whether a questionnaire or a computer-based test to establish the students' level of knowledge regarding fractions would help them, and they all agreed that it would be useful to them. Indeed, Angelo stated that "there should be stages on the test; five stages to see how many students are in each stage". Furthermore, the teachers stated that they would prefer to use a ready-made test instead of creating it themselves, in order to save time, but also because, as one teacher said, "skilled professionals should develop such a test". Moreover, the participants believe that such tests should be developed for the fourth, fifth, and sixth grade of primary school, in the form of an electronic application, ideally using images and sound.

Regarding how the teachers explore the students' learning styles, through the discussion it became evident that teachers do not use any specialized test or questions to identify the most appropriate learning modalities (visual, auditory, tactile, and kinesthetic) for their students. However, teachers stated that they get such information based on their experience and the use of various examples. One teacher, Mary (pseudonym), said that "we are trying to understand the student's learning style, without knowing exactly what the actual subject that each student needs to perceive is". Another teacher, Peter (pseudonym), said that when a student fails to understand, he prefers to explain the material twice, or three times, or more times, in different ways, even during the recess (break), but does not use any material to identify the student's learning style.

The participants were also asked whether they use any ready-made tests, consisting of simple questions, to help them identify the students' learning style, and they agreed that this would be a useful tool. However, Angelo raised objections saying that "it will help you know whether a student has an auditory, visual, or whatever style, but this means that you should have the ability to personalize teaching". Another teacher, Anna (pseudonym), said that it is not necessary to identify the students' style, because she can obtain that information based on her experience.

8.4.2 Teachers' methods for teaching fractions

Regarding how the teachers introduce fractions to their students, the teachers said that they teach fractions using examples from everyday life. More specifically, the presentation of the fractional unit is done through reference to orange, pizza, or chocolate. Indeed, Anna said that "there must be a practical way, to have an orange, cut it, and then move to the theoretical part". Kostas also said that "for the fractional unit, you can also have a bag with twenty candies and you can share one-fifth, you do not share a single piece". The same teacher added that he has even referred to the fractional unit, using chestnuts. Another teacher, Sophia, employs paper folding to teach the half, the quarters, the eights and sixteenths. Peter said that he is using paper folding, apples, or a circular container with triangle-shaped servings of cheese.

Teachers reported that students have great difficulty understanding the fractional unit. They also stated that if students understood the fractional unit, then it would be easier for them to understand everything else following that. Moreover, the fractional unit would have been well-comprehended before the students get stuck in the comparison and the equivalent fractions. Indeed, fraction equivalence constitutes a significant difficulty for students [201], and this is the reason why all the participants pay particular attention in explaining the fractional unit.

It is worth noting that two of the participant teachers argued that a computer based approach would be a perfect solution, and there are online tools available, which present fractions in a playful way. However, one teacher disagreed with this statement.

In respect of the methods that the teachers use to compare fractions, the participants stated that to enhance the students' comprehension, they visualize fractions on the whiteboard, using various techniques. They begin by comparing the fractional unit, then they compare fractions having the same denominator, and finally, they compare fractions with the same numerator. The example of pizza is the most commonly used representation for fractions.

Sofia said that if there are difficulties in understanding these examples, she will repeat the examples as a method to enhance the students' understanding. At the same time, another teacher stated that he is using different ways to explain the comparison of fractions, converting them to like fractions and decimals.

Also, the teachers agree that up to two representations would assist students, for example, a circle and a bar graph, but more than two representations would confuse them. The use of the circle is the most commonly used representation because students can relate it with everyday examples, such as cake or pizza. Some teachers also believe that when there is a problem with the comparison of fractions, the students may not have comprehended the comparison of whole numbers or decimals and that the students have prior knowledge gaps regarding fractions. Although it would be better if the students' prior knowledge had been refreshed, this takes time, and the teacher could be behind schedule with the curriculum; as Angelo said: "it's always a double-edged knife".

The teachers also agreed that using software with animation, allowing the representation of up to two fractions at the same frame, would significantly help students comprehend the comparison. Moreover, the teachers do not use the number line, because they believe that it will further confuse the students and impede their understanding.

On the issue of the strategies that the teachers use for teaching unlike fractions, the participants staged that for the basic operations (addition and subtraction) of unlike fractions, the students have first to find the Least Common Multiple (LCM), i.e. students have to find a number to multiply each fraction to transform it into a like fraction. Teachers seem to use numbers only and not a graphic representation of the fractions because it would take a long time to draw it on the whiteboard; however, one male teacher stated that he prefers to use a circle representation such as the pizza example.

Kate (pseudonym), stated that she uses another approach for some students. More specifically, if students do not respond to the teaching of fractions, then she will refer to previous knowledge, for example, repeat teaching of equivalent fractions. Anthony (pseudonym), added that regarding unlike fractions, in particular, the software could be beneficial because it could suggest specific exercises and theoretical sessions for the students to catch up with the required level of knowledge and Angelo agreed that the visualization through software would be beneficial. Sophia commented that students have difficulty in adding and subtracting unlike fractions, because they do not understand how they can find the LCM. Thus, students often are not aware why they need to follow the specific order of actions, i.e. the algorithm of a process or operation.

8.4.3 Difficulties that the students encounter with fractions

All the teachers reported that the fractional unit concept is challenging for students and students also face difficulties with comparing fractions and with the equivalent fractions. Indeed, the fraction equivalence is one of the most important mathematical concepts in primary school, and students often face difficulties [201]. Peter added that the students face difficulties in comparing fractions because they have to use an algorithm to make the comparison. The addition of unlike fractions is also tricky for the students because they need to know the specific algorithm to do this operation. Furthermore, Kostas stated that the students also face difficulties when they are asked to do operations with improper fractions.

Additionally, problems involving fractions are difficult for students. Finally, the multiplication and division are two of the most challenging operations for students to perform because they do not know when to multiply and when to divide. Of course, one teacher said that he sometimes struggles with fractions.

Another aspect of fractions that the students find difficult to understand, as Angelo said, is "what is the meaning of the number that was produced as the result; they add, let's say, like fractions when the numerator is too big concerning the denominator, and they cannot tell how close this number is to the whole number". Kostas added that "students cannot visualize fractions, as an image, in their minds; they cannot make an object, they cannot shape it; I try to draw it; this is the major problem, that they cannot think what the problem is asking for; its meaning".

Furthermore, the teachers pointed out that the students also face difficulties when they have to represent the fraction as a decimal or as a mixed number. Converting numbers from one format to another is difficult for the students. Angelo noted that the students have not understood that "the fraction is a division; the 3/4 is a division, three divided by 4; it is also equal to 0.75 as a decimal or to 75% as a percentage". The same example of 3/4 was used in [213] with five sub-constructs (part-whole, ratio, quotient, measure, and operator), to explain why the multifaceted construct is one of the factors contributing to the students' difficulties in learning fractions.

8.4.4 What kind of representations teachers use for fractions

The teachers reported that certain fraction representations are more helpful to the students than others. More specifically, they believe that the pizza and the jug (graded filling) representations help the students more. They also believe that the simultaneous representation of fractions (i.e. pizza and bars) can be more helpful for the students to understand fractions. Anna added that when the parts of the fractions are represented, then the students will better understand the fractions comparison.

Moreover, the participants commented that animation looks like a game to the students, and thus it stimulates their interest. Generally, a static image does not stimulate interest, said Kostas, who has used a game "that had three questions and for every correct answer, you took a step to cross a river and reach a rock". He also referred to another, snowball multiplication game, where the player could, with every correct answer, throw snowballs across; however, if the answer were wrong, then the player would get snowballs thrown at him instead. Angelo added that software regarding fraction is necessary and can help students understand fractions.

8.5 Discussion

8.5.1 What difficulties do students encounter with fraction?

The teachers argued that the aspects of fractions that the students face particular difficulties with include the concept of the fractional unit, fraction comparison, equivalent fractions, multiplication and division of fractions. This is consistent with other studies suggesting that the students face problems in comparing fractions [300], in the division [55], and also in multiplication and division of fractions smaller than the unit [263]. Furthermore, in [26] equivalent fractions, comparing fractions, operations of fractions, are some of the obstacles and difficulties that the students face with fractions. Also, the participants in our study, believe that the interpretation of the outcomes from exercises with fractions is also tricky for the students to understand, as well as the representation of a fraction as a decimal or a mixed number.

The strategies that the participants employ to explain fraction comparison is the graphic representation on the whiteboard, mainly drawing objects like a pizza. When understanding fractions are complicated, the teachers employ repetition and do not use the number line. Past research has shown that the number line, when used as a representation of fractions, does not significantly help students solve fraction exercises [276]. The difficulty in understanding fractions may indicate that the students have not comprehended the comparison between whole numbers or decimals either, and the representation of fractions is considered overly complicated compared to the representation of whole numbers. The participants believe that the software-based representation of fractions would help them when teaching fractions and they think that up to two different representations may help the students.

The teachers participating in this study believe that the critical element regarding operations with unlike fractions is the understanding of the LCM. They stated that the students do not know the algorithm of the process for transforming unlike into like fractions to perform operations (addition or subtraction) then. This finding is also consistent with another survey suggesting that students have difficulty finding the LCM [56]. Although the participants believe that the graphic representation of a circle on the whiteboard would help students understand fractions, they try to avoid it, because it is time-consuming. Also, if a student does not respond effectively, they repeat the teaching of equivalent fractions. In this context, they believe that the use of software for unlike fractions would undoubtedly help students, especially if it would suggest exercises for the students to practice.

8.5.2 Which strategies, methods and tools do primary teachers use when teaching fractions?

In our research we found that the teachers do not consider that identifying the students' level of knowledge regarding fractions is necessary and they believe that repetition is enough to fill this gap. A possible explanation is because the prior knowledge test will have to be performed by themselves; they believe that such testing would be difficult for them to perform and it would require a lot of time as well. This finding is in line with [210], in which teachers were found to avoid the use of diagnostic tests that would guide teaching decisions, because of the time that they would have to spend for administering, interpreting and implementing test results. However, if the teachers who participated in our study had such a diagnostic test, developed by experts, available in electronic format, they would, of course, use it. Moreover, in [150], the use of computers to efficiently administer and score Cognitively Diagnostic Assessments are strongly supported. Another study has also found that cognitive assessment for fractions can be successfully performed through digital learning games [133]. In any case, if the teachers know their students' prior level of knowledge regarding fractions, they would be able to help their students more efficiently [195].

The participants also believe that they do not need to identify the students' learning style because, based on their experience and the various ways and examples that they are using, they can help each student understand fractions. Indeed, students understand fractions better when fractions were taught in different ways [104]. Again, if the participants had a ready-made test available for identifying the students' learning style, they would use it and believe that it would be a useful tool for them to help their students.

Our data shows that the participants only use area models and sets to represent fractions when introducing fractions to their students for the first time. They also use real-life examples (e.g. pizza, orange, chocolate), and this finding is in agreement with previous research [26]. The participants believe that the area models (circles and rectangles) and sets are better forms for the first representation of fractions to the students. Of course, the use of technology and appropriate software would be the right solution. Moreover, the teachers pay particular attention to the teaching of the fractional unit, because they consider it as a significant factor for the further development of their students' mathematical thinking.

Teachers also try to visualize fractions on the whiteboard, when they teach the comparison of fractions. They prefer to represent fractions with pizza and do not use the number line. If they identify difficulty in understanding fractions, they repeat the examples. Another approach is that they use different representations of fractions, and they use two simultaneous representations (e.g. a circle and a bar graph) to help their students.

Finally, the strategy implemented by teachers for unlike fractions is the LCM. Teachers only use numbers and not representations when teaching operations for unlike fractions. Another technique used by the participants in the repetition of previous knowledge (e.g. equivalent fractions) and the teachers also repeats equivalent fractions for the students who do not respond.

Our research findings also show that when teachers teach the fractional unit to their students, they prefer to use circle and rectangle (bar), utilizing real-life examples such as pizza, orange, and chocolate. The use of such shapes, and especially the use of the circle and the rectangle, is referred to in other studies [276] which also support that they help students understand fraction exercises (part-whole). Another survey [269] also refers to the example of pizza as the material with the highest contribution in the effective teaching of fractions. Furthermore, the examples of apple, cake, and watermelon, are used to introduce fractions to the students [26], and the teachers also employ paper folding to teach halves, quarters, eights, and sixteenths.

Also, the participants stated that they do not use the number line for fractions and this finding is in agreement with another survey [276], showing that the students could solve fraction exercises with the representation on the number line. Furthermore, only 20% of fifth-grade students could place 3/4 in the correct position on a number line, marked from zero to one [105]. The everyday life examples of pizza and cake are also used by other teachers in the teaching of part-whole, as reported in another study [56]. Moreover, in another research study [53] the students' performance was found to be better in part-whole tasks (using parallelogram) than in tasks measuring the ratio, operator, quotient, and measure (using the number line). Another study [201] found that students perform better with the area models than the set models and number lines. Finally, another study [128] found that the students tend to fail in representations such as the number line and perform better when other representations are used.

Regarding the representation of the fractions, the participants in this study believe that these representations are useful (up to two), mainly in the form of a circle (e.g. pizza) or a rectangle/bar (e.g. chocolate). This finding is consistent with past research showing that multiple fraction representations for multiplication help the students [297], and when teaching employs many ways of representation, this helps students understand Mathematics [303]. Furthermore, the use of multiple representations in the teaching of Mathematics helps the students understand and improve their problem-solving skills [128]. The participants also believe that the students are motivated, and their interest is stimulated by software because of the animation and sound help students. Furthermore, previous research suggests that the representation of fractions through digital applications, using virtual manipulatives, help students in the conceptual, but also procedural, learning of fractions [241].

The participants also believe that software in Greek, with specific features, could help them teach fractions more effectively to their students. Virtual manipulatives, animated exercises with fractions, exercises with an increasing degree of difficulty, help, and feedback, are some of the fundamental principles that must be taken into consideration for designing and implementing software for the teaching of fractions. Another survey [269] has also found that the use of audio-visual material positively contributed to the teaching of fractions.

Finally, the teachers agree that multiple representations help the students, provided that there are no more than two representations used at the same time, and acknowledge that digital application with animation and sound would spark the students' interest and would certainly help them understand fractions.

8.6 Conclusions

The teachers who participated in our study were in-service teachers, with many years of teaching experience in the classroom. Selecting these teachers as our respondents were not accidental but were based on a study [141] which has found that the teachers' age, experience, and pedagogical knowledge, had a positive effect on the students.

The data analysis produced useful conclusions and scientific findings that were consistent with the findings of past research. More specifically, to teach the fractional unit, the teachers use everyday life examples that the students are familiar with. The use of the orange example for the representation of fractions was impressive; this can be explained by the fact that orange is a widespread fruit in Greece. Furthermore, teachers prefer examples of pizza, apple, chocolate, and paper folding to teach fractions. Also, the teachers want an electronic questionnaire or test to identify their students' level of knowledge regarding fractions, provided that experts develop this questionnaire. Tests for the identification of their students' learning styles, in the form of digital applications, would also be useful for the teachers.

Students, on the other hand, encounter difficulties understanding the concept of the fractional unit, equivalent fractions, multiplying and dividing fractions, as well as difficulties with fractions problem solving. Teachers use graphic representations, drawing circles on the whiteboard to teach fraction comparisons. They also believe that software, presenting up to two different representations, could help the students. Furthermore, the teachers consider the understanding of LCM to be very important as a significant prerequisite for teaching, unlike fractions. They also believe that a digital application, with a graphical representation of unlike fractions, would help students. Finally, the teachers suggest several specific features that such software should include to help students.

All these conclusions will help us design a web-based platform for the teaching of fractions. More specifically, we will be taking into consideration that representation is a crucial element. We will also take into account the specific difficulties that the students face, and thus, the software would provide training regarding the fractional unit and the comparison of fractions. The software would also include exercises related to the fractional unit, employing different representations (e.g. pies and rectangles). In our software, the comparison of fractions would be represented in different ways (e.g. pies, rectangles, lines) and two representations of fractions will be shown at the same time, i.e. in the same frame.

Moreover, the software could represent fractions dynamically, using sliders for nominators and denominators.

The researchers intend to expand this study, including a larger sample of in-service teachers (using questionnaires), to confirm the findings of the present study. It is also the researchers' intention to develop software that will help in the teaching of fractions, taking into consideration the features suggested by the participants, since their views and remarks are the results of their long teaching experience in Greek primary schools.

Proposed System

9

Chapter Nine

9.1 Introduction

The previous chapters presented the difficulties that students have in fractions. They usually encounter problems when solving exercises in fraction comparisons, as well as in the basic operations of fractions. In fact, in Chapter Five, the difficulties that students have in comparing fractions were presented. This results in stress and anxiety and ultimately leads to poor performance in this vital area of Mathematics. The role of the teacher is limited in time and aims mainly at providing knowledge to students, as well as to acquire skills in handling fractions.

Also, in Chapter Eight in-service teachers presented their views on the methodology, teaching method and strategies they apply, the means they use, but also what they would like to be able to help students in the classrooms. From the reading of the positions of the teachers, it emerged that the presentation of the fractions with shapes in the traditional way in the whiteboard is the method that they usually use for the representation of the fractions. Of course, this process of representation is time-consuming and is usually done by the teacher, with the result that the student is usually a passive recipient.

Existing online software offers the possibility to each student individually, if there is a computer or tablet for each one, to be able to become the same operator of the application and perform the available tasks. In other words, the student’s participation in the lesson changes and from passive, who only watched the teacher draw the fractions with surface models on the board, he becomes active, since he handles the program himself. The software that exist for fractions, and presented in Chapter Six, while offering many ways of presenting fractions, cannot recognize the student’s emotional state while students solve fractions’ tasks. Besides, Chapter Seven introduced the Affective Tutoring Systems, which can detect students’ emotional state, developed for various areas of Mathematics, but not for fractions.

The findings mentioned above were the main motivation for the design and implementation of the online educational environment for fractions **S**tudent‘s **K**nowledge and **A**ffective level for **F**ractions in an **O**pen **S**ystem (SKAFOS). This system was designed to help understand the comparison of fractions. It uses multiple representations of fractions and recognizes the emotions they create in students when solving fraction comparison exercises. The teacher can use this information to design a series of exercises with the type of fraction representation that helps the student in solving fraction comparison tasks.

9.2 Features of the environment of SKAFOS

A review of the literature of e-learning platforms for fractions and the Affective Tutoring Systems for Mathematics presented in Chapter Three, as well as the views and strategies applied by in-service teachers in the classroom, has shown that an effective environment for the teaching of Mathematics should have the following characteristics:

a) Be online and run through the web so that it can be run on all known web browsers.

b) Be an independent platform and digital device so that it can run on both personal computers and smart devices (tablets or mobile phones).

c) Be able to visually provide mathematical activities with different representations, both individually and simultaneously.

d) Be able to recognize emotion when performing mathematical activities with different representations so that the teacher can take this into account in the process of assigning mathematical activities to the students.

e) To enable the student to be actively involved, to explore and to experiment with the mathematical activities assigned to him each time.

f) Enable the student to choose the representation he prefers for mathematical activities.

So far, several software and web applications have been developed to teach fractions that provide multiple representations with various objects. However, there seems to be no system for fractions that provides and cumulatively meets the above standards internationally. Especially in Greece, there is no other system that recognizes the emotions of students when solving problems with fractions in primary education. The system so far supports the comparison of fractions through five different representations.

9.3 Characteristics relating to the fractions

From the analysis of the results of the research that was done for the difficulties that the students face in the fractions, but also from the difficulties that the teachers identify in the students, some characteristics that SKAFOS should have emerged:

a) Students learn differently so that they can provide the representation of fractions with different representations;

b) Make a dynamic representation of the fractions so that the student can understand the changes he makes to the formation of the fractions;

c) To be able to experiment with the formation of the fraction when solving activities and problems;

d) Provide dynamic representations in the comparison of fractions;

e) Provide different representations when comparing fractions;

f) Provide fractions to be compared with different properties (e.g. common denominator, common numerator) so that students can understand differences visually;

g) To be able for the student to compare two fractions of his choice and to be able to see their difference simultaneously in the same representation;

h) To be able to recognize the feeling experienced by the student when comparing fractions;

9.4 Dynamic characteristics of fraction comparison representations

To solve fraction activities, students can actively engage in the formation of fractions. This feature initially enables them to experiment with the formation of fractions in visual form with various representations (numbers, circle, number line, and rectangle). This makes them familiar with the form that a fraction can have when a particular representation is selected. For example, when they have the fraction 2/3, they visually see that the circular disk has three equal parts, and only the two parts are colored.

The system is designed to make it easier for the student to form fractions so that they are not distracted by the screen showing the fractions. Also, it provides the ability for the student to be able to experiment easily, without spending any special time, in the formation of the two fractions that need to be compared. In addition, the student dynamically sees the change in the format of the representation each time he changes the denominator or numerator. In this way, the student approaches the process of forming the fractions and builds the knowledge that will be needed afterwards to compare the fractions.

Also, the student can, at any time, focus on the representation of only one fraction, hiding the other. He can even hide both representations of the fractions if he prefers. Also, the system enables the student to have a new representation of fractions in the same frame, further facilitating the understanding of the act of comparing two fractions.

9.4.1 Design option I: Slider for fraction formation

The first essential design choice that had to be made was to form a fraction.

Various software use text boxes to create a fraction where the user enters the numerator or denominator value, as indicated in the text box, as shown in Figure 23.

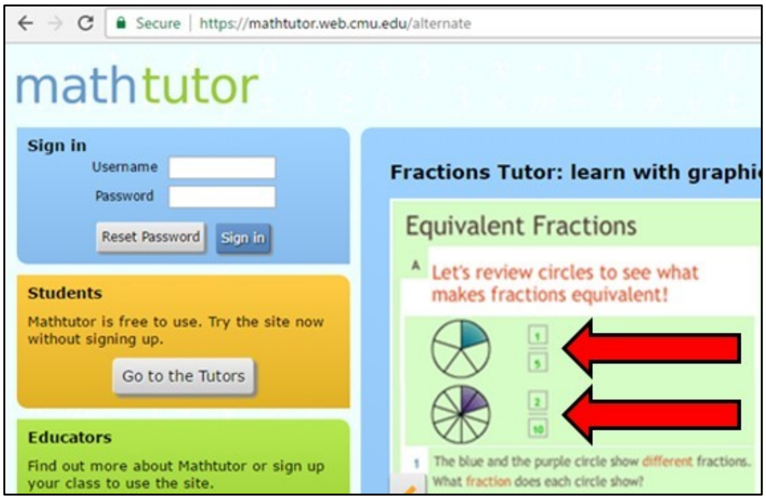

Figure 23: Fraction value fluctuation in a text box.

Other fractional software provides two two-button arrows, up and down. Based on what the user presses, the number indicated on the numerator or denominator increases or decreases, as shown in Figure 24. In both cases, the user is distracted by the fraction and can sometimes devote considerable time to the formation of fractions.

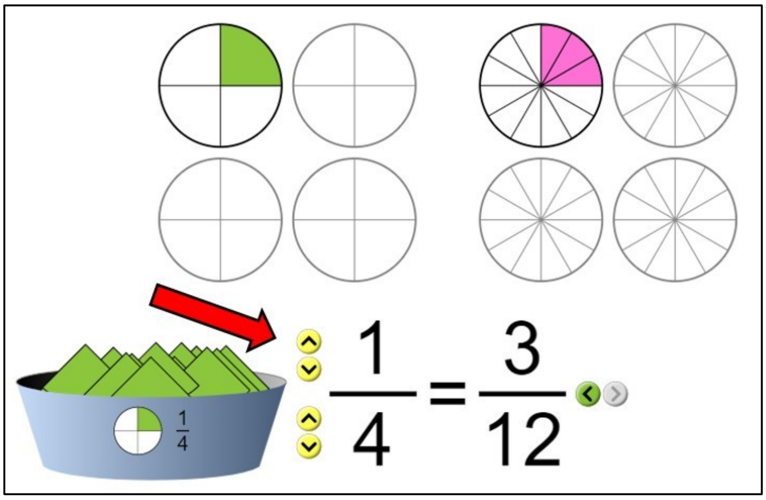

Figure 24: Fraction value fluctuation with arrows.

Taking into account the above considerations, something different was chosen for the proposed system to change the value of the fraction. What was selected in SKAFOS is the use of a slider to change the value of the numerator and denominator. Specifically, as shown in Figure 25, the denominator is initially formed by selecting the slider, which essentially slices a circular disk into as many sections as the slider shows. To make the value of the denominator more distinct and clear to the student, a small black circle, as shown in Figure 26, is placed on each piece of the circular disc.

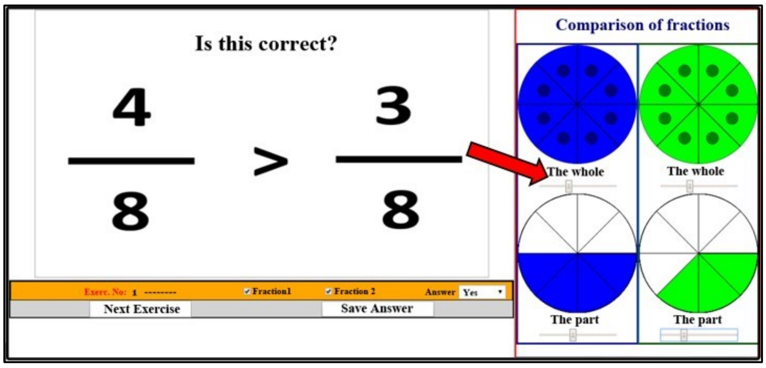

Figure 25: Denominator value fluctuation with a slider.

In this way, the student is focused on understanding only the formation of the value of the fraction. Adjusting the sliders is very easy by moving the corresponding button to the right or left. At the same time, there is a dynamic change in the representation of the fraction and those parts of the circular disk that the numerator has are displayed in color.

9.4.2 Design option II: Representation with a circular disk of the two comparable fractions in the same frame

The second fundamental design choice that had to be made was how to present the comparison of the two fractions. Until now, most software in the fraction comparison process provided the two fractions separately next to each other, as shown in Figure 23. This shows each fraction separately, but in cases where one fraction is very close to the other makes it difficult for the student to make the comparison or sometimes confuses him.

What SKAFOS offers is to present the two fractions separately, next to each other, but to compare them in parallel in a new frame where both will appear, as shown in Figure 27 on the right. The advantage of this representation is that it dynamically displays the two fractions in the same frame, and the student sees their difference at the same time. Specifically in Figure 27, seeing only the two fractions, someone cannot clearly understand which fraction is larger than the two having the two representations in separate boxes. But, if he have the right representation in Figure 27 where the two fractions appear together, he can clearly understand which fraction is larger. In this way of representation, the student can structure his knowledge of fractions, capture representations and make connections between the value of the fraction and its graphical representation. Also, he can experiment with the sliders of the two fractions to be compared, form other different fractions, see their difference and slowly build his knowledge of the meaning of fractions. This develops students' intuitive ability to perceive approximately the value of fractions through the simultaneous representation of fraction comparison.

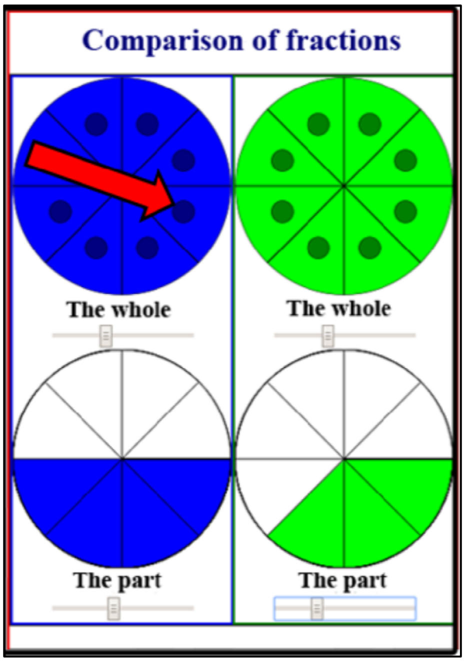

Figure 26: Discrete circles for the denominator value.

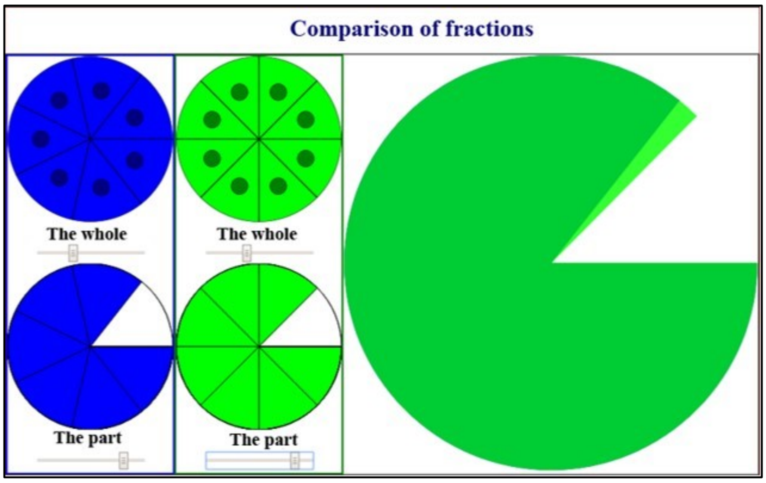

Figure 27: Representation of the two fractions in the same frame.

9.4.3 Design option III: Representation with rectangles of the two comparable fractions in the same frame

The third essential design choice that had to be made was that of presenting the comparison of the two fractions with a rectangle. So far, most software in the rectangle fraction comparison process has provided the two fractions separately next to each other, as shown in Figure 28. This shows each fraction separately, but in cases where one fraction is very close in the other, it makes it difficult for the student to make the comparison or sometimes it confuses him.

What SKAFOS offers is to present the two fractions with a rectangle separately, next to each other, but to compare them in parallel in a new frame where both fractions will appear, as shown in Figure 29 at the bottom. The advantage of this representation is the dynamically display of the two fractions in the same frame, and the student sees their difference at the same time.

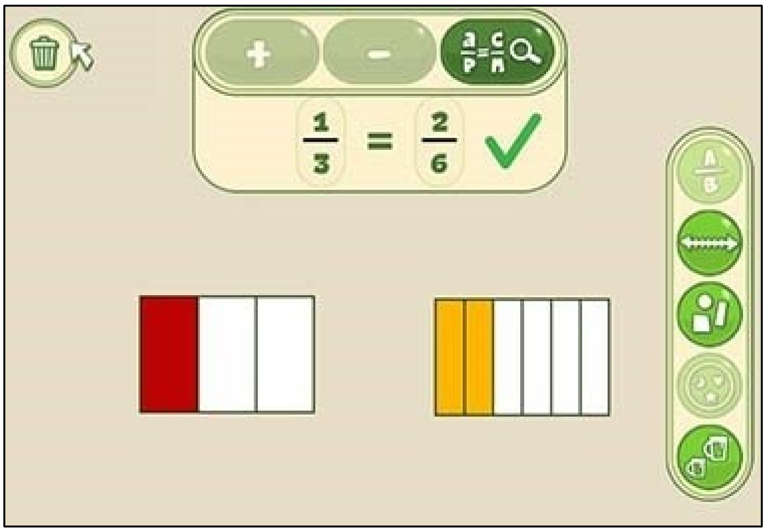

Figure 28: Comparison of two fractions with rectangles.

Specifically in Figure 29, seeing only the two fractions it does not clearly show a fraction is larger than the two having the two representations in separate frames, but looking at the representation in the lower frame where the two fractions appear together it is clear which is larger. In this way of representation, the student can structure his knowledge of fractions, capture representations and make connections between the value of the fraction and its graphical representation. Also, he can experiment with the sliders of the two fractions to be compared, form other different fractions, see their difference and slowly build his knowledge of the meaning of fractions. Thus intuitively develops the ability of students to perceive approximately the value of fractions through the simultaneous representation of the comparison of fractions.

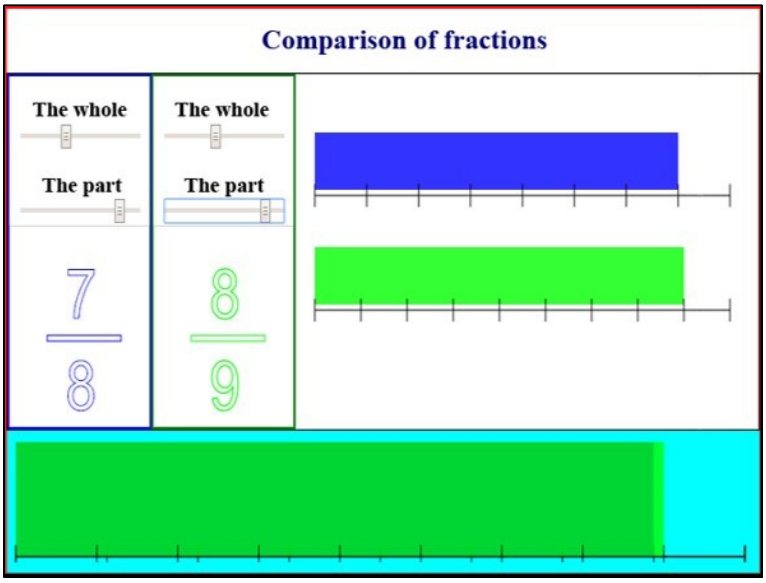

Figure 29: Representation of the two fractions with two rectangles in the same frame.

9.5 Subsections of the SKAFOS

The developed system provides a web interface where students perform comparisons of fractions. Fractions are visualized on a virtual canvas with different representations. Also, the system can recognize emotions during the execution of tasks. In brief, the proposed system consists of three subsystems. The first subsystem is responsible for the graphical representation of fractions. There are five different representations, and they use virtual manipulatives such as circles and rectangles. The second subsystem is responsible for capturing the student's facial expression. Final, the third subsystem which is mainly responsible for analyzing the emotional state of students. The facial expression subsystem can recognize seven affective states (i.e. anger, disgust, fear, happiness, sadness, surprise, and neutral) while students perform a set of exercises on fraction comparisons. The choice of the seven emotional states was made because these seven emotions were used by other systems (e.g., Fermat) for mathematical concepts.

9.5.1 Sub-system for Fractions' Graphical Representations

The first subsystem is responsible for drawing graphical representations of fractions, as an area model (circle) or a rectangle. The objects of circles and rectangles are created dynamically and they are represented on a canvas.

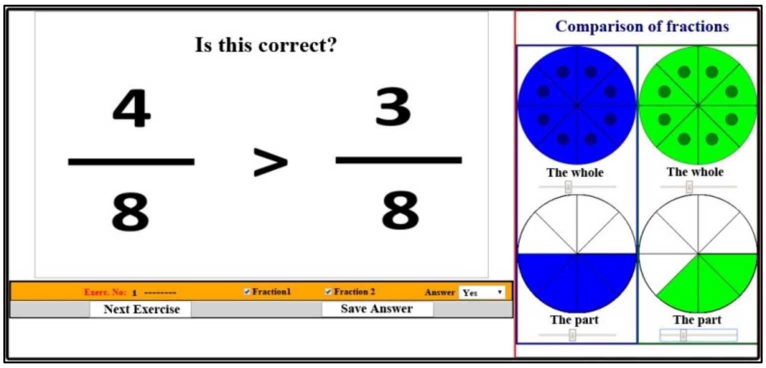

Figure 30: Screenshot of a random problem in fraction comparison in the second representation with circles.

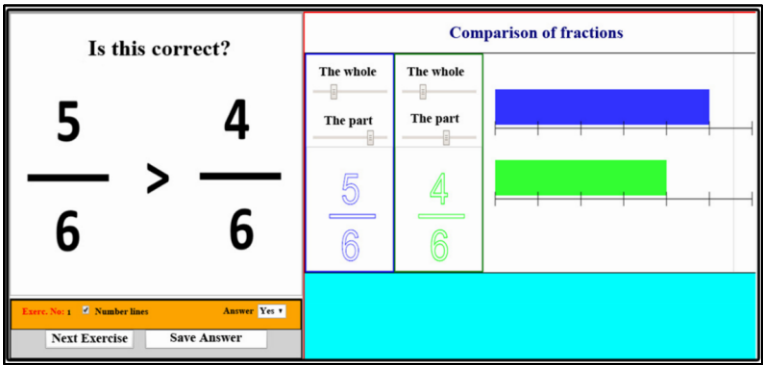

Figure 31: Screenshot of a random problem in fraction comparison in the third representation with rectangles.

The circles are part of area models. To represent the part-whole fractions, the circle is divided into equal parts as the denominator. Then the parts of the whole, which represent the nominator, are colored (with either blue or green), as illustrated in Figure 30. The student can choose and represent a proper fraction, which is a fraction greater than 0 but less than 1 (e.g., 8/9).

The rectangles have a number line based in a graduated straight line. To represent the fraction in rectangle with a number line, the straight line is divided into several parts equal to the denominator's ratio. Then, a colored line with width as the nominator is appeared on top with a color (either blue or green), as illustrated in Figure 31.

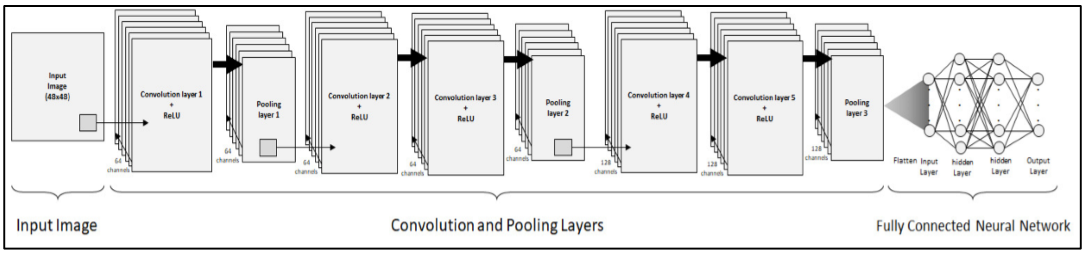

Figure 32: Architecture of the CNN model for emotion recognition from an image.

9.5.2 Sub-system for Emotion Recognition

The second component is responsible for capturing a facial image of the user, once the answer of the current question is selected. The image with a face is saved in a Portable Network Graphics (PNG) format, of size 256×256. A simple web-camera is used for capturing the facial image in real-time.

9.5.3 Sub-system for Emotions Analysis

The third component of the system is responsible for performing the analysis of facial emotion. This part was developed using deep learning. This subsystem takes as input the facial image of the user. It produces as output a seven dimensional vector that represents the following emotions: anger, disgust, fear, happiness, sadness, surprise, and neutral.

Deep Learning solutions have yielded outstanding results in different machine learning applications including image recognition [147], image retrieval [224], scene recognition [311], emotion recognition [77], [200], [159], [183], and speech recognition.

Our Emotions Analysis subsystem is developed using a CNN, which is a modified version of the traditional neural networks. This CNN classifies images according to the emotions recognized by analyzing a facial image. The architecture of the CNN model for emotions recognition is presented in Figure 32. The CNN model is split into three parts: (a) the Input Image part; where a 48×48 matrix with the face in black and white is used as input to the system, (b) the Convolution and Pooling Layers; where a set of three convolution layers and three pooling layers is responsible for detecting features, and (c) the Fully Connected Neural Network; used by the system to classifies the extracted features.

More specifically, the CNN is a feed-forward neural network which extracts features from an image and uses the back-propagation algorithm to classify the extracted features. The system is feeding with a two-dimension grey scale face image (48×48 pixels), in the form of 48×48 matrix. This matrix feeds a series of convolutional and pooling layers in the second part of the system, as it is analyzed above.

The second layer of the architecture, as shown in Figure 32, consists of three convolutional layers and three pooling layers utilized to extract several feature maps. The pooling layers are used to reduce dimensionality, the number of parameters, computational time and control over-fitting. More specifically, the first convolutional layer consists of 64 channels with 5×5 kernel size with stride one, using the Rectified Linear Units (ReLU) as an activation function. The step that the convolution filter moves each time is called stride. An extra layer of zero-value pixels is added around the feature map to prevent shrinking.

Thus, the size of feature map is (48−5+1)×(48−5+1) = 44×44. Also, the first pooling layer has a 5×5 window size with a stride of two, using the max function, and this means that in the 5×5 matrix, it stores in the feature map only the maximum value of the 25 numbers of the matrix each time, whereas, the feature map is 20×20. The second convolutional layer consists of 64 channels with 3×3 kernel size, with stride one using the ReLU function, where the size of the feature map is (20−3+1)×(20−3+1) = 18×18. The third convolutional layer consists of 64 channels with 3×3 kernel size, with a stride of one using ReLU function, whereas, the size of the feature map is 16×16. The second pooling layer has a 3×3 window size with a stride of two, using the average function, and this means that in the 3×3 matrix saves in the feature map the average value of the nine numbers of the matrix each time, where the feature map is 7×7. The fourth convolutional layer consists of 128 channels with 3×3 kernel, with a stride of one using the ReLU function, whereas, the size of the feature map is (7−3+1)×(7−3+1) = 5×5. The fifth convolutional layer consists of 128 channels with 3×3 kernel, with a stride of one using the ReLU function. The size of the feature map is 3×3. The third pooling layer has a 3×3 window size with a stride of two, using an average function, whereas, the feature map is 1×1.

The general expression of a feature map in a convolutional layer, as defined in [298] is,

$$y_j^l = \theta\left(\sum_{i=1}^{N_j^{l-1}} w_{i,j} \otimes x_i^{l-1} + b_j^l\right), j = 1, 2, \dots, M \qquad (1)$$

where, j is current feature map of the current layer; l is current layer; θ is the activation function; $w_{i,j}$ is the convolution kernel; x_i^{l-1} is the i^{th} feature map of the previous layer; b_j^l is the bias of the j^{th} feature map of the current layer and M is the number of feature maps of the current layer.

The ReLU function activation function is defined as follows:

$$f(x) = \max(0, x) \qquad (2)$$

The fully connected neural network is consisted of the classification part of the system and uses as input a one-dimensional array. This means that the two-dimensional array needs to transform to one-dimension, using a flatten function. The neurons in the fully connected layer are fully connected to all of the activations of the previous layer. The function of the output value of neurons is defined as follows:

$$h_w^b = f(w^T x + b) \qquad (3)$$

where, $f(x)$ is the activation function; x is the neuron's input feature vector; w^T is the transpose of w, which is weight vector; b is the bias.

Furthermore, the number of neurons in the input layer of the fully connected neural network is 128. Also, there are two hidden layers with 2048 neurons, whereas the output layer contains seven neurons.

Finally, the implemented system uses a Softmax classifier, to classify the facial expression into the seven facial expression types. Note that the classifier prevents values higher than one and lower than zero. This means that the final output will be a seven-value vector V with values between 0 and 1. It is worth noting that:

$$\sum_t V_t = 1, t \in [0,6] \qquad (4)$$

Initially, the implemented model detects patterns that are fed to the neural network. Our model uses a pre-trained Haar Classifier for face detection (for more details the reader is referred to [285]) that uses grey-scale images. At a later step, the original image is cropped and resized in a 48×48 pixel image, only with the face and without any unnecessary information that may appear on the original image. The cropped version of the image is used as the input matrix 48×48 to the CNN model.

For training and testing the CNN model, we have used the FER-2013 facial expression dataset [99]. The FER-2013 dataset consists of 28,709 images suitable for original training data, 3589 images for validation, and 3589 images for testing [200]. Also, it contains 4953 images that capture the "Anger" emotion, 547 images with the "Disgust" emotion, 5121 images with the "Fear" emotion, 8989 images with the "Happiness" emotion, 6077 images with the "Sadness" emotion, 4002 images with the "Surprise" emotion, and 6198 images with the "Neutral" emotion [99].

We choose this database of images, because it has a big dataset of images and we can train better our CNN because a deep learning model requires an extensive database for training from scratch [132], [155]. Furthermore, we choose this database which contains adult facial images, because of the limited amount of facial images of children. Also, this was the reason than in research for affective robot tutors focused on primary school children [70] used the FER2013 dataset. Additionally, there are only three publicity databases that contain children emotional images [132] with a small number of images from a small number of different children. Specifically, (a) The NIMH Child Emotional Faces Pictures Set (NIMH-ChEFS) has 482 emotional frames of children between 10 and 17 years old [80], (b) The Dartmouth Database of Children's Faces with a set of photographs of 40 male and 40 female Caucasian children between 6 and 16 years old [71], (c) The Child Affective Facial Expression (CAFE) has 1192 images of children between 2 and 8 years old [163]. Furthermore, in [132] it is proposed a new emotional database, called LIRIS-CSE that contains movie clips/dynamic images of 12 ethnically diverse children between 6 and 12 years old.

Our CNN model was trained with FER-2013 dataset, a batch size of 128 and for 300 epochs. The validation of our CNN was made using the FER2013 validation dataset images. The FER-2013 used in a lot of researches, i.e. [70], [199], for training and validating of CNN models. Moreover, to avoid overfitting, we use dropout layers, a technique that is also used in [70]. During the training, our model obtained an accuracy of 65.21% on the test set and 64.36% on the validation set. In a survey, [70] the model's accuracy in FER2013 was 66.15% and 65.47% in test and validation, respectively. The human accuracy on FER-2013 was 65 ± 5% [99].

9.6 Features of the environment of SKAFOS

The cognitive and emotional environment for fractions, called SKAFOS (Student's Knowledge and Affective level for Fractions in an Open System), is shown in Figure 33, which shows a snapshot of the comparison of two fractions with a representation of circular disks through a web browser.

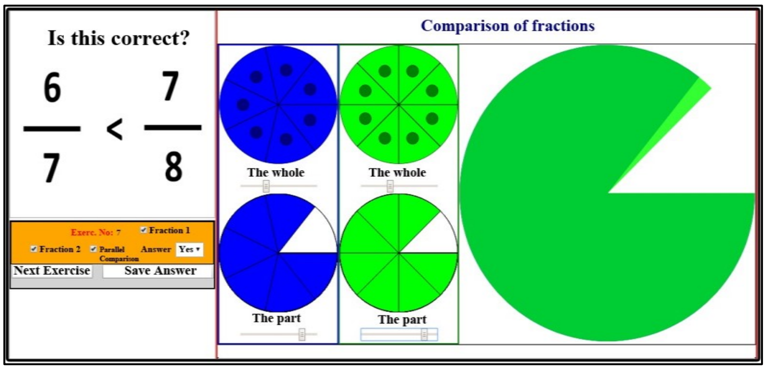

Figure 33: A snapshot of SKAFOS comparing two fractions using circular disks.

9.6.1 Interface

Initially, the student will register if entering the system for the first time. The information required is a username, an email account and a password so that the student can log in, as shown in Figure 34. Once the student information has been entered in the appropriate database system data, the student can use the system with this information and does not need to re-register. Then the student, when he enters the system for the first time and only once in the beginning, does a diagnostic test. This test concerns the student's level of knowledge in fractions, so that he can solve appropriate level exercises and problems.

Immediately after the student enters the system and depending on his level of knowledge, performs the exercises or problems through a large selection of exercises for all levels of knowledge in fractions. The system enables students to solve problems with various representations of fractions. At this stage, only the comparison of two fractions with five different representations has been developed. In the future, other operations of fractions (addition, subtraction, multiplication, division) with multiple representations will be incorporated.

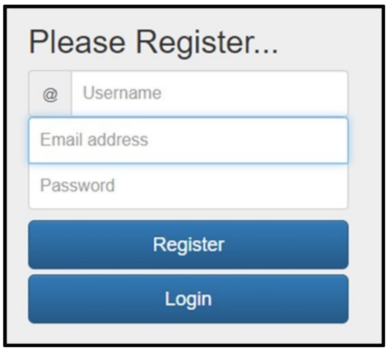

Figure 34: Screenshot of the initial login screen of the system.

9.6.2 Architecture

The proposed system follows the architecture of a typical Intelligent Tutoring System (ITS) and includes [203]: (a) the cognitive/cognitive model, (b) the student model, (c) the teaching model, and (d) the user interface model.

Figure 35 shows the architecture of the SKAFOS online learning environment. In particular, the system consists of three basic units, the Student Unit, the Cognitive Unit and the Emotional Unit. These three units constitute the AKUS (**A**ffective and **K**nowledge **U**nit of **S**tudent). Each of the three component units works with the database that contains the corresponding unit information. When the student enters the system for the first time, the level of knowledge in the fractions is recorded, through a diagnostic test for fractions. Then the relevant data are stored in the appropriate unit of AKUS.

Figure 36 shows the AKUS unit, which consists of the Student Unit, the Cognitive Unit and the Emotional Unit. The Student Unit, which is essentially the Student Model, processes information about the elements that refer to its cognitive level in fractions, as well as its emotional state for the multiple representations of fractions but also fractions in general. The Cognitive Unit includes the various exercises of the fractions provided by the system. Finally, the Emotional Unit records and provides data on the students' emotional state, after the completion of the exercises and the problems.

Figure 37 shows the User Interface Model of the SKAFOS system, which consists of four units: (a) the Exercise Presentation Unit, (b) the Multiple Representation Unit of the fractions, (c) the Response Recording Unit, and (d) the Emotion Recording Unit of the student. The Exercise Presentation Unit is responsible for showing the student the exercises he is required to solve. The Multiple Representations Unit of Fractions displays the available representations of each fraction so that the student can use them in solving each exercise.

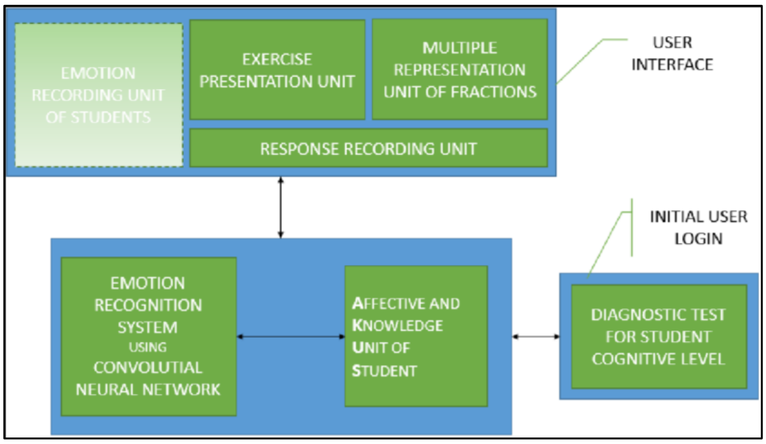

Figure 35: Architecture of the online learning environment SKAFOS.

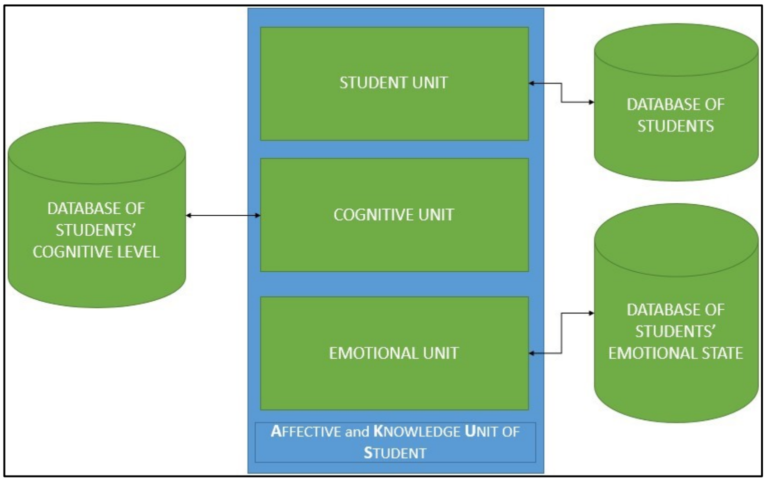

Figure 36: The AKUS unit.

In the Answer Recording Unit, which communicates with the AKUS Unit, the student's answers to the specific exercise are recorded.

Finally, there is the Student Emotional State Recording Unit where, with the help of the computer camera, data are sent for processing and recorded data about the student's emotions during the specific exercise and the specific fraction representation. The recognition of the student's emotions is done with the help of a Convolutional Neural Network, which recognizes seven different emotions: anger, disgust, fear, happiness, sadness, surprise and neutral emotion. The architecture of the Convolutional Neural Network is shown in Figure 38. CNN is described in detail previously in the 9.5.3 paragraph.

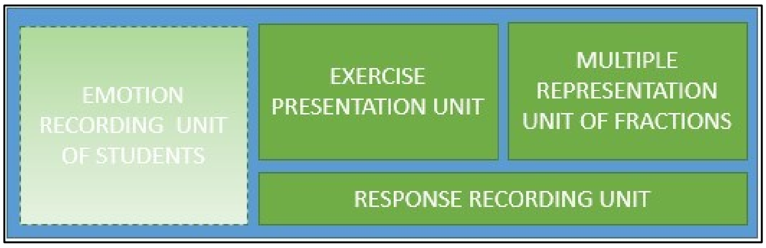

Figure 37: The User Interface Model of the proposed SKAFOS system.

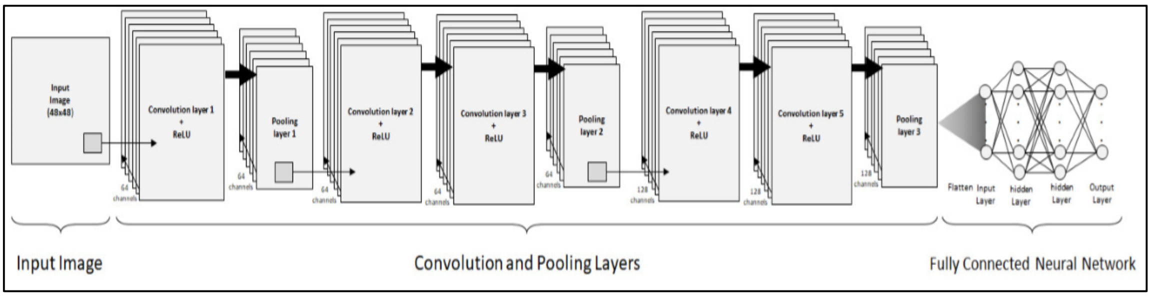

Figure 38: Architecture of the CNN model for emotion recognition from an image.

9.7 Content

Immediately after the student enters the system, depending on his level of knowledge of fractions, he performs the exercises or problems through a large selection of graded exercises in fractions. The system enables students to solve problems with various representations of fractions. At this stage, the system only supports the comparison of two fractions with five different representations. In the future, the four operations (addition, subtraction, multiplication, division) will be integrated with multiple representations of fractions.

9.7.1 Representation with text only

Initially, the problem is given in text only, without the use of another fraction representation for help, as shown in Figure 39. In this representation, the student sees the comparison he has to make in the form of a text, without being given any additional help. From a drop-down list, the student chooses whether he agrees with the answer or not. This answer is stored in the system in the Student Unit. The student can then select the next exercise he wants to do with this representation by selecting the "Next Exercise" button. Each time the student selects the "Save Answer" button to save his answer. Also, in each answer, a picture of the student is saved at the moment when he saves his answer in the specific exercise. It is sent to the Emotional Unit, processing and extracting the percentages of each of the seven emotions it can recognize. The percentages of these seven emotions are stored in the Student Emotional Database. The teacher can later use this Database of Students' Emotional State to see if the particular representation helped the student. Then, the teacher can take into account the student's answers as well as the student's emotional state from the particular representation, design an exercise strategy for the student that will be able to help him understand the fractions.

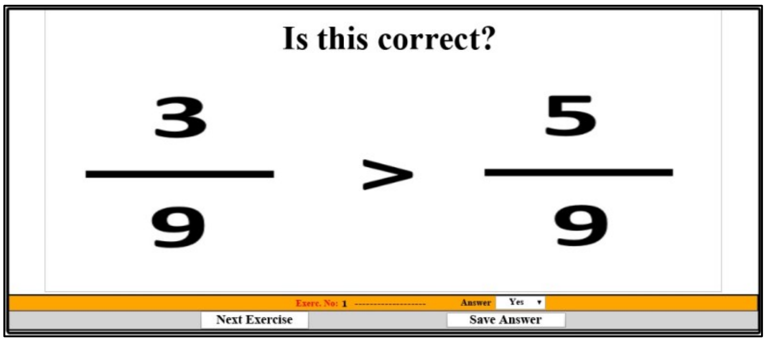

Figure 39: Screenshot of the system with the comparison of fractions with text.

9.7.2 Representation with circular disks

The second form of fractions that the proposed system can represent is that of circular disks. Figure 40 shows a screenshot of the system with the formulation of the problem and the corresponding representations of the fractions in the form of circular disks and parts of the circular disks. In this way of representation, the student has the opportunity to see the two comparable fractions in the form of circular disks.

The student can initially form the first fraction. The student moving the slider left or right forms the denominator, i.e. the "whole", from the fraction he wants to create. The appearance of the form is dynamic, and at the same time, every change it makes in the denominator value is displayed. He can then form the numerator by moving the corresponding slider and see the parts of the circular disk corresponding to the numerator value appear dynamically. In this way, a student can form the first fraction by representing the use of the circular disk, as in Figure 40 representing 4/8.

In the same way, he can form the second fraction. He will first move the slider left or right to form the value of the denominator he wants while seeing this change in the shape of the fraction being formed. He will then do the same with the numerator until the desired fraction is formed. In this way, the second fraction is formed with a representation of the use of the circular disk, as in Figure 40 represented by 3/8.

When the student completes the creation of the two fractions that he wants to compare, then he can choose from a drop-down list, "Yes" or "No", if he agrees with the question for the comparison of the fraction that has been done to him. The student’s answer is stored in the system in the Student’s Unit. The student could continue in the next exercise by pressing the button "Next Exercise". To save the answer, the student must select the command "Save Answer" by pressing the appropriate button.

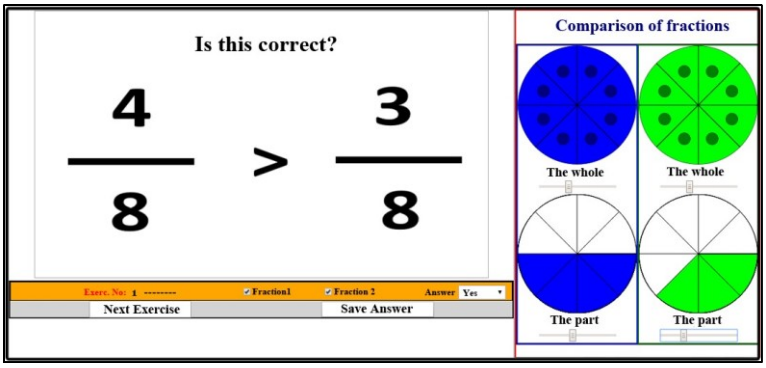

Figure 40: Screenshot of the system with the comparison of fractions in the form of circular disks.

Also, in each answer of the student, an image of the student is saved at the moment when he saves his answer in the specific exercise. The system sends this picture to the Emotional Unit. It then processes it and extracts the percentages of each of the seven emotions it can recognize from that image. The values of the percentages of the seven emotions are stored in the Student Emotional Database. The teacher can use this particular Student Emotional Database later to see if the particular representation helped the student. The teacher can take into account the student’s responses and his emotional state from the specific representation to design an exercise strategy for the student that will be able to help him understand the fractions.

9.7.3 Representation with rectangles

The third form of fractions, that the proposed system can represent, is that of rectangles. Figure 41 shows a screenshot of the system with the formulation of the problem and the corresponding representations of the fractions in the form of rectangles. In this way, the student can compare two fractions which can be represented by a rectangle.

Initially, the student creates the first fraction. Moving the slider left or right forms the value of the denominator, i.e. the "whole", from the fraction it wants to create. The appearance of the form is dynamic, and at the same time, every change it makes in the denominator value is displayed. He can then form the numerator by moving the corresponding slider and see the rectangle corresponding to the numerator value appear dynamically. In this way, it can form the first fraction by representing the use of the rectangle, as in Figure 41 representing 5/6.

The second fraction is formed in the same way. The student can move the slider left or right to form the value of the denominator he wants while seeing this change in the shape of the fraction being formed. The student will follow the same procedure to form the value of the numerator he wants. In this way, the second fraction is formed with a representation of the use of the rectangle, as in Figure 41 represented by 4/6.

After the student has created the two fractions he wants, then he can answer the initial question of comparing the fractions using a drop-down list, which has two "Yes / No" options. The answer that the student will give is stored in the system in the Student Unit. In order the student to continue to the next exercise he wants to select the command "Next Exercise" by pressing the appropriate button. To save the answer, the student must select the command "Save Answer" by pressing the corresponding button.

Additionally, the system stores an image of the student in each answer given. The system then sends this image to the Emotional Unit, it processes and extracts the percentages of each of the seven emotions it can recognize from that image and stores them in the Students' Emotional State Database. The teacher can use the Student Emotional Database to determine if this representation helped the student. Also, the teacher, taking into account the emotional state and the answers given by the student to each representation, to design an exercise strategy for the student that will be able to help him understand the fractions.

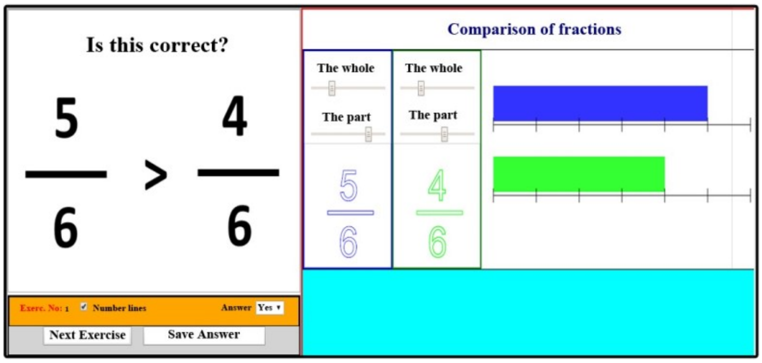

Figure 41: Screenshot of the system with the comparison of fractions in the form of rectangles.

9.7.4 Representation with circular disks and simultaneous comparison in the same frame

The fourth form in the comparison of the fractions offered by the proposed system is the one with the use of the circular disks, as in the second representation. Still, there is also a new representation of the two fractions as shown on the right in Figure 42. So the student while he can see the two fractions, at the same time he has a new representation of them that directly sees their difference.

In the beginning, the student draws the first fraction, moving the sliders left or right to form the denominator and numerator precisely, as he did in the second form presented in section 9.7.2. The form of the first fraction is done dynamically, as in the example of Figure 42 with the formation of 6/7. In the right part of the window, the form of the fraction in the form of a circular disk appears again, as shown in Figure 42.

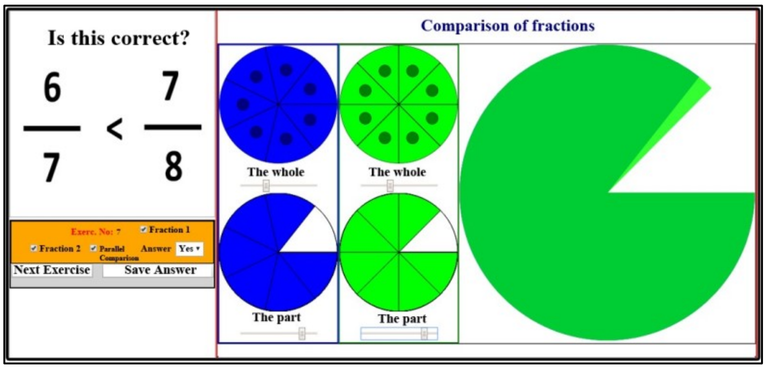

Figure 42: Screenshot of the system with the comparison of fractions in the form of circular disks and simultaneous comparison in the same frame.

Then the student forms the second fraction in precisely the same way. Thus, the student forms the second fraction, such as 7/8 of Figure 42. At the same time, the circular disk corresponding to this fraction appears in the right part of the window, so that the two fractions can be visually compared, as shown in Figure 42. In this example, looking at the visual difference of the two fractions, it seems that 7/8 is greater than 6/7.

After the student completes the comparison of the two fractions, then he can answer the initial question using a drop-down list, which has two options "Yes / No", and his answer will be stored in the system in the Student Unit. To continue to the next exercise, the student selects the "Next Exercise" command by pressing the corresponding button. To save the answer, the student selects the command "Save Answer" by pressing the appropriate button.

In addition, the system stores an image of the student's face in each answer it gives and sends it to the Emotional Unit. Then it processes and extracts the percentages of each of the seven emotions it can recognize from that image and is stored in the Students' Emotional State Database immediately after the image is deleted from its database. The teacher can use the Student Emotional Database to determine if this representation helped the student. Also, the teacher, taking into account the emotional state and the answers given by the student to each representation, to design an exercise strategy for the student that will be able to help him understand the fractions.

9.7.5 Representation with rectangles and simultaneous comparison in the same frame.

The fifth form in the comparison of the fractions offered by the proposed system is the one with the use of rectangles, as in the third representation. Still, also, there is also a new representation of the two fractions, as shown in the lower right in Figure 43. So the student, while having the two comparable fractions, has at the same time a new representation of them that directly sees their difference.

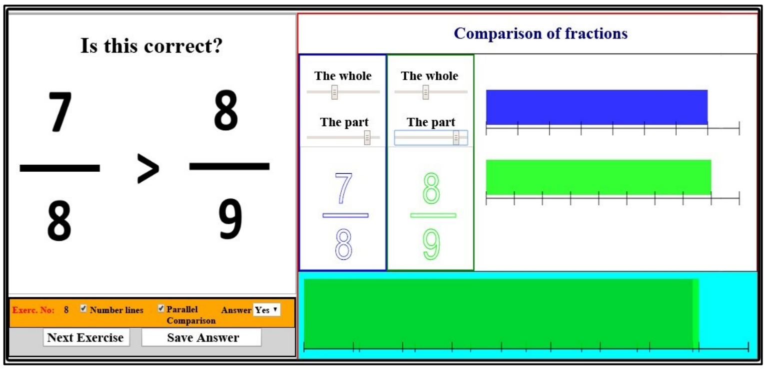

Figure 43: Screenshot of the system with the comparison of fractions in the form of rectangles and simultaneous comparison in the same frame.

At the beginning the student forms the first fraction, moving the sliders left or right to form the denominator and numerator precisely as he did in the third form presented in section 9.7.3. The creation of the first fraction is done dynamically, as in the example of Figure 43 with the formation of 7/8. In addition, in the lower right part of the window, the shape of the fraction in the form of rectangles appears again, as shown in Figure 43.

After that, the student forms the second fraction in precisely the same way as the 8/9 of Figure 43. At the same time, the rectangle corresponding to this fraction appears in the right part of the window, so that the two fractions can be visually compared, as shown in Figure 43. In this example, looking at the optical difference of the two fractions it appears that 8/9 is greater than 7/8.

Completing the comparison of the two fractions, the student can answer the initial question using a drop-down list, which has two options "Yes / No", stored in the system in the Student Unit. He can move to the next exercise by pressing the button labelled "Next Exercise". If the student wants to save the answer, then he presses the button labelled "Save Answer".

It also stores an image of the student's face in each response it sends to the Emotional Unit. After processing it, the system extracts the percentages of each of the seven emotions that it can recognize from the specific image and are stored in the Database of Students' Emotional State. Then the image is deleted from its database, the teacher by checking the Database of Students' Emotional State, can determine the student's performance in the specific representation and whether this representation helped the student. Thus, taking into account the student's emotional state and the answers to the questions of each representation, the teacher can design an activity strategy for the student that will help him to understand the fractions.

Survey on proposed system

10

Chapter Ten

10.1 Introduction

Mathematics is a discipline in which students have difficulties. More specifically, recent research [49] revealed that less than 10% of elementary-level students have the understanding to solve simple mathematical exercises, such as 8+4=_+5. Additionally, a study [190] disclosed that most of the students that participated failed to estimate the quantities in everyday life. In another study, [240] it is suggested that students who have misconceptions in learning fractions are prone to facing difficulties in other mathematical topics, such as ratios and proportions.

The conceptual understanding of fractions and their symbolism is causing an unpleasant mathematical experience for many students [29]. This is to be expected if we consider the fact that even teachers find that fractions are difficult to teach and understand [58]. For instance, one of the National Assessment of Educational Progress (NAEP) tests showed that students experienced difficulties in correctly ordering fractions and solving problems with fraction division [85], [255]. Also, as extensively argued in the literature [39], [53], [72]. However, instruction of fractions is carried out during elementary studies, many students fail to grasp a basic fundamental understanding of fractions, and they shift their misconceptions of fractions to upper secondary and higher education. Students often fail to use their imagination to describe, visualize, and understand fractions because of the abstract concepts of fractions. As a result, these difficulties accompany students during their next grades and levels of education, resulting in 33.3% of students failing to add fractions with different denominators [27]. On another note, other studies indicate that although fraction comparison is simple, i.e., 8/9 and 11/12, elementary school students consider this to be a challenging exercise [300], [39], [53], [72]. Additionally, problems involving division are difficult for many students [55]. The lack of adequacy in mathematical knowledge leads students to a variety of negative emotions and attitudes. Often students without reading the problem in question are likely to develop a negative attitude that prevents them from performing the exercise [23].

To sum up, students are likely to: (a) have misconceptions in mathematical topics including fractions, (b) face difficulties in solving exercises that relate with fraction operations, and (c) develop high anxiety with regards to mathematical tasks.

As a response to these negative consequences, teachers use a variety of techniques. For example, they use different manipulative objects and multiple representations for teaching mathematical concepts and understanding fractions. The use of length models (e.g., number lines), area models (e.g., circles and rectangles), and sets of objects are some of the tools that teachers use as representations for fractions, their operations and problems that related to fractions. As discussed in [82], the region/area model, length model, and set model are essential in learning and understanding fractions. There is no single recommended representation of manipulatives for fractions. Number lines are used in [85] to find improvements in children's knowledge of fraction magnitudes. Also, the number line was used in [134] for designing a digital game-based math test for evaluating cognitive and affective outcomes. Furthermore, fractions that are represented as number lines, rectangles, circles, other graphical representations, and virtual manipulatives are used in several e-learning platforms [172].

Virtual manipulatives are replicas of physical manipulatives [241], and they are used in computer applications. Virtual manipulative, as mentioned in [193], is *"best defined as an interactive, Web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge"*. Thus, virtual manipulatives constitute a suitable tool for the understanding of mathematical concepts, and they are applied for the conceptual understanding of fractions, fractions' properties, and operations of fractions. Nowadays, with the use of Information and Communications Technologies (ICT) and their tools, teachers are provided with the ability to use virtual manipulatives as a medium for communicating knowledge to students that also leverages a hands-on dimension transferable to a virtual environment. In a study [313] mentioned that students comprehend better graphs with the use of ICT (specifically using a web-based application for graphs) than without using this software. The majority of e-learning platforms make use of different types of virtual manipulatives that also apply for learning and studying fractions. A notable example is Fractions Lab, an online platform [101], [102], which virtually provides representation of fractions in four different types (e.g., area models, number lines, and sets of objects) [79]. From the other hand Fractions Tutor makes use of circles, number lines, and sets of objects to represent fractions [238]. Furthermore, virtual illustrations presented by the Physics Education Technology (PhET) platform for fractions use a variety of representations, such as number lines, area models, and liquid measurement [106].

This paper contributes an additional dimension to the development of such systems by introducing the ability to recognize the emotions of students using affective computing techniques that built on a Convolutional Neural Network when students solve fractions tasks with multiple virtual manipulatives. The experiments presented in this work focus on measuring the performance of students on various learning tasks. More specifically, the emotions of a representative sample of fifth-grade students are recorded and recognized on the execution of various operations in fractions. The aim of our experiments is set to infer additional knowledge for the following questions:

a) How students respond to various visual graphical representations used for fractions comparisons; and

b) To what extent does this system provide valuable information about student's emotions while engaging in various tasks;

10.2 Related work

As already mentioned, there is enough evidence to conclude that the domain of Mathematics is considered challenging both for teachers – for communicating mathematical concepts – and students for developing mathematical knowledge [153]. In this paper, we focus on the challenges faced by students during the conceptual understanding of fractions [300]. Recent studies indicated that positive psychological attitudes and predisposition to Mathematics are considered an essential factor (among others) on influencing the performance of students for the mathematical domain [54]. Additionally, as discussed in [20], the recognition of the emotional state of students and reacting on that state using various interventions provides evidence that students are likely to understand mathematical concepts better. At the same time, this strategy is likely to shift their perspective for approaching various mathematical tasks [20]. A variety of systems for mathematical concepts have been proposed in the literature for recognizing and interpreting the emotional state of students, and further assist them with appropriate strategies towards developing a positive attitude on mathematical concepts.

PAT2Math (Personal Affective Tutor to Math), is a web-based system which takes into consideration the emotional state of students by using traditional input methods [119]. PAT2Math uses the model presented by Orthony, Clore, and Collins (OCC) that can recognize seven emotions: joy/distress, satisfaction/disappointment, gratitude/anger, and shame [118]. These emotions come from students' actions while interacting with the interface of the system [119]. A significant module of PAT2Math is the tutoring agent that can determine the emotional state of a student [118]. However, the system does not make use of any visual equipment (e.g., a web-camera for recognize emotions). The PAT2Math focuses on algebra, especially on the field of linear and square equations, and it is designed for elementary school students [182].

Another system that can recognize students' emotions for educational purposes is Fermat. Fermat is an Intelligent Tutoring System (ITS) for teaching and learning mathematics [46]. It was developed to solve the main problems of Mathematics for secondary education in Mexico because a substantial percentage of students had an inadequate or elementary level mathematical skills [23]. The system was initially developed to recognize seven emotion types (i.e., anger, disgust, fear, happiness, sadness, surprise, and neutrality) [46], but finally concluded to recognize five emotions (anger, happiness, sadness, surprise, and neutrality) [307]. One of the characteristics of the system is that it combines a Self-Organizing Map or a Kohonen Network to detect learning styles and a Neural Network for detecting the emotional state of students using a web-camera [310]. The system is used for multiplication, division, and fraction issues [46].

Easy with Eve is another Affective Tutoring System (ATS) for teaching mathematical concepts that are mostly used for primary school students [248] with the use of real-time face expression analysis [250]. Easy with Eve can recognize eight emotional states (neutral, smiling, laughing, surprised, angry, fearful, sad, and disgusted) [250]. A web-camera is used for capturing images [7] and a modular system consisting of: an artificial neural network for face detection, the extraction of facial features, and the fuzzy face-expression classifier to detect student's emotional state [247]. Similarly, in [249] a neural network was used to identify facial expressions. The detection of emotions is made possible with the use of a set of fuzzy rules for eyebrows, eyes, and mouth. The Easy with Eve was designed for supporting basic mathematical operations for addition, subtraction, multiplication, and division [249].

Prime Climb is another system that helps students to learn factorization [60]. It uses a Dynamic Decision Network (DDN) to model a student's emotional state based on the OCC cognitive theory of emotions [60]. Prime climb recognizes six out of the 22 OCC's emotions (i.e., joy, distress, admiration, reproach, pride, and shame) [62]. The cognitive field of Prime Climb is mainly the factorization of natural numbers, and it was evaluated on a sample of sixth and seventh-grade students [14].

The work presented in the paper proposes an alternative system for presenting different representations of fractions using virtual manipulatives to students, along with a technique for analyzing the emotional state of students. A detailed account of the modules developed for the proposed system is presented in subsequent sections. In contrary to the systems mentioned above, our system builds on a Convolutional Neural Network (CNN) for classifying images according to the emotions recognized from the facial image. CNN is used successfully for image-related tasks because they can extract good representations from data [200].

10.3 Method

10.3.1 Participants

The proposed system was evaluated on a sample of 30 fifth grade students, 10-11 years old pupils. The sample consisted of an equal number of boys and girls. Before participating in the experiment, the objective was explained to all participants asking for their consent. The facilitators overlooking the experiment demonstrated the different forms and visualizations and permitted actions.

10.3.2 Tasks

It was given to each participant a set of 5 incremental tasks (or interventions) to complete, each one with different graphical representations of fractions and using different types of virtual manipulatives. Every task had eight proper fraction comparisons, and the ability to choose a Yes/No answer from a drop-down list. The order of the incremental tasks was as follows: (a) Fraction comparison with text only, (b) Fraction comparison with circular disks, (c) Fraction comparison with rectangles with number lines, (d) Fraction comparison with circular disks and extra comparison, and (e) Fraction comparison with rectangles with number lines and extra comparison. The five tasks were organized with this way because students were familiar in their books with the first, the second, and the third form. The fourth and fifth forms were something new to students, and we did not know how students will react to them, and we decided to present them lastly in order not to confuse students from the beginning. Maybe in future research, we could change the order of the five tasks, in a random way or from easiest to harder form, or we could create additional form with other virtual manipulatives to research if the order of tasks affects students' performance in fractions tasks.

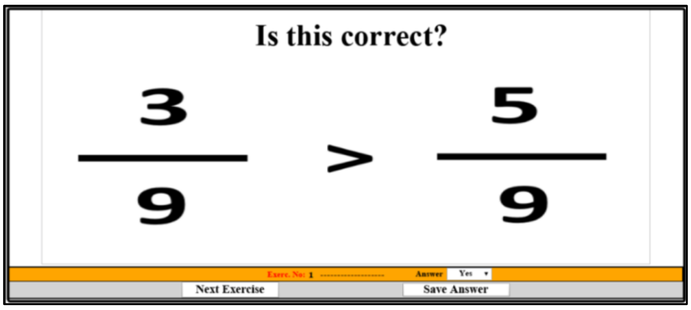

Figure 44: Screenshot of a random problem in fraction comparison in the first representation with text only.

Fractions' comparison only with text

At this stage of the intervention, participants had to solve a set of fraction comparison exercises represented only with text. This task consisted of eight exercises, divided into two difficult levels: four fractions with common or multiple denominators, and four fractions with different denominators. Participants were presented with eight pairs of fractions for comparison: 3/9 and 5/9, 7/8 and 6/8, 3/4 and 6/8, 2/6 and 2/5, 3/5 and 2/3, 6/7 and 7/8, and 6/8 and 7/9. The eight pairs of fractions were presented randomly to the students, and the students picked up the answer from a drop-down list (Yes/No). The system then collected participants' mathematical test scores and the students' emotions (i.e., anger, disgust, fear, happiness, sadness, surprise, and neutral) while solving each exercise. Figure 44 illustrates an example of this stage.

Fraction comparison with circular disks

The second stage of the intervention included comparison exercises of fractions represented with circular disks. This task included eight exercises, divided into two difficult levels: four fractions with common or multiple denominators, and four fractions with different denominators. At this stage, each participant had two representations of circular disks representing the two fractions. Furthermore, in each representation, each participant had to form the fraction using the part-whole representation of circular disks. The following eight pairs of fractions were used for comparison: 4/8 and 3/8, 5/9 and 4/9, 2/3 and 4/9, 2/4 and 3/8, 3/5 and 3/7, 3/7 and 4/6, 7/8 and 8/9, and 5/6 and 7/9.

Similarly to the previous stage, the pairs of fractions presented randomly to the participants. Students picked up an answer from a drop-down list (Yes/No). At the end of this stage, the system collects the mathematical achievement test scores for each student along with the student's emotions (angry, disgust, fear, happy, sad, surprise and neutral) when they solve each exercise. Figure 30 shows a sample of this stage.

Fraction comparison with rectangles with number lines

The third stage of the intervention included fraction comparison tasks represented with rectangles with number lines. This task included eight exercises, divided into two difficult levels: four fractions with common or multiple denominators, and four fractions with different denominators. In this stage of the process, the student had two representations of number lines for the two fractions. Furthermore, in each representation, students had to form the fraction using the part-whole representation of the number line. Participants were presented with eight pairs of fractions for comparison: 5/6 and 4/6, 3/5 and 4/5, 2/4 and 5/8, 1/3 and 2/6, 5/8 and 5/7 , 4/5 and 3/6, 6/7 and 8/9, and 7/8 and 8/9. The eight pairs of fractions were presented randomly to the students. The students picked up an answer from a drop-down list (Yes/No). At the end of this stage, the system collects the mathematical achievement test scores for each student along with the student's emotions (i.e. angry, disgust, fear, happy, sad, surprise and neutral) when they solve each exercise. Figure 31 shows a sample of this stage.

Fraction comparison with circular disks and extra comparison

The fourth stage of the intervention was similar to the second one, but with an extra feature. There was a third representation of two fractions in the same circle. If the first fraction were bigger than the second one, then the color of the first fraction would occupy a larger space in the common place. Respectively, if the second fraction was greater than the first. Participants were presented with eight pairs of fractions for comparison: 4/8 and 3/8, 5/6 and 4/6, 3/4 and 6/8, 2/4 and 3/8, 3/6 and 3/5, 3/7 and 4/6, 6/7 and 7/8, and 5/6 and 7/9. The eight pairs of fractions were presented randomly to the students, and the students picked up an answer from a drop-down list (Yes/No). At the end of this stage, the system collects the mathematical achievement test scores for each student along with the student's emotions (angry, disgust, fear, happy, sad, surprise and neutral) when they solve each exercise. Figure 45 shows a sample of this stage.

Fraction comparison with rectangles with number lines and extra comparison

The fifth stage of the intervention was similar to the third one, but with an extra feature. There was a third representation of the two fractions in the same rectangle with number line. If the first fraction were greater than the second one, then the color of the first fraction would occupy a larger space in the common place. Respectively, if the second fraction was greater than the first one. Participants were presented with eight pairs of fractions for comparison:

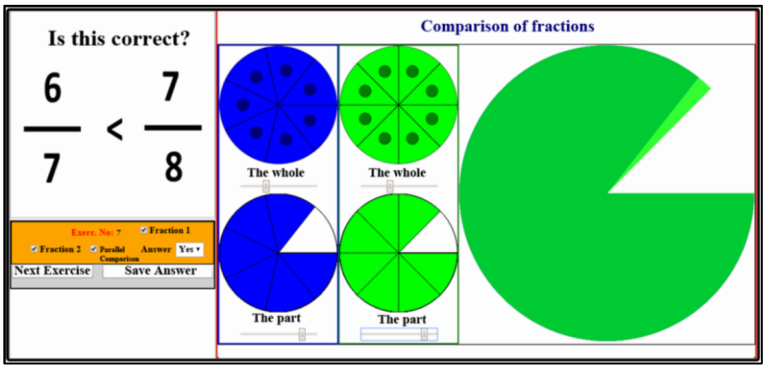

Figure 45: Screenshot of a random task in fraction comparison in the fourth representation with circular disks and the extra representation.

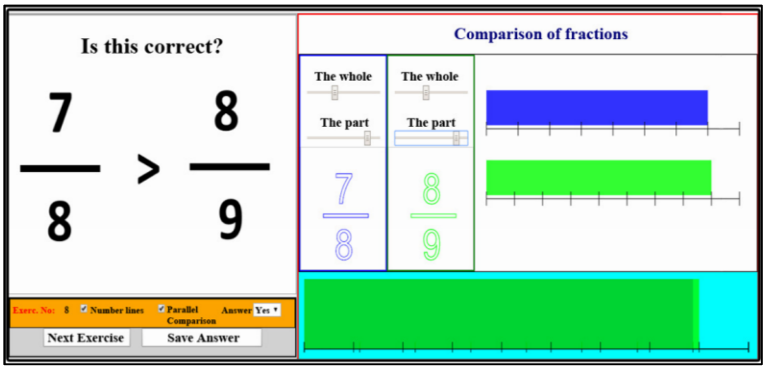

Figure 46: Screenshot of a random problem in comparison of fractions in fifth representation with rectangles with number lines and the extra representation.

4/8 and 3/8, 5/6 and 4/6, 3/4 and 6/8, 2/4 and 3/8, 3/6 and 3/5, 3/7 and 4/6, 6/7 and 7/8 , and 5/6 and 7/9. The eight pairs of fractions were presented randomly to the students, and the students picked up an answer from a drop-down list (Yes/No). At the end of this stage, the system collects the mathematical achievement test scores for each student along with the student's emotions (i.e. angry, disgust, fear, happy, sad, surprise and neutral) when they solve each exercise. Figure 46 shows a sample of this stage.

10.3.3 Instructions

Considering that the participants in this experiment had no previous experience with virtual manipulatives for fractions, the web-based system presented various helpful instructions; consisted of four frames. The first frame, on the left upper side of the system, illustrated the fraction's problem. In the bottom left side, the system had the button controls: (a) a button to navigate to the next question, (b) a button to choose the answer (Yes/No) from a drop-down list, and (c) a button to save current answer. The frame on the right half of the canvas shows fraction representation as circular disks or rectangles with number lines, respectively.

Furthermore, an additional circular disk or a rectangle with number line was shown, when the fourth and fifth task was chosen (e.g., Figure 45).

10.3.4 Procedure

For all experiments, a personal computer equipped with a web-camera was used. The facilitator orally explained and demonstrated the interface of the system to the students. After this brief introduction, the set of incremental tasks was assigned to each of the participants. Every student worked alone on a computer. Actions from each participant were recorded in a log file one for each task. Students had to repeat the procedure five times.

10.4 Results

In all the tasks presented during our experimentation, proper fractions were examined, i.e., where the numerator is less than the denominator. There were four sets of proper fractions. The first set consisted of two easy fraction comparisons having the same denominator (e.g., 3/9 and 5/9). The second set comprised of two fraction comparisons where the two compared fractions had a denominator and a multiple number respectively (e.g., 2/3 and 4/9). The third set had two medium difficulty proper fraction comparisons, where one of them had fractions with the same numerator (e.g., 5/8 and 5/7). The fourth set had two difficult proper fraction comparisons (e.g., 7/8 and 8/9). All exercises used for the tasks were based on elementary level educational books following the Greek educational system. Individual data from the testimonies were collected and analyzed using descriptive statistics.

10.4.1 Comparing fractions with and without manipulatives

The first of the five tasks had fractions' comparisons without manipulatives. The other four tasks had comparisons of fractions using virtual manipulatives (circular disks and rectangles with number lines). We compare the emotions captured during these two methods (with and without manipulatives), and present the results in the Figure 47.

The three emotions that prevail in both cases, with up to 75.00% percent are: neutral, happy, and sad. Furthermore, the dominant Emotion in both cases is neutral (with 31.79% and 31.82%, respectively). Also, in cases without manipulatives, the Emotion of sadness was reported greater than the Emotion of happiness (24.14% and 21.23%, respectively). When fractions are presented to students with virtual manipulatives, then the Emotion of happiness reported is greater than the Emotion of sadness (24.65% and 21.05%, respectively).

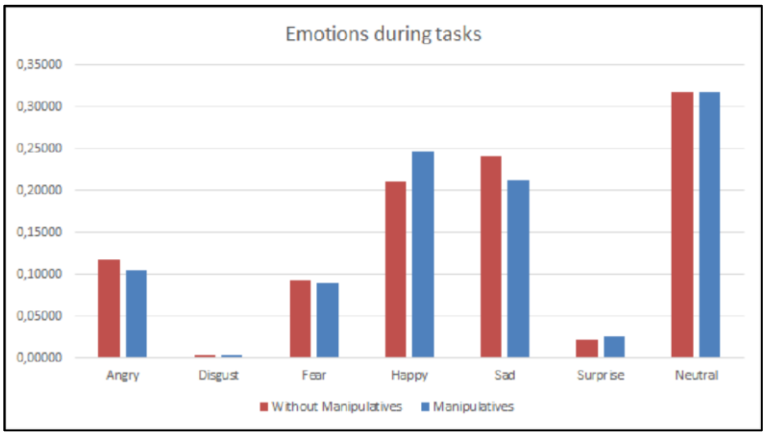

Figure 47: Emotions that recognized during tasks with and without manipulatives.

10.4.2 Comparing circular disks and rectangles with number lines

The second and fourth tasks use circular disks to compare fractions. The third and fifth tasks use rectangles with number lines to compare fractions. The results comparing circular disks and rectangles with number lines are presented in Figure 48.

Additionally, we observed that the rectangles with number lines representation used to compare fractions are slightly more enjoyable for students than the circular disks (26.15% and 23.15%, respectively).

10.4.3 Emotions that was recognized during tasks

The system also captured the emotional-state of students. It seems that students enjoy fractions more when they use the rectangles with number of lines and its extra comparison of fractions. In this form (rectangles with number lines and extra comparison), the emotion of happiness reported to be 28.64%, neutrality reported as 29.37% and sadness as 21.52%, as presented in Figure 49.

10.4.4 Students' performance in different fraction representations

Students solved eight problems in each of the five different representations of fractions' comparison. It is indicated from the results that students improved their scores when they used virtual manipulatives (circular disks and rectangles with number lines) of fractions. Considering the average score of 5.20 in the first form, students improved their average score performance to 6.60 and 6.80 in the second and third form, respectively. Furthermore, students showed improvement when they used the fourth and fifth form with an extra parallel comparison in the same area. The average score in the fourth and fifth form were 7.07 and 6.93, respectively. The average scores in five different representations are shown in Figure 50.

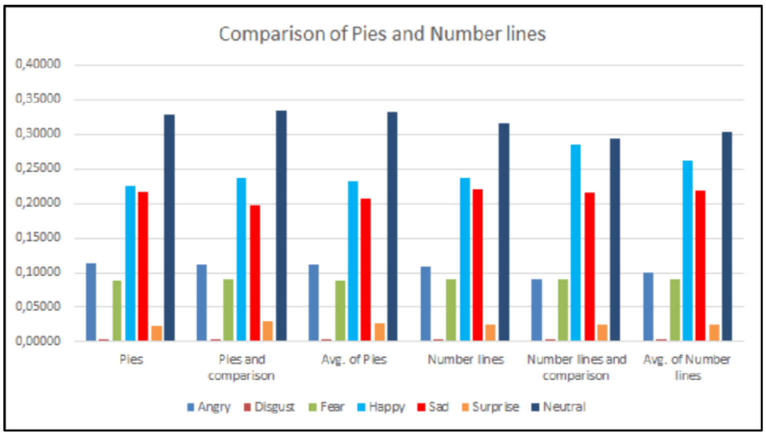

Figure 48: Comparison of circular disks and rectangles with number lines tasks.

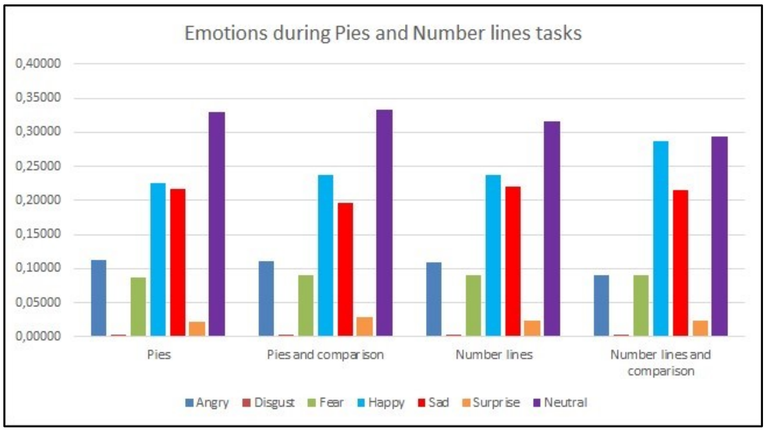

Figure 49: Emotions that were recognized during tasks with circular disks and rectangles with number lines.

10.4.5 Discussion with students

At the end of the experiment, the facilitators interviewed each student individually. Feedback was received on the following questions: (a) their opinion with regards to the five forms of the graphical representations, and (b) to describe their experiment with virtual manipulatives. Two students reported that the fourth and fifth form of the graphical representations, had too much information and as a result, it confused them. Additionally, three students found the second (only circular disks) and the third (only rectangles with number lines) forms helpful. Also, they found that the fourth form (circular disks and extra parallel comparison) had too many processes. This extra information got them confused.

Furthermore, one student said that the fourth form prevents students from clearly thinking because it provides too many helpful options. Also, this student suggested that the system should provide scaffolding help and that it would be better in assisting the thinking process. Finally, a student asked if there was an option to configure only the part (nominator) of the whole (denominator) in the virtual manipulatives.

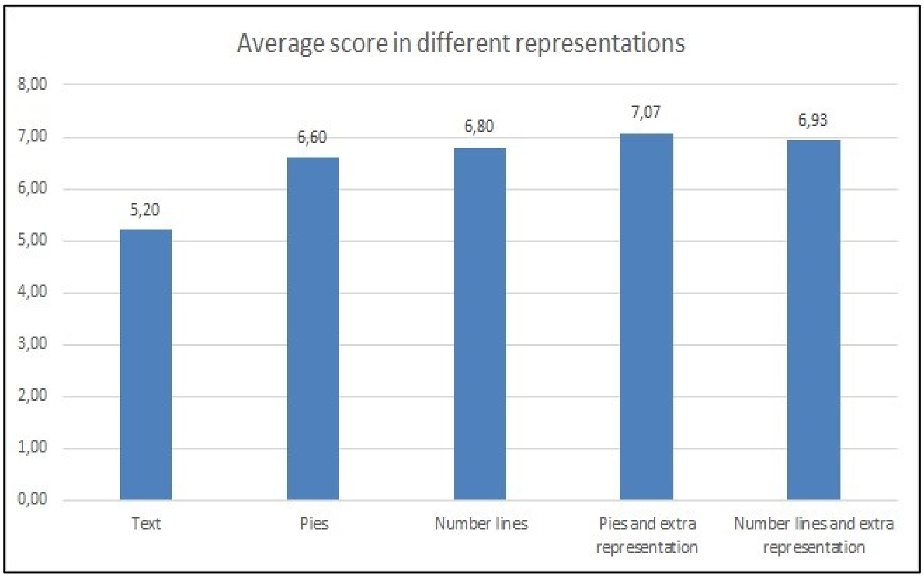

Figure 50: Students' performance in different fraction representations.

10.5 Conclusions

The main clues relate to virtual manipulatives, the students' preferences in specific virtual manipulatives, and analyzing students' emotions. Specifically, the main findings are: (a) students increase their positive emotions when they use virtual manipulatives, (b) students when comparing fractions are happier using rectangles with number lines than circular disks, and (c) students are more likely to improve their performance in mathematics when they use virtual manipulatives than without using them.

More specifically, it seems that participated students of the study have positive emotions when using virtual manipulatives in fraction comparison tasks (ignoring the neutral emotion). This is according to a study [241], which indicates that virtual manipulatives increase students' enjoyment while learning Mathematics. Furthermore, students prefer the use of circular disks and rectangles with number lines instead of using plain text. Additionally, participated students were slightly happier when they used virtual manipulatives of circular disks with extra comparison than when they used only manipulatives with circular disks. Also, precisely the same was observed with the rectangles with number lines.

Furthermore, participated students when comparing fractions are happier using rectangles with number lines (third and fifth forms) than circular disks (second and fourth forms). The difference between the two visual graphical representations is small, but it is indicative that students tend to prefer the rectangle with number line as a tool when they compare proper fractions. Of course, we cannot generalize this finding because of the small number of our sample size.

Also, the performance of students is improving with the use of virtual manipulatives than without using them. Thus, the multiple representations of fractions with rectangles with number lines and circular disks, seem to help students in understanding various mathematical tasks. This finding aligns with the conclusions discussed in [236], where multiple representations of fractions with graphical manipulatives helped students in the learning process of fractions. Additionally, multiple representations of fractions help primary school students to understand fractions [276]. Also, a study presents how applying multiple representations in fractions multiplication can enhance students' learning [297]. Another study in fifth graders found that virtual manipulatives help students understand equivalent fractions and fraction addition [272]. Hence, this agrees with our study, where multiple graphical representations lead students to better conceptual learning than a single graphical representation [239]. Also, when they use the fourth form (circular disks with extra feature), students report the best score. This agrees with a survey [276], where students had to use area models (circles and rectangles) and number lines in various problems. In this survey students' performance was significantly lower when they use number lines than area models.

Moreover, having discussed with each student separately, some students have objections in some features of the system. Perhaps, the sub-system in the use of virtual manipulatives requires a reconstruction to increase user-friendliness. Additionally, the system could benefit from implementing a scaffolding help mechanism to provide the opportunity for students to use all the features simultaneously from the beginning. Also, there is a chance the form with the extra parallel comparison of fractions has too much information. Maybe the two forms (fourth and fifth) only need to provide the two fractions in the same area in the beginning.

One limitation of the study is perhaps the sample size; additional experiments need to be conducted with a larger sample for generalizing our findings. Maybe upcoming research with bigger sample size, a further and detailed analysis using more virtual manipulatives and statistic tools, could be conducted. Also, further research consisting of an experimental group and a control group could be performed to study attitudes, difficulties and emotions of students that are likely to be developed while solving exercises and problems with fractions.

10.6 Discussion

Students often develop a negative attitude towards learning mathematical concepts. Developing an understanding of fractions their properties and operations remains a difficult subject for many students due to their abstract nature. However, appropriate instruction methods and useful graphical manipulatives are proven effective in supporting the learning process for students to understand fractions better.

In this paper, we studied the various emotions caused by students when exposed to a set of tasks using virtual manipulatives in an appropriate e-learning web-based system. One of the main findings is that virtual manipulatives increase student's positive emotion (happy) towards fractions. For future research, we plan to expand the current system with additional manipulatives for representing fractions (e.g., bars or rectangles).

Another finding is that three basic emotions were recognized during fraction comparison tasks, summing up to 75.00% of the total emotions, which are sadness, neutrality and happiness. Also, there are emotions (disgust and surprise) that are lower than 5% percent. These emotions could probably be removed from the face recognition sub-system if it is proven that they do not cause any malfunction to the whole process and learning process. Thus, the system could recognize less than seven emotions without losing any critical information. Also, there is proposed system (e.g., Fermat) which initially could recognize seven types of emotions (anger, disgust, fear, happiness, sadness, surprise, and neutral) [46], and finally conclude to recognize five types of emotions (anger, happiness, sadness, surprise, and neutral) [307].

Furthermore, our experimental study indicated that students prefer rectangles with number lines and circular disks in fractions' comparison than plain text without manipulatives. This finding indicates that it is appropriate for such systems to provide various virtual manipulatives for fraction comparisons. In other words, students should have alternatives when exposed to mathematical exercises. Thus, a personalized system for fractions could further support the learning experience for students.

Finally, our study showed positive evidence that the performance of students in understanding fractions improved using virtual manipulatives. Circular disks and rectangles with number lines are improving students' scores more than plain text when used in fraction comparison problems. The proposed system could be used as a tool for teaching and practicing various exercises in fractions. In our plans is to redesign the e-learning system and make it friendlier and more effective for students and embed other useful features for teaching and learning purposes.

Discussion

11

Chapter Eleven

11.1 Basic findings of the dissertation

The core of the dissertation was on the didactic and drills of fractions using ICT. Specifically, the main aim of the dissertation was the study and design of an online visualization system for fractions. This system would be used for learning the fractional unit, the comparison of fractions and the simultaneous recognition of emotions when student solving exercises in fractions. For this purpose was conducted a literature review for systems that were designed for fractions and a bibliographic study on ATS. Also, a survey of elementary school students on the difficulty of comparing fractions, and an ongoing interview of in-service elementary school teachers on the teaching of fractions were conducted. Finally, a survey was conducted on primary school students with the implementation of the proposed web system in the comparison of two fractions.

Analytically, the first survey was conducted on 115 students in the fifth and sixth grades of primary school and aimed to detect students' difficulties during the comparison of fractions. Also, it was investigated any relationship between students' performance depending on their gender or level of the grade. The students compared 12 pairs of fractions with proper fractions, and they had enough time to solve the tasks. Useful data emerged from the analysis of the data. Specifically, the main finding of this research was that students encounter difficulties when comparing two fractions. The fifth and sixth graders experienced equally difficulties, but the fifth graders appeared to have had more difficulties and achieved a lower score than sixth graders. Of course, this finding is according to other studies for fractions. Also, one of the research findings was that gender does not play a significant role in students' performance in comparing fractions. Still, the level of grade in which they study plays a vital role in students' performance in comparing fractions. Also, an important finding was that students perform better when comparing fractions with congruent items and when pairs of fractions to be compared have common elements. The research findings were used to design the content of the fraction comparison that would be provided by the proposed system. Comparison activities were created with both common and non-common elements (same numerator or denominator) of fractions, but also fractions with congruent and incongruent items. Also, it was taking into consideration the fact that fifth grade students encounter more difficulties and the content of fractions it was from the curricula for the fifth grade. Finally, the gender of the students was not taken into account by the researcher regarding the design of the system.

After the first survey of the students followed a study on fraction systems that exist, the first study concerned the literature review of online systems dealing with fractions. Thus, a survey was carried out on large electronic databases for systems dealing with fractions. The research focused mainly on online systems that had documentation in the literature. The results of the survey showed that there were not too many documentation for systems dealing with fractions, so the subsequent study was limited to only some of them. Specifically we studied: (a) the fractions simulations on PHET, (b) the Fractions Lab, and (c) the Fractions Tutor [172]. The study of the specific three systems revealed essential elements of formation and representation of the fractions, feedback information with the student as well as the way of comparing the fractions. One of the main findings of this research was that the design of a modern platform should have some specific features. More specifically, the designed system should: (a) be developed on the web (using HTML5 or Javascript), (b) can be supported in the most well-known web browsers and smart devices, (c) utilize multiple representations, (d) be highly interactive, (e) provide basic operations of fractions, (f) provide dynamic content through a repository of exercises, and (g) had a tutor during solving tasks [172]. From the data that this research concluded that a modern system should have, in the online fraction system that we developed we tried to integrate most of these features. Specifically the system for the fractions we developed applied several of these features such as: that it was online, it could be executed in the most well-known browsers, it provided multiple representations of the fractions, it had dynamic content, and they could (in the first phase) perform the operation of comparing two fractions. This research was beneficial because it showed us what was already in the literature so far in terms of the systems dealing with fractions and highlighted the gap that existed to try to fill it with our proposed system.

Considering the type of software that exists for fractions, the next step was to research how emotions are recognized in systems that deal with fractions. Thus, the third study was a literature review on the Affective Tutoring Systems for Mathematics. So we searched large online databases for systems in teaching mathematical concepts that recognize the student's emotional state while solving exercises. The research was limited over the last fifteen years and mainly to literature that had articles that had been reviewed.

Specifically, the following ATSs were studied: (a) PAT2Math, (b) Fermat, (c) Easy with Eve, (d) WaLLis, and (e) Prime Climb. The study of the five ATSs revealed essential data on the architecture, the way of feedback, the number of emotions they can recognize as well as the way students' emotions are recognized. By analyzing this information for each system individually, we observed that each system has a separate way of designing, recognizing students' emotions and presenting the mathematical subjects it deals with. Of course, there was a degree of difficulty because in most of them we did not have direct access to the system, so we could study it and see how it works, but the study was done mainly by the study of the literature review. So, we found enough useful information about these systems. One of the main findings of the research was that there is a minimum number of emotions, five to seven that ATSs could recognize during mathematical problem solving [171]. Also, another finding of this study is that there is no choice of a specific strategy for recognizing emotions. Still, the use of Artificial Intelligence techniques as well as the use of a camera is considered necessary [171]. In addition, the use of an animated pedagogical agent can motivate students in Mathematics [171]. The findings of our study helped us design a system that adopts ATS principles, recognizes seven emotions that can be made up of five, uses a camera as a media to recognize emotions while solving fractional activities and uses Deep Learning to identify emotions.

Having studied the literature for both fraction systems and ATSs dealing with mathematical concepts, we also had to research with teachers to see mainly how they teach fractions, the difficulties they encounter and how they overcome them, but also other issues related to the teaching of fractions. Thus, the first interview was conducted using the focus group method in groups of in-service teachers. The focus group method was used, mainly because it enables the interviewer to develop his thoughts about the use of a questionnaire freely. Specifically, two groups of primary school teachers, a total of 12 teachers, with considerable teaching experience took part in the interview. The thematic areas of the discussion were mainly the strategies they implement for the teaching of fractions, the difficulties that students encounter in fractions and actions of fractions, as well as the representations they use for the teaching of fractions. From the study of the interviews, useful conclusions were drawn, and useful findings emerged. Specifically, one of the findings of the interviews was that teachers identify students' difficulties in different areas of fractions (fractional unit, fraction comparison, fraction equivalence, fraction multiplication and division, fraction problem solving), use to teach fractions. Every day students use examples such as pizza, orange or chocolate to represent the fractional unit. In addition to the research findings, teachers need a test to identify students' level of knowledge and agree that multiple representations, up to a maximum of two, help students. Another finding is that teachers accept that software with design and sound is beneficial to students. Finally, one of the most important findings is that teachers use circular disks to represent fractions on the whiteboard. The findings of this research helped us to design a system that uses circular disks to create fractions, implements design-motion in the representation of fractions, provides up to two different representations of fractions (circular disk, rectangle) and is designed to present, for the teaching of the fractional unit and the comparison of two fractions.

All the findings of the above research and studies were useful, and most of them tried to be implemented in an online system for fractions. Thus it was developed a system based on the web that can be run on the most popular web browsers and mobile devices (i.e. smartphones, tablets). In the first phase it provides the ability to form fractions to represent the fractional unit and compare two fractions using circular disks and rectangles. There is interactivity, and the representation of the fractions is done dynamically with drawing movement. The system also provides activities for comparing two fractions with fractions that have common elements (same numerator or same denominator), and also fractions with congruent and incongruent items. Also, this system recognizes seven emotions of students when solving activities using Deep Learning.

The final survey was conducted on 30 students in the fifth grade of primary school and aimed at how students respond to the various visual representations of fractions, and what information the proposed system provides for students' emotions while doing various activities. Significant data emerged from the data analysis. Specifically, one of the research findings that emerged was that virtual manipulatives increase students' positive emotions towards fractions. A second important finding was that the three most basic emotions with a large percentage, over 75.00%, were: sadness, neutrality, and happiness. Another useful finding was that students preferred the circular disks and the rectangles with number lines to compare fractions. Also, one of the most important findings was that student performance improved with the use of virtual manipulatives.

11.2 Comparison with corresponding studies and fraction representation environments

The modern software that has been developed for fractions provide the ability to visualize fractions and represent them in various forms (circular disks, rectangles, bar charts, containers, object sets, etc.). Representing fractions with digital manipulatives seems to help in understanding them [241]. Providing multiple representations of fractions leads to better learning of fractions than a single graphical representation [235], [236].

The study of the literature showed that there is no detailed research to evaluate e-learning systems for fractions. From the data we have studied, it appears that the research we have conducted on the evaluation of software dealing with fractions is one of the first, if not the first to be done in this field. The study of e-learning systems for fractions showed that existing software uses multiple representations to represent fractions. Interactive representations are more effective in improving students' fractional learning than static graphics with fractions [86].

One of the advantages of SKAFOS is that it provides the ability to present the form of two comparable fractions in the same frame. This is done using both the shape of the circular disks and the rectangles. In this way, the two fractions are presented simultaneously, and their difference is seen. To date, most software what has been quoted in the process of comparing two fractions has been the two representations of the fractions next to each other. Also, the main advantage of the SKAFOS system is that it provides the ability to recognize emotions in the process of solving fraction comparison exercises. This recognition of emotions enables the teacher to detect the possible form of representation that helps the student the most and to design a strategy with exercises of this form of representation.

The analysis of the results of the dissertation showed that the multiple representations of the presentation of fractions seem to help students learn fractions, which is in line with other research [235], [236]. It was seen when the additional representation of the two fractions is presented for comparison at the same time in the same position. The students have better performance and have an increased feeling of joy compared to when this extra representation does not appear.

11.3 Utilization of SKAFOS in education

The SKAFOS system can be used in education to help the teaching of fractions. It is designed for primary education, but exercises and tasks can be modified and used in secondary education. Students using multiple representations can explore the answer to the exercises for comparing two fractions.

At the present stage, the SKAFOS system can be used to understand the fractional unit as well as to compare two fractions through the representation of five forms for primary education. Different representations can help students understand fractions more than if they only had the representation of fractions with numbers. Since every student learns with a different way, the multiple representations of fractions could eliminate the difficulties that students face in fractions.

The use of the SKAFOS system can be done by the class teacher using it for the student to initially understand the fractional unit. The different representations (text, circular disk, and rectangle) enable the teacher to present the fractional unit differently and the student to see the same fraction in different forms. So if a student cannot understand the fractions through a specific representation, then he can choose to see a different representation of fractions and be able to understand the concepts in this way.

Besides the teacher can use it for the learning process of comparing two fractions. Experimenting with students with different representations during fraction comparison can help develop an intuitive understanding of the value of the fraction and gradually build knowledge about fraction comparison. Also, the teacher, knowing the emotions of the students that appeared when solving exercises with different representations, can design learning activities of fractions with the representations of the fractions that the students were most successful.

Guided by the findings of the present study, exercises and tasks in fractions can be designed with examples from the daily life of students regarding the fractional unit and the comparison of two fractions.

11.4 Suggestions for additional research and further expansion of SKAFOS

The research carried out with SKAFOS had some limitations. One of the limitations is the number of participating students, the sample size of students. In the current survey, it was thirty students. This number is at least the minimum number of a case study research to do some statistical analysis [59]. Thus, research with more students could give us more data and better results that could support our findings. Also, the survey could be extended to students of secondary and tertiary, since as it emerged from the literature review, these students also encounter problems with fractions, but in other fields. Additionally, small changes could be made to the content of fraction comparisons to make them available to higher grade students.

Furthermore, the research could be carried out in teachers of elementary schools. As teachers are the ones who provide the knowledge, it is a good thing to know how SKAFOS works so that they can help their students understand fractions. They could perform precisely the same fraction comparison activities that students perform in the SKAFOS system, observe the environment and experiment with the exercises. In the end, they could suggest changes or corrections that could be made to the system to make it better and essentially help students understand fractions.

Also, the use of the SKAFOS system in real conditions, in a classroom, by teachers would be of particular research interest. That is, some teachers to integrate the SKAFOS system in the teaching process, to see in practice how it meets the needs of teaching and to monitor the involvement of students with the specific system. Teachers' views on the use of this tool in the classroom would undoubtedly be of research interest, and their feedback would undoubtedly help to improve and streamline the system. Of course, it's completely different to do the fraction activities yourself and otherwise have to do them during classroom time.

Also, future research on different groups of students would undoubtedly come to useful conclusions. An experimental group and a control group could be surveyed to compare the results for the effectiveness of the SKAFOS system. First of all, a pre-test could be given to both teams to record their initial level of performance. After that, the intervention would take place. Specifically, the control group would continue to follow the traditional way of fractions teaching, but the experimental group would use the SKAFOS system. After the completion of the intervention, a post-test would be provided to record the performance of the students. So a comparison of pre-test and post-test could be made using statistical analysis. In this way, useful conclusions could be drawn from the comparison of the results of the two groups for the use of the SKAFOS system in the teaching process.

Finally, there are in our plans changes or extensions of the system on various axes. In particular, other operations (i.e. addition, subtraction, multiplication and division) in fractions could be added to the system, so that the student has a complete help for fractions. Exercises could initially be added to the addition and subtraction. Then the multiplication and division of fractions could be added. All these additions also require enough changes to the system so that it can support all of them. Of course, it might be necessary to redesign the system so that the system can be more functional for all the basic operations of fractions.

Furthermore, some elements will be incorporated that will help the student during the solution of exercises with fractions. One of them is the feedback process. It could be provided instructions during the solving of tasks. These instructions could be as directions in a text box frame or as an audio message. Of course, under no circumstances would the provision of instructions distract the student from the activities he had to do.

Another feature of the system that could be changed is the shape of the manipulatives (circular disks and number line). The size of the objects are fixed, and the user could not change them. A good option would be for the user to be able to resize shapes to different sizes (small, medium and large) and color them with different colors. Another option would be the use of real images of objects (e.g. picture of sliced pizza into pieces).

The current version of the system could represent two kinds of digital manipulatives (circular disks and rectangles with number lines). The system could allow the user to see fractions in other forms of representation in two or three dimensions. There could be a rectangle divided into pieces, such as chocolate or a container that fills up, as it has been presented in some other existing software.

Besides, there are ideas for integrating problems with fractions on everyday issues that are of real interest to students. Specifically, situations from the daily activities of students (such as sports activities, shopping in the canteen) could be turned into fraction tasks to be solved.

Finally, the reduction of the number of emotion recognition from seven to five is studied, since the research showed that only five are the dominant emotions and those that play a crucial role in shaping students' emotions. This reduction could improve the system's performance in detecting emotions faster.

Published work

12

Chapter Twelve

In this Chapter it is presented the list of scientific publications - presentations (not simple participation), in scientific conferences, as well as the list of scientific publications in scientific journals.

1. Mastorodimos, D., Chatzichristofis, S. A., Jimoyiannis, A., & Christodoulou, K. (2018, July). Making Sense of Fractions using E-learning Platforms-A Survey. In 2018 Innovations in Intelligent Systems and Applications (INISTA) (pp. 1-6). Thessaloniki, Greece. 3-5 July, 2018. IEEE.

2. Mastorodimos, D., & Chatzichristofis, S. A. (2019). Studying affective tutoring systems for mathematical concepts. Journal of Educational Technology Systems, 48(1), 14-50.

3. Mastorodimos, D., Jimoyiannis, A., & Chatzichristofis, S. A. (2019, December). Design and development of an online platform for learning and recognizing the emotional state of the student when solving problems with fractions. In Proceedings of 2nd Panhellenic Conference: Open Educational Resources and E-Learning (ELOER2019). Korinth, Greece. 13-14 December, 2019. (in greek).

4. Mastorodimos, D., Jimoyiannis, A., & Chatzichristofis, S. A. (2020). Design and development of an online platform for learning and recognizing the emotional state of the student when solving problems with fractions. Themes of Sciences and Technology in Education, 13 (1/2), 7-32. (in greek)

5. Mastorodimos, D., Jimoyiannis, A. & Chatzichristofis, S. A. Investigating the teachers' perceptions of the challenges and the difficulties they face in teaching fractions to young students. (Under Review).

6. Mastorodimos, D., Christodoulou, K., Jimoyiannis, A. & Chatzichristofis, S. A. Evaluating Fractions' Manipulative using Deep Learning. (Under Review).

7. Mastorodimos, D., & Chatzichristofis, S. A. Difficulties on comparing pairs of proper fractions for fifth and sixth grade Greek students. (Under Review).

References

[1] Adams, W. K., Reid, S., LeMaster, R., McKagan, S. B., Perkins, K. K., Dubson, M., and Wieman, C. E. A study of educational simulations part i-engagement and learning. *Journal of Interactive Learning Research 19*, 3 (2008), 397–419.

[2] Aksoy, N. C., and Yazlik, D. O. Student errors in fractions and possible causes of these errors. *Journal of education and training studies 5*, 11 (2017), 219–233.

[3] Aksu, M. Student performance in dealing with fractions. *The Journal of Educational Research 90*, 6 (1997), 375–380.

[4] Alepis, E., and Virvou, M. Automatic generation of emotions in tutoring agents for affective e-learning in medical education. *Expert Systems with Applications 38*, 8 (2011), 9840–9847.

[5] Aleven, V., McLaren, B. M., and Sewall, J. Scaling up programming by demonstration for intelligent tutoring systems development: An open-access web site for middle school mathematics learning. *IEEE Transactions on Learning Technologies 2*, 2 (2009), 64–78.

[6] Alexander, S. Emulating human tutor empathy. In *Proceedings of the IIMS Postgraduate Conference, Albany, New Zealand* (2004), Citeseer, pp. 26–32.

[7] Alexander, S. Modeling emotions from non-verbal behaviour in an affective tutoring system. In *International Conference on User Modeling* (2005), Springer, pp. 493–495.

[8] Alexander, S., and Sarrafzadeh, A. Interfaces that adapt like humans. In *Asia-Pacific Conference on Computer Human Interaction* (2004), Springer, pp. 641–645.

[9] Alexander, S., Sarrafzadeh, A., and Hill, S. Easy with eve: A functional affective tutoring system. In *Workshop on Motivational and Affective Issues in ITS. 8th International Conference on ITS* (2006), Citeseer, pp. 5–12.

[10] Alexander, S., Sarrafzadeh, A., and Hill, S. Foundation of an affective tutoring system: Learning how human tutors adapt to student emotion. *International journal of intelligent systems technologies and applications 4*, 3-4 (2008), 355–367.

[11] Alkhatlan, A., and Kalita, J. Intelligent tutoring systems: A comprehensive historical survey with recent developments. *arXiv preprint arXiv:1812.09628* (2018).

[12] Alqahtani, F., and Ramzan, N. Comparison and efficacy of synergistic intelligent tutoring systems with human physiological response. *Sensors 19*, 3 (2019), 460.

[13] Ambrósio, A. P. L., and Costa, F. M. Evaluating the impact of pbl and tablet pcs in an algorithms and computer programming course. In *Proceedings of the 41st ACM technical symposium on Computer science education* (2010), ACM, pp. 495–499.

[14] Amershi, S., Conati, C., and Maclaren, H. Using feature selection and unsupervised clustering to identify affective expressions in educational games. In *Proceedings of the Workshop" Motivational and Affective Issues in ITS". In conjunction with ITS2006, 8th International Conference on Intelligent Tutoring Systems (2006), Jhongli* (2006), Citeseer.

[15] Ammar, M. B., Neji, M., Alimi, A. M., and Gouardères, G. The affective tutoring system. *Expert Systems with Applications 37*, 4 (2010), 3013–3023.

[16] Aremu, A., and Adebagbo, A. Games, game flow, and gender as they affect mathematics achievement of pupils in nigeria. *Bulgarian Journal of Science & Education Policy 10*, 1 (2016).

[17] Armstrong, R. D. An area model for solving probability problems. *Teaching statistics and probability* (1981), 135–142.

[18] Arroyo, I., Cooper, D. G., Burleson, W., Woolf, B. P., Muldner, K., and Christopherson, R. Emotion sensors go to school. In *AIED* (2009), vol. 200, Citeseer, pp. 17–24.

[19] Ashcraft, M. H. Math anxiety: Personal, educational, and cognitive consequences. *Current directions in psychological science 11*, 5 (2002), 181–185.

[20] Aslan, S., Alyuz, N., Okur, E., Mete, S. E., Oktay, E., and Esme, A. A. Effect of emotion-aware interventions on students' behavioral and emotional states. *Educational Technology Research and Development 66*, 6 (2018), 1399–1413.

[21] Banilower, E. R., Smith, P. S., Weiss, I. R., Malzahn, K. A., Campbell, K. M., and Weis, A. M. Report of the 2012 national survey of science and mathematics education. *Horizon Research, Inc.(NJ1)* (2013).

[22] Barrón-Estrada, M. L., Zatarain-Cabada, R., Aranda-Ortega, M., Pérez, H. E. G., and Mejía-Arredondo, J. An intelligent, affective and configurable tutor for 3rd grade natural numbers teaching. *Research in Computing Science 87* (2014), 35–41.

[23] Barrón-Estrada, M. L., Zatarain-Cabada, R., Beltrán V. J.A., Cibrian R. F.L., and Pérez, Y. H. An intelligent and affective tutoring system within a social network for learning mathematics. In *Ibero-American Conference on Artificial Intelligence* (2012), Springer, pp. 651–661.

[24] Barrón-Estrada, M. L., Zatarain-Cabada, R., ZatarainCabada, R., and Estrada, A. B. Design and implementation of an affective its. *Research in Computing Science 56* (2012), 61–68.

[25] Barrón-Estrada, M. L., Zatarain-Cabada, R., Zatarain-Cabada, R., Verdugo, J. A. B., Robles, F. L. C., and López, M. I. Q. Fermat: An intelligent social network for mathematics. *Research in Computing Science 47* (2012), 73–79.

[26] Basturk, S. Primary student teachers' perspectives of the teaching of fractions. *Acta Didactica Napocensia 9*, 1 (2016), 35–44.

[27] Behr, M., and Post, T. Teaching rational number and decimal concepts. *Teaching mathematics in grades K-8: Research-based methods 2* (1992), 201–248.

[28] Behr, M. J., Harel, G., Post, T. R., and Lesh, R. Rational number, ratio, and proportion. In *Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics*, D. A. Grouws, Ed. Macmillan Publishing Co, Inc, 1992, pp. 296–333.

[29] Behr, M. J., Lesh, R., Post, T., and Silver, E. A. Rational number concepts. *Acquisition of mathematics concepts and processes* (1983), 91 – 126.

[30] Behr, M. J., Wachsmuth, I., Post, T. R., and Lesh, R. Order and equivalence of rational numbers: A clinical teaching experiment. *Journal for research in mathematics education* (1984), 323–341.

[31] Biber, A., Tuna, A., and Aktaş, O. Students' misconceptions of fractions and its effect on solving fractions problems. *Trakya University Journal of Education 3*, 2 (2013), 152–162.

[32] Biggs, J. B., and Collis, K. F. *Evaluation the quality of learning: the SOLO taxonomy (structure of the observed learning outcome)*. Academic Press, 1982.

[33] Bloom, B. S., Englehart, M., Furst, E. J., Hill, W. H., and Krathwohl, D. R. *Taxonomy of educational objectives: Handbook 1 Cognitive domain*. David McKay, 1956.

[34] Βούργιας, Χ., and Χρυσοστόμου-Βούργια, Σ. Στάσεις και απόψεις των μαθητών και των γονιών για το μάθημα των μαθηματικών: Αρωγός ή τροχοπέδη; In *9ο Παγκύπριο Συνέδριο Παιδαγωγικής Εταιρείας Κύπρου* (2006), pp. 135–144.

[35] Booker, G. Constructing mathematical conventions formed by the abstraction and generalization of earlier ideas: The development of initial fraction ideas. In *Theories of Mathematical Learning*, L. P. Steffe, P. Nesher, P. Cobb, G. A. Goldin, and B. Greer, Eds. Lawrence Erlbaum Associates, 1996, pp. 393–408.

[36] Βοσνιάδου, Σ. *Η ψυχολογία των μαθηματικών.* Gutenberg, 2005.

[37] Βοσνιάδου, Σ. *Παιδεία, σχολεία, και υπολογιστές. Προοπτικές, προβλήματα και προτάσεις για την αποτελεσματικότερη χρήση των νέων τεχνολογιών στην εκπαίδευση.* Gutenberg, 2006.

[38] Bostan, C.-G. Focus-group research on modern techniques and multimedia tools implementation in teaching practice. *Procedia-Social and Behavioral Sciences 180* (2015), 1444–1450.

[39] Boulet, G. Didactical implications of children's difficulties in learning the fraction concept. *Focus on Learning Problems in Mathematics 20*, 4 (1998), 19–34.

[40] Bragg, L. A. Hey, i'm learning this. *Australian Primary Mathematics Classroom 11*, 4 (2006), 4 – 9.

[41] Braun, V., and Clarke, V. Thematic analysis. In *APA handbook of research methods in psychology, Vol. 2. Research designs: Quantitative, qualitative, neuropsychological, and biological*, H. Cooper, P. M. Camic, D. L. Long, A. T. Panter, D. Rindskopf, and K. J. Sher, Eds. American Psychological Association, 2012, pp. 57–71.

[42] Brousseau, G., Brousseau, N., and Warfield, V. Rationals and decimals as required in the school curriculum: Part 1: Rationals as measurement. *The Journal of Mathematical Behavior 23*, 1 (2004), 1–20.

[43] Brown, G., and Quinn, R. J. Algebra students' difficulty with fractions: An error analysis. *The Australian Mathematics Teacher 62*, 4 (2006), 28.

[44] Brusilovsky, P., and Su, H.-D. Adaptive visualization component of a distributed web-based adaptive educational system. In *International Conference on Intelligent Tutoring Systems* (2002), Springer, pp. 229–238.

[45] Buxton, L. *Do you Panic about Maths?: Coping with maths anxiety*. Heinemann Educational Books, 1981.

[46] Cabada, R. Z., Estrada, M. L. B., Beltrán, V. J. A., Cibrian, R. F. L., Garcia, C. A. R., and Pérez, Y. H. Fermat: merging affective tutoring systems with learning social networks. In *Advanced Learning Technologies (ICALT), 2012 IEEE 12th International Conference on* (2012), IEEE, pp. 337–339.

[47] Cabada, R. Z., Estrada, M. L. B., Hernández, F. G., and Bustillos, R. O. Intelligent tutoring system with affective learning for mathematics. In *Mexican International Conference on Artificial Intelligence* (2014), Springer, pp. 483–493.

[48] Carpenter, P. T., Levi, L., Berman, P. W., and Pligge, M. Developing algebraic reasoning in the elementary school. In *Understanding mathematics and science matters*, T. A. Romberg, T. P. Carpenter, and F. Dremock, Eds. Lawrence Erlbaum Associates, 2005, pp. 81–98.

[49] Carpenter, T. K., Levi, L., Berman, P. W., and Pligge, M. Developing algebraic reasoning in the elementary school. In *Understanding Mathematics and Science Matters,* T.A. Romberg, T.P. Carpenter and F. Dremock, Eds. Lawrence Erlbaum Associates, Inc., 2005, pp. 81–98.

[50] Carstens, A., and Beck, J. Get ready for the gamer generation. *TechTrends 49*, 3 (2004), 22–25.

[51] Cetin, I., and Andrews-Larson, C. Learning sorting algorithms through visualization construction. *Computer Science Education 26*, 1 (2016), 27–43.

[52] Chang, W.-L., Yuan, Y., Lee, C.-Y., Chen, M.-H., and Huang, W.-G. Using magic board as a teaching aid in third grader learning of area concepts. *Educational Technology & Society 16*, 2 (2013), 163–173.

[53] Charalambous, C. Y., and Pitta-Pantazi, D. Drawing on a theoretical model to study students' understandings of fractions. *Educational studies in mathematics 64*, 3 (2007), 293–316.

[54] Chen, L., Bae, S. R., Battista, C., Qin, S., Chen, T., Evans, T. M., and Menon, V. Positive attitude toward math supports early academic success: Behavioral evidence and neurocognitive mechanisms. *Psychological science 29*, 3 (2018), 390–402.

[55] Chinnappan, M., and Desplat, B. Contextualisation of fractions: Teachers pedagogical and mathematical content knowledge for teaching. *Journal of Science and Mathematics Education in Southeast Asia 35*, 1 (2012), 43–59.

[56] Choy, B. H. Teachers' productive mathematical noticing during lesson preparation. In *Proceedings of the Joint Meeting of PME* (2014), vol. 38, pp. 297–304.

[57] Clarke, D. M., and Roche, A. Students' fraction comparison strategies as a window into robust understanding and possible pointers for instruction. *Educational Studies in Mathematics 72*, 1 (2009), 127 – 138.

[58] Clarke, D. M., Roche, A., and Mitchell, A. Year six fraction understanding: A part of the whole story. In *Proceedings of the 30th annual conference of the Mathematics Education Research Group of Australasia: Mathematics; Essential research, essential practice*, J. Watson and K. Beswick, Eds. Citeseer, 2007, 207–216.

[59] Cohen, L., Manion, L., and Morrison, K. *Research methods in education*. Routledge, 2013.

[60] Conati, C. Probabilistic assessment of user's emotions in educational games. *Applied artificial intelligence 16*, 7-8 (2002), 555–575.

[61] Conati, C. How to evaluate models of user affect? In *Tutorial and Research Workshop on Affective Dialogue Systems* (2004), Springer, pp. 288–300.

[62] Conati, C., Chabbal, R., and Maclaren, H. A study on using biometric sensors for monitoring user emotions in educational games. In *Workshop on Assessing and Adapting to User Attitudes and Affect: Why, When and How* (2003), Citeseer.

[63] Conati, C., and Maclare, H. Evaluating a probabilistic model of student affect. In *International Conference on Intelligent Tutoring Systems* (2004), Springer, pp. 55–66.

[64] Conati, C., and Maclaren, H. Data-driven refinement of a probabilistic model of user affect. In *International Conference on User Modeling* (2005), Springer, pp. 40–49.

[65] Conati, C., and Maclaren, H. Modeling user affect from causes and effects. In *International Conference on User Modeling, Adaptation, and Personalization* (2009), Springer, pp. 4–15.

[66] Conati, C., and Manske, M. Evaluating adaptive feedback in an educational computer game. In *International Workshop on Intelligent Virtual Agents* (2009), Springer, pp. 146–158.

[67] Conati, C., and Zhao, X. Building and evaluating an intelligent pedagogical agent to improve the effectiveness of an educational game. In *Proceedings of the 9th International Conference on Intelligent User Interfaces* (2004), ACM, pp. 6–13.

[68] Conati, C., and Zhou, X. Modeling students' emotions from cognitive appraisal in educational games. In *International Conference on Intelligent Tutoring Systems* (2002), Springer, pp. 944–954.

[69] Cramer, K., and Henry, A. Using manipulative models to build number sense for addition of fractions. In *Making sense of fractions, ratio and proportions*, B. Litwiller, Ed. NCTM, 2002, pp. 41–48.

[70] Cuadrado, L.-E. I., Riesco, Á. M., and de la Paz López, F. Fer in primary school children for affective robot tutors. In *International WorkConference on the Interplay Between Natural and Artificial Computation* (2019), Springer, pp. 461–471.

[71] Dalrymple, K. A., Gomez, J., and Duchaine, B. The dartmouth database of children's faces: acquisition and validation of a new face stimulus set. *PloS one 8*, 11 (2013), e79131.

[72] Davis, G., Hunting, R. P., and Pearn, C. What might a fraction mean to a child and how would a teacher know? *The Journal of Mathematical Behavior 12*, 1 (1993), 63–76.

[73] De Jong, T., and Van Joolingen, W. R. Scientific discovery learning with computer simulations of conceptual domains. *Review of Educational Research 68*, 2 (1998), 179–201.

[74] Desmet, L., Grégoire, J., and Mussolin, C. Developmental changes in the comparison of decimal fractions. *Learning and Instruction 20*, 6 (2010), 521–532.

[75] Dewey, J. *Democracy and education: An introduction to the philosophy of education*. Free Press, 1997.

[76] DeWolf, M., and Vosniadou, S. The whole number bias in fraction magnitude comparisons with adults. In *Proceedings of the annual meeting of the cognitive science society* (2011), vol. 33, pp. 1751–1756.

[77] Ding, H., Zhou, S. K., and Chellappa, R. Facenet2expnet: Regularizing a deep face recognition net for expression recognition. In *2017 12th IEEE International Conference on Automatic Face & Gesture Recognition (FG 2017)* (2017), IEEE, pp. 118–126.

[78] D'Mello, S., Picard, R. W., and Graesser, A. Toward an affect-sensitive autotutor. *IEEE Intelligent Systems 22*, 4 (2007), 53–61.

[79] Duval, R. A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational Studies in Mathematics 61*, 1-2 (2006), 103–131.

[80] Egger, H. L., Pine, D. S., Nelson, E., Leibenluft, E., Ernst, M., Towbin, K. E., and Angold, A. The nimh child emotional faces picture set (nimh-chefs): a new set of children's facial emotion stimuli. *International Journal of Methods in Psychiatric Research 20*, 3 (2011), 145 – 156.

[81] Ernest C. Davenport, J., Davison, M. L., Kuang, H., Ding, S., Kim, S.-K., and Kwak, N. High school mathematics course-taking by gender and ethnicity. *American Educational Research Journal 35*, 3 (1998), 497–514.

[82] Ervin, H. K. Fraction multiplication and division models: A practitioner reference paper. *International Journal of Research in Education and Science 3*, 1 (2017), 258–279.

[83] Εξαρχάκος, Γ. Τ. *Διδακτική των μαθηματικών: Εκπαίδευση και μαθηματικά-Ειδική διδακτική των μαθηματικών-Ειδικά θέματα διδακτικής μαθηματικών.* ΕΛΛΗΝΙΚΑ ΓΡΑΜΜΑΤΑ, 1993.

[84] Fan, C., Johnson, M., Messom, C., and Sarrafzadeh, A. Machine vision for an intelligent tutor. In *Proceedings of the International Conference on Computational Intelligence, Robotics and Autonomous Systems, Singapore* (2003), Citeseer.

[85] Fazio, L. K., Kennedy, C. A., and Siegler, R. S. Improving children's knowledge of fraction magnitudes. *PloS one 11*, 10 (2016), e0165243.

[86] Feenstra, L., Aleven, V., Rummel, N., Rau, M., and Taatgen, N. Thinking with your hands: interactive graphical representations in a tutor for fractions learning. In *International Conference on Artificial Intelligence in Education* (2011), Springer, pp. 453–455.

[87] Fennema, E. The study of affect and mathematics: A proposed generic model for research. In *Affect and mathematical problem solving*. Springer, 1989, pp. 205–219.

[88] Fischbein, E., Deri, M., Nello, M. S., and Marino, M. S. The role of implicit models in solving verbal problems in multiplication and division. *Journal for research in mathematics education* (1985), 3–17.

[89] Fryer Jr, R. G., and Levitt, S. D. An empirical analysis of the gender gap in mathematics. *American Economic Journal: Applied Economics 2*, 2 (2010), 210–40.

[90] Gagne, R. M., Briggs, L. J., and Wager, W. W. *Principles of instructional design (4th ed.)*. Holt, 1992.

[91] Gallistel, C. R., and Gelman, R. Preverbal and verbal counting and computation. *Cognition 44*, 1-2 (1992), 43–74.

[92] Γαγάτσης, Α., Λοίζου, Α., Στυλιανού, Μ., and Τόφαρου, Σ. Διδακτικό συμβόλαιο και μάθηση μαθηματικών. In *9ο Παγκύπριο Συνέδριο Παιδαγωγικής Εταιρείας Κύπρου* (2006), pp. 39–53.

[93] Γαγάτσης, Α., Σιημήτρα-Κωνσταντίνου, Α. and Χριστοδουλίδου, Ο. Γιατί οι μαθητές δυσκολεύονται στα κλάσματα; In *9ο Παγκύπριο Συνέδριο Παιδαγωγικής Εταιρείας Κύπρου* (2006), pp. 99–110.

[94] Garcia, I., and Pacheco, C. A constructivist computational platform to support mathematics education in elementary school. *Computers & Education 66* (2013), 25–39.

[95] Gómez, D. M., and Dartnell, P. Middle schoolers' biases and strategies in a fraction comparison task. *International Journal of Science and Mathematics Education 17*, 6 (2019), 1233–1250.

[96] Gómez, D. M., Jiménez, A., Bobadilla, R., Reyes, C., and Dartnell, P. Exploring fraction comparison in school children. *North American Chapter of the International Group for the Psychology of Mathematics Education* (2014).

[97] González-Forte, J. M., Fernández, C., and Van Dooren, W. Gap and congruency effect in fraction comparison. In *Proceedings of the 42 nd Conference of the International Group for the Psychology of Mathematics Education (Vol. 2)* (2018), PME; Umea, Sweden, pp. 459–466.

[98] González-Forte, J. M., Fernández-Verdú, C., and Van Dooren, W. Is there a gap or congruency effect? A cross-sectional study in students' fraction comparison. *Studia Psychologica 62*, 2 (2020), 109–122.

[99] Goodfellow, I. J., Erhan, D., Carrier, P. L., Courville, A., Mirza, M., Hamner, B., Cukierski, W., Tang, Y., Thaler, D., Lee, D.-H., et al. Challenges in representation learning: A report on three machine learning contests. In *International Conference on Neural Information Processing* (2013), Springer, pp. 117–124.

[100] Gordon, G., Spaulding, S., Westlund, J. K., Lee, J. J., Plummer, L., Martinez, M., Das, M., and Breazeal, C. Affective personalization of a social robot tutor for children's second language skills. In *Thirtieth AAAI Conference on Artificial Intelligence* (2016), AAAI.

[101] Grawemeyer, B., Gutierrez-Santos, S., Holmes, W., Mavrikis, M., Rummel, N., Mazziotti, C., and Janning, R. Talk, tutor, explore, learn: Intelligent tutoring and exploration for robust learning. In *Proceedings of the 17th International Conference on Artificial Intelligence in Education (AIED)* (2015).

[102] Grawemeyer, B., Mavrikis, M., Holmes, W., Hansen, A., Loibl, K., and Gutiérrez-Santos, S. Affect matters: Exploring the impact of feedback during mathematical tasks in an exploratory environment. In *International Conference on Artificial Intelligence in Education* (2015), Springer, pp. 595–599.

[103] Gunawan, M. S., Putri, R. I. I., and Zulkardi, M. Learning fractions through swimming context for elementary school students. In *5th SEA-DR (South East Asia Development Research) International Conference 2017 (SEADRIC 2017)* (2017), Atlantis Press.

[104] Hackenberg, A., Creager, M., Eker, A., and Lee, M. Understanding how to differentiate instruction for middle school students. In *Paper presented at the annual research conference of the National Council of Teachers of Mathematics* (2016).

[105] Hannula, M. S. Locating fraction on a number line. *International Group for the Psychology of Mathematics Education 3* (2003), 17–24.

[106] Hansen, A., Mavrikis, M., and Geraniou, E. Supporting teachers' technological pedagogical content knowledge of fractions through codesigning a virtual manipulative. *Journal of Mathematics Teacher Education 19*, 2-3 (2016), 205–226.

[107] Hasemann, K. On difficulties with fractions. *Educational Studies in Mathematics 12*, 1 (1981), 71–87.

[108] Haser, Ç., and Ubuz, B. Conceptual and procedural performance in fractions. *Education and Science 27*, 126 (2002), 53–61.

[109] Herbst, P. The number-line metaphor in the discourse of a textbook series. *For the Learning of Mathematics 17* (1997), 36–45.

[110] Hiebert, J. Children's mathematics learning: The struggle to link form and understanding. *The Elementary School Journal 84*, 5 (1984), 497–513.

[111] Hill, S. Troublesome knowledge: why don't they understand? *Health Information & Libraries Journal 27*, 1 (2010), 80–83.

[112] Hoyles, C., Noss, R., and Pozzi, S. Proportional reasoning in nursing practice. *Journal for Research in Mathematics Education* (2001), 4–27.

[113] Hyde, J. S., Lindberg, S. M., Linn, M. C., Ellis, A. B., and Williams, C. C. Gender similarities characterize math performance. *Science 321*, 5888 (2008), 494–495.

[114] Inamdar, P., and Kulkarni, A. Hole-in-the-wall computer kiosks foster mathematics achievement-a comparative study. *Educational Technology & Society 10*, 3 (2005), 170–179.

[115] Ivanova, M. Researching affective computing techniques for intelligent tutoring systems. In *2013 International Conference on Interactive Collaborative Learning (ICL)* (2013), IEEE, pp. 596–602.

[116] Jackson, C. D., and Leffingwell, R. J. The role of instructors in creating math anxiety in students from kindergarten through college. *The Mathematics Teacher 92*, 7 (1999), 583–586.

[117] Janning, R., Schatten, C., and Schmidt-Thieme, L. Feature analysis for affect recognition supporting task sequencing in adaptive intelligent tutoring systems. In *European Conference on Technology Enhanced Learning* (2014), Springer, pp. 179–192.

[118] Jaques, P. A., Lehmann, M., and Pesty, S. Evaluating the affective tactics of an emotional pedagogical agent. In *Proceedings of the 2009 ACM symposium on Applied Computing* (2009), ACM, pp. 104–109.

[119] Jaques, P. A., Seffrin, H., Rubi, G., de Morais, F., Ghilardi, C., Bittencourt, I. I., and Isotani, S. Rule-based expert systems to support step-by-step guidance in algebraic problem solving: The case of the tutor pat2math. *Expert Systems with Applications 40*, 14 (2013), 5456–5465.

[120] Jaques, P. A., Vicari, R., Pesty, S., and Martin, J.-C. Evaluating a cognitive-based affective student model. In *International Conference on Affective Computing and Intelligent Interaction* (2011), Springer, pp. 599 – 608.

[121] Jimoyiannis, A. Computer simulations and scientific knowledge construction. In *Gaming and simulations: Concepts, methodologies, tools and applications*. IGI Global, 2011, pp. 57–74.

[122] Jonassen, D. H. *Learning to solve problems: A handbook for designing problem-solving learning environments*. Routledge, 2010.

[123] Κόμης, Β. Οι νέες τεχνολογίες στη διδακτική και μαθησιακή διαδικασία. In *1ης Πανηπειρωτικής ημερίδας «Πληροφορική και Εκπαίδευση»* (1998), pp. 23–34.

[124] Kaklauskas, A., Kuzminske, A., Zavadskas, E. K., Daniunas, A., Kaklauskas, G., Seniut, M., Raistenskis, J., Safonov, A., Kliukas, R., Juozapaitis, A., et al. Affective tutoring system for built environment management. *Computers & Education 82* (2015), 202–216.

[125] Καλογιαννάκης, Μ., Παπαδάκης, Σ., and Ζαράνης, Ν. Φορητές εκπαιδευτικές εφαρμογές για παιδιά. Εκτός από φορητές είναι και εκπαιδευτικές. In *9ο Πανελλήνιο Συνέδριο με Διεθνή Συμμετοχή «Τεχνολογίες της Πληροφορίας & Επικοινωνιών στην Εκπαίδευση»* (2014), Πανεπιστήμιο Κρήτης, pp. 1054–1061.

[126] Kaput, J. Technology and mathematics education. *Handbook of research on mathematics teaching and learning* (1992), 515–556.

[127] Kaput, J. J. Creating cybernetic and psychological ramps from the concrete to the abstract: Examples from multiplicative structures. *Software goes to school: Teaching for understanding with new technologies* (1995), 130–154.

[128] Kara, F., and Incikabi, L. Sixth grade students' skills of using multiple representations in addition and subtraction operations in fractions. *International Electronic Journal of Elementary Education 10*, 4 (2018), 463–474.

[129] Käser, T., Baschera, G.-M., Busetto, A. G., Klingler, S., Solenthaler, B., Buhmann, J. M., and Gross, M. Towards a framework for modelling engagement dynamics in multiple learning domains. *International journal of artificial intelligence in education 22*, 1-2 (2013), 59–83.

[130] Kautzmann, T. R., Carlotto, T., and Jaques, P. A. Adaptive training of the metacognitive skill of knowledge monitoring in intelligent tutoring systems. In *International Conference on Intelligent Tutoring Systems* (2016), Springer, pp. 301–306.

[131] Ketterlin-Geller, L. R., and Yovanoff, P. Diagnostic assessments in mathematics to support instructional decision making. *Practical Assessment, Research, and Evaluation 14*, 1 (2009), 16.

[132] Khan, R. A., Crenn, A., Meyer, A., and Bouakaz, S. A novel database of children's spontaneous facial expressions (LIRIS-CSE). *Image Vision Computing 83-84* (2019), 61–69.

[133] Kiili, K., and Ketamo, H. Evaluating cognitive and affective outcomes of a digital game-based math test. *IEEE Transactions on Learning Technologies 11*, 2 (2017), 255–263.

[134] Kiili, K., and Ketamo, H. Evaluating cognitive and affective outcomes of a digital game-based math test. *IEEE Transactions on Learning Technologies 11*, 2 (2018), 255–263.

[135] Kilpatrick, J., Swafford, J., and Findell, B. *Adding it up: Helping children learn mathematics*. National Academy Press, 2001.

[136] Kinash, S., Brand, J., Mathew, T., and Kordyban, R. *Uncoupling mobility and learning: When one does not guarantee the other. In* International Conference on ICT in Teaching and Learning *(2011), Springer, pp. 342–350.*

[137] Kocaoglu, Y., and Yenilmez, K. 5th grade students' mistakes and misconceptions in solving problems about fractions. *Dicle Üniversity Journal of Ziya Gökalp Education Faculty 14* (5), 71–85.

[138] Koedinger, K., and Tanner, M. *Things you should know about intelligent tutoring systems.* In *Proceedings of the EDUCAUSE Learning Initiative (ELI),* Denver, CO, USA *(2013), 4 – 6.*

[139] Koedinger, K. R., and Corbett, A. Cognitive tutors: Technology bringing learning sciences to the classroom. *Cambridge University Press, 2006.*

[140] Kong, P. P. Understanding the teachers' perspectives on the role of teacher autonomy in english classrooms in chinese secondary schools. *Educational Studies* (2020), 1–11.

[141] Koopman, M., Thurlings, M., and den Brok, P. Factors influencing students' proficiency development in the fraction domain: the role of teacher cognitions and behaviour. *Research Papers in Education 34*, 1 (2019), 14–37.

[142] Kort, B., Reilly, R., and Picard, R. W. *An affective model of interplay between emotions and learning: Reengineering educational pedagogybuilding a learning companion. In* Proceedings IEEE International Conference on Advanced Learning Technologies *(2001), IEEE, pp. 43–46.*

[143] Κοτίνη, Ι., and Τζελέπη, Σ. Σχεδιάζοντας ένα μάθημα Πληροφορικής που βασίζεται στις αρχές του εποικοδομητισμού, της υπολογιστικής σκέψης και της παιχνιδοποίησης. In *5th Conference on Informatics in Education – Η Πληροφορική στην εκπαίδευση (5th CIE 2013)* (2013), pp. 1–14.

[144] Kotini, I., and Tzelepi, S. A gamification-based framework for developing learning activities of computational thinking. In *Gamification in Education and Business*, Springer, 2015, pp. 219–252.

[145] Krathwohl, D. R. A revision of bloom's taxonomy: An overview. *Theory into practice 41*, 4 (2002), 212–218.

[146] Krathwohl, D. R., Bloom, B. S., and Masia, B. B. *Taxonomy of educational objectives, the classification of educational goals. Handbook II: affective domain*. David McKay, New York, 1964.

[147] Krizhevsky, A., Sutskever, I., and Hinton, G. E. Imagenet classification with deep convolutional neural networks. In *Advances in neural information processing systems* (2012), pp. 1097–1105.

[148] Ku, O., Chen, S. Y., Wu, D. H., Lao, A. C. C., and Chan, T.-W. The effects of game-based learning on mathematical confidence and performance: High ability vs. low ability. *Journal of Educational Technology & Society 17*, 3 (2014), 65–78.

[149] Ku, O. Y. M., Wu, J. C. Y., Yang, B. C. J., Lin, C.-J., and Chan, T.-W. Maintaining student engagement in extensive practice by implanting gaming factors. In *The 16th International Conference on Computers in Education (ICCE 2008)* (2008), pp. 721–728.

[150] Kuo, B.-C., Chen, C.-H., Yang, C.-W., and Mok, M. M. C. Cognitive diagnostic models for tests with multiple-choice and constructedresponse items. *Educational Psychology 36*, 6 (2016), 1115–1133.

[151] Λεμονίδης, Χ. *Μια νέα πρόταση διδασκαλίας των Μαθηματικών στις πρώτες τάξεις του Δημοτικού Σχολείου.* Πατάκης, 2003.

[152] Λεοντίδης, Ε. *Συναισθηματικό μαθησιακό μοντέλο για εξ αποστάσεως εκπαίδευση: Εφαρμογή στη διδακτική της Πληροφορικής.* PhD thesis, Εθνικό και Καποδιστριακό Πανεπιστήμιο Αθηνών (ΕΚΠΑ), Σχολή Θετικών Επιστημών, 2011.

[153] Lamon, S. J. *Teaching fractions and ratios for understanding: Essential content knowledge and instructional strategies for teachers.* Routledge, 2012.

[154] Lazaridis, V., Samaras, N., and Sifaleras, A. An empirical study on factors influencing the effectiveness of algorithm visualization. *Computer Applications in Engineering Education 21*, 3 (2013), 410–420.

[155] LeCun, Y., Bengio, Y., and Hinton, G. Deep learning. *Nature 521*, 7553 (2015), 436-444.

[156] Lemonidis, C., and Kaiafa, I. The effect of using storytelling strategy on students' performance in fractions. *Journal of Education and Learning 8*, 2 (2019), 165–175.

[157] Lesh, R., Post, T. R., and Behr, M. Dienes revisited: Multiple embodiments in computer environments. In *Development in school mathematics education around the world*. National Council of Teachers of Mathematics, 1987, pp. 647–680.

[158] Letrouvé, T., Bouscayrol, A., Lhomme, W., Dollinger, N., and Calvairac, F. M. Different models of a traction drive for an electric vehicle simulation. In *2010 IEEE Vehicle Power and Propulsion Conference* (2010), IEEE, pp. 1 – 6.

[159] Li, S., Deng, W., and Du, J. Reliable crowdsourcing and deep localitypreserving learning for expression recognition in the wild. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition* (2017), pp. 2852–2861.

[160] Lin, C.-Y. Beliefs about using technology in the mathematics classroom: Interviews with pre-service elementary teachers. *Online Submission 4*, 2 (2008), 135–142.

[161] Lin, H.-C. K., Chao, C.-J., and Huang, T.-C. From a perspective on foreign language learning anxiety to develop an affective tutoring system. *Educational Technology Research and Development 63*, 5 (2015), 727 – 747.

[162] Lindberg, S. M., Hyde, J. S., Petersen, J. L., and Linn, M. C. New trends in gender and mathematics performance: a meta-analysis. *Psychological bulletin 136*, 6 (2010), 1123.

[163] LoBue, V., and Thrasher, C. The child affective facial expression (cafe) set: Validity and reliability from untrained adults. *Frontiers in psychology 5* (2015), 1532.

[164] Loong, E. Y.-K., and Herbert, S. Student perspectives of web-based mathematics. *International Journal of Educational Research 53* (2012), 117–126.

[165] Lopez-Morteo, G., and López, G. Computer support for learning mathematics: A learning environment based on recreational learning objects. *Computers & Education 48,* 4 (2007), 618–641.

[166] Luo, X., Wang, F., and Luo, Z. Investigation and analysis of mathematics anxiety in middle school students. *Journal of mathematics Education 2,* 2 (2009), 12–19.

[167] Ma, L. *Knowing and Teaching Elementary Mathematics: Teacher's Understanding Of Fundamental Mathematics in China and the United States.* Lawrence Erlbaum Associates, Incorporated, 1999.

[168] Ma, L. *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Routledge, 2010.

[169] Ma, X. A meta-analysis of the relationship between anxiety toward mathematics and achievement in mathematics. *Journal for research in mathematics education 30,* 5 (1999), 520–540.

[170] Mao, X., and Li, Z. *Agent based affective tutoring systems: A pilot study.* Computers & Education 55*, 1 (2010), 202–208.*

[171] Mastorodimos, D., and Chatzichristofis, S. A. Studying affective tutoring systems for mathematical concepts. Journal of Educational Technology Systems 48, 1 (2019), 14–50.

[172] Mastorodimos, D., Chatzichristofis, S. A., Jimoyiannis, A., and Christodoulou, K. Making sense of fractions using e-learning platforms-a survey. In *2018 Innovations in Intelligent Systems and Applications (INISTA)* (2018), IEEE, pp. 1 – 6.

[173] Mavrikis, M. Logging, replaying and analysing students' interactions in a web-based ile to improve student modelling. In *AIED* (2005), p. 967.

[174] Mavrikis, M., and Lee, J. Towards more affective tutoring systems. In *Proceedings of the 6th International Conference on Intelligent Tutoring Systems* (2002), Springer-Verlag, p. 1003.

[175] Mavrikis, M., and Maciocia, A. Incorporating assessment into an interactive learning environment for mathematics. *Maths CAA Series* (2003), 1–17.

[176] Mavrikis, M., and Maciocia, A. Wallis: a web-based ile for science and engineering students studying mathematics. In *Workshop of Advanced Technology for Mathematics Education in the 11th International Conference on Artificial Intelligence in Education* (2003), pp. 505–512.

[177] Mavrikis, M., Maciocia, A., and Lee, J. Towards predictive modelling of student affect from web-based interactions. In *Proceedings of the 13th International Conference on Artificial Intelligence in Education* (2007), pp. 169–176.

[178] Mavrikis, M. P. Modelling students' behaviour and affect in ILE through educational data mining. *PhD thesis,* The University of Edinburgh, 2008.

[179] Meece, J. L., Wigfield, A., and Eccles, J. S. Predictors of math anxiety and its influence on young adolescents' course enrollment intentions and performance in mathematics. *Journal of educational psychology 82,* 1 (1990), 60.

[180] Meert, G., Grégoire, J., and Noël, M.-P. Comparing 5/7 and 2 / 9: Adults can do it by accessing the magnitude of the whole fractions. *Acta psychologica 135*, 3 (2010), 284–292.

[181] Meert, G., Grégoire, J., and Noël, M.-P. Comparing the magnitude of two fractions with common components: Which representations are used by 10-and 12-year-olds? *Journal of Experimental Child Psychology 107*, 3 (2010), 244–259.

[182] Mello, G., Carlotto, T., Rubi, G., Seffrin, H., and Jaques, P. A. Implementando o agente de base de domínio do sistema tutor inteligente pat2math. *RENOTE 7,* 1 (2009).

[183] Meng, Z., Liu, P., Cai, J., Han, S., and Tong, Y. Identity-aware convolutional neural network for facial expression recognition. In *2017 12th IEEE International Conference on Automatic Face & Gesture Recognition (FG 2017)* (2017), IEEE, pp. 558–565.

[184] Merenluoto, K., and Lehtinen, E. Conceptual change in mathematics: Understanding the real numbers. In *Reconsidering conceptual change: Issues in theory and practice.* Springer, 2002, pp. 232–257.

[185] Miller, D. J., and Robertson, D. P. Using a games console in the primary classroom: Effects of brain training' programme on computation and self-esteem. *British Journal of Educational Technology 41*, 2 (2010), 242–255.

[186] Miller, L. D., and Mitchell, C. E. Mathematics anxiety and alternative methods of evaluation. *Journal of instructional psychology 21*, 4 (1994), 353.

[187] Μιχαηλίδου, Ε. Ρ. Η χρήση του γεωμετρικού μοντέλου της αριθμητικής γραμμής, για την αναπαράσταση της ισοδυναμίας και πρόσθεσης κλασμάτων: Εφαρμογή σε μαθητές πέμπτης δημοτικού. *PhD thesis*, Πανεπιστήμιο Κύπρου, Σχολή Κοινωνικών Επιστημών και Επιστημών της Αγωγής, 2004.

[188] Mitchell, T. M. Machine Learning. McGraw Hill, 1997.

[189] Mitrovic, A., and Suraweera, P. Evaluating an animated pedagogical agent. In *International Conference on Intelligent Tutoring Systems (2000),* Springer, pp. 73–82.

[190] Mohyuddin, R. G., and Khalil, U. Misconceptions of students in learning mathematics at primary level. *Bulletin of Education and Research 38*, 1 (2016).

[191] Mondragon, A. L., Nkambou, R., and Poirier, P. Evaluating the effectiveness of an affective tutoring agent in specialized education. In *European conference on technology enhanced learning* (2016), Springer, pp. 446–452.

[192] Moreno, R., Mayer, R. E., Spires, H. A., and Lester, J. C. The case for social agency in computer-based teaching: Do students learn more deeply when they interact with animated pedagogical agents? *Cognition and instruction 19*, 2 (2001), 177–213.

[193] Moyer, P. S., Bolyard, J. J., and Spikell, M. A. What are virtual manipulatives? *Teaching Children Mathematics 8*, 6 (2002), 372–378.

[194] Μπερδούσης, Ι. Αξιοποίηση των κινητών υπολογιστικών συσκευών (tablets) στο δημοτικό σχολείο: Μια μελέτη περίπτωσης. *PhD thesis,* Πανεπιστήμιο Πάτρας, Σχολή Κοινωνικών και Ανθρωπιστικών Επιστημών, 2014.

[195] Naiser, E. A., Wright, W. E., and Capraro, R. M. *Teaching fractions: Strategies used for teaching fractions to middle grades students. Journal of research in childhood education 18, 3 (2003), 193–198.*

[196] National Council of Teachers of Mathematics. *Principles and standards for school mathematics, executive summary.* https://www.nctm.org/uploadedFiles/Standards_and_Positions/PSSM_ExecutiveSummary.pdf, 2016. Accessed: 15/12/2016.

[197] National Mathematics Advisory Panel. *Foundations for success: The final report of the National Mathematics Advisory Panel.* US Department of Education, 2008.

[198] Newton, K. J. An extensive analysis of preservice elementary teachers' knowledge of fractions. *American educational research journal 45,* 4 (2008), 1080–1110.

[199] Nezami, O. M., Hamey, L., Richards, D., and Dras, M. *Deep learning for domain adaption: Engagement recognition*. CoRR abs/1808.02324 (2018).

[200] Ng, H.-W., Nguyen, V. D., Vonikakis, V., and Winkler, S. Deep learning for emotion recognition on small datasets using transfer learning. In *Proceedings of the 2015 ACM on international conference on multimodal interaction* (2015), ACM, pp. 443–449.

[201] Ni, Y. Semantic domains of rational numbers and the acquisition of fraction equivalence. *Contemporary educational psychology 26*, 3 (2001), 400–417.

[202] Ni, Y., and Zhou, Y.-D. Teaching and learning fraction and rational numbers: The origins and implications of whole number bias. *Educational Psychologist 40,* 1 (2005), 27–52.

[203] Nkambou, R., Mizoguchi, R., and Bourdeau, J. *Advances in intelligent tutoring systems*, vol. 308. Springer Science & Business Media, 2010.

[204] Nunes, T. M., Bittencourt, I. I., Isotani, S., and Jaques, P. A. Discouraging gaming the system through interventions of an animated pedagogical agent. In *European Conference on Technology Enhanced Learning* (2016), Springer, pp. 139–151.

[205] OECD. *Are boys and girls equally prepared for life?*OECD, 2014.

[206] OECD. *The abc of gender equality in education: Aptitude, behaviour, confidence.* OECD, 2015.

[207] Oldewurtel, F., Parisio, A., Jones, C. N., Morari, M., Gyalistras, D., Gwerder, M., Stauch, V., Lehmann, B., and Wirth, K. *Energy efficient building climate control using stochastic model predictive control and weather predictions. In Proceedings of the 2010 American control conference* (2010), IEEE, pp. 5100–5105.

[208] Olney, A. M., D'Mello, S., Person, N., Cade, W., Hays, P., Williams, C., Lehman, B., and Graesser, A. Guru: A computer tutor that models expert human tutors. In *International Conference on Intelligent Tutoring Systems* (2012), Springer, pp. 256–261.

[209] Olson, A. M., and Stoehr, K. J. From numbers to narratives: Preservice teachers experiences' with mathematics anxiety and mathematics teaching anxiety. *School Science and Mathematics 119*, 2 (2019), 72–82.

[210] Oosterhof, A. *Developing and using classroom assessments* (3rd ed.). Pearson Education Ltd, 2003.

[211] Ortony, A., Clore, G. L., and Collins, A. *The cognitive structure of emotions*. Cambridge University Press, 1988.

[212] Paleari, M., Lisetti, C., and Lethonen, M. VALERIE: A virtual agent for learning environment reacting and interacting emotionally. In *AIED 2005, 12th International Conference on Artificial Intelligence in Education, July 18-22, 2005, Amsterdam, The Netherlands* (2005), IOS Press, pp. 3 – 5.

[213] Pantziara, M., and Philippou, G. Levels of students' "conception" of fractions. *Educational Studies in mathematics 79*, 1 (2012), 61–83.

[214] Pantziara, M., and Philippou, G. N. Students' motivation in the mathematics classroom. Revealing causes and consequences. *International Journal of Science and Mathematics Education 13*, 2 (2015), 385–411.

[215] Pareto, L. *Mathematical literacy* for everyone using arithmetic games. In *Proceedings of the 9th International Conference on Disability, Virtual Reality and Associated Technologies* (2012), vol. 9, University of Readings Reading, UK, pp. 87–96.

[216] Pashler, H., McDaniel, M., Rohrer, D., and Bjork, R. Learning styles: Concepts and evidence. *Psychological science in the public interest 9*, 3 (2008), 105–119.

[217] Patterson, J., and Gibson, A. *Deep learning: A practitioner's approach.* O'Reilly Media, Inc., 2017.

[218] Payne, J. N. Curricular issues: Teaching rational numbers. *Arithmetic Teacher 31*, 6 (1984), 14–17.

[219] Peker, M. Pre-service teachers' teaching anxiety about mathematics and their learning styles. *Eurasia Journal of Mathematics, Science and Technology Education 5,* 4 (2009), 335–345.

[220] Perkins, K., Adams, W., Dubson, M., Finkelstein, N., Reid, S., Wieman, C., and LeMaster, R. Phet: Interactive simulations for teaching and learning physics. *The Physics Teacher 44*, 1 (2006), 18–23.

[221] Perkins, K., and Wieman, C. Free on-line resource connects real-life phenomena to science. *Physics Education* (2005), 93–95.

[222] Petrovica, S., Anohina-Naumeca, A., and Ekenel, H. K. Emotion recognition in affective tutoring systems: Collection of ground-truth data. *Procedia Computer Science 104* (2017), 437–444.

[223] Petrovica, S., and Ekenel, H. K. Emotion recognition for intelligent tutoring. In *BIR Workshops* (2016), vol. 1.

[224] Petscharnig, S., Lux, M., and Chatzichristofis, S. A. Dimensionality reduction for image features using deep learning and autoencoders. In *Proceedings of the 15th International Workshop on Content-Based Multimedia Indexing, CBMI 2017, Florence, Italy, June 19-21, 2017* (2017), pp. 1–6.

[225] Φιλίππου, Γ., and Χρίστου, Κ. Συναισθηματικός τομέας, αριθμοφοβία και διδασκαλία των μαθηματικών. In *Καΐλα, Μ. (επ.), Η σχολική αποτυχία. Από την «Οικογένεια» του Σχολείου στο «Σχολείο» της οικογένειας*, Εκδόσεις Ελληνικά Γράμματα, Αθήνα, 1998.

[226] Φιλίππου, Γ., and Χρίστου, Κ. *Διδακτική των μαθηματικών.* Εκδόσεις Γιώργος Δαρδανός, 2002.

[227] Pilli, O., and Aksu, M. The effects of computer-assisted instruction on the achievement, attitudes and retention of fourth grade mathematics students in North Cyprus. *Computers & Education 62* (2013), 62–71.

[228] Plant, E. A., Baylor, A. L., Doerr, C. E., and Rosenberg-Kima, R. B. Changing middle-school students' attitudes and performance regarding engineering with computer-based social models. *Computers & Education 53*, 2 (2009), 209–215.

[229] Porayska-Pomsta, K., Mavrikis, M., and Pain, H. Diagnosing and acting on student affect: the tutor's perspective. *User Modeling and User Adapted Interaction 18,* 1-2 (2008), 125–173.

[230] Post, T., Behr, M., Harel, G., and Lesh, R. Rational numbers: Towards a semantic analysis–emphasis on the operator construct. In *Rational numbers: An integration of research* T.P. Carpenter, E. Fennema, and T.A. Romberg, Eds. Lawrence Erlbaum Associates, 1993, pp. 13–47.

[231] Post, T. R. Fractions: Results and implications from national assessment. *The Arithmetic Teacher 28*, 9 (1981), 26–31.

[232] Ρουμελιώτης, Μ. *Μοντελοποίηση και Προσομοίωση.* Ελληνικό Ανοιχτό Πανεπιστήμιο, 2001.

[233] Rahayu, C., Putri, R. I. I., and Zulkardi, M. Multiplication of fraction with natural number by using hurdles. In *5th SEA-DR (South East Asia Development Research) International Conference 2017 (SEADRIC 2017)* (2017), Atlantis Press.

[234] Ratliff, M. S., and Patterson, E. Emotion recognition using facial expressions with active appearance models. In *Proceeding of HRI* (2008), Citeseer.

[235] Rau, M., Rummel, N., Aleven, V., Pacilio, L., and Tunc-Pekkan, Z. How to schedule multiple graphical representations? A classroom experiment with an intelligent tutoring system for fractions. In *The future of learning: Proceedings of the 10th international*

conference of the learning sciences (2012), International Society of the Learning Sciences (ISLS), pp. 64–71.

[236] Rau, M. A., Aleven, V., and Rummel, N. Intelligent tutoring systems with multiple representations and self-explanation prompts support learning of fractions. In *AIED* (2009), pp. 441–448.

[237] Rau, M. A., Aleven, V., and Rummel, N. Blocked versus interleaved practice with multiple representations in an intelligent tutoring system for fractions. In *International conference on intelligent tutoring systems* (2010), Springer, pp. 413–422.

[238] Rau, M. A., Aleven, V., and Rummel, N. Interleaved practice in multi-dimensional learning tasks: Which dimension should we interleave? *Learning and Instruction 23* (2013), 98–114.

[239] Rau, M. A., Aleven, V., and Rummel, N. Successful learning with multiple graphical representations and self-explanation prompts. *Journal of Educational Psychology 107*, 1 (2015), 30.

[240] Razak, F. A., Noordin, N., Alias, R., and Dollah, R. How do 13 year olds in malaysia compare fractions? *Procedia-Social and Behavioral Sciences 42* (2012), 100–105.

[241] Reimer, K., and Moyer, P. S. Third-graders learn about fractions using virtual manipulatives: A classroom study. *Journal of Computers in Mathematics and Science Teaching 24*, 1 (2005), 5–25.

[242] Research Advisory Committee of the National Council of Teachers of Mathematics. Curriculum and evaluation standards for school mathematics. *Journal for Research in Mathematics Education 19,* 4 (1988), 338–344.

[243] Resnick, L. Do the math: Cognitive demand makes a difference. *Research Points: Essential Information for Educational Policy 4*, 2 (2006), 1 – 4.

[244] Reys, R., Lindquist, M., Lambdin, D. V., and Smith, N. L. *Helping children learn mathematics.* John Wiley & Sons, 2014.

[245] Rinne, L. F., Ye, A., and Jordan, N. C. Development of fraction comparison strategies: A latent transition analysis. *Developmental Psychology 53*, 4 (2017), 713.

[246] Roussou, M. Learning by doing and learning through play: an exploration of interactivity in virtual environments for children. *Computers in Entertainment (CIE) 2*, 1 (2004), 10–10.

[247] Sarrafzadeh, A., Alexander, S., Dadgostar, F., Fan, C., and Bigdeli, A. See me, teach me: Facial expression and gesture recognition for intelligent tutoring systems. In *Innovations in Information Technology, 2006* (2006), IEEE, pp. 1 – 5.

[248] Sarrafzadeh, A., Alexander, S., Dadgostar, F., Fan, C., and Bigdeli, A. "how do you know that i don't understand?" a look at the future of intelligent tutoring systems. *Computers in Human Behavior 24,* 4 (2008), 1342–1363.

[249] Sarrafzadeh, A., Fan, C., Dadgostar, F., Alexander, S., and Messom, C. Frown gives game away: affect sensitive systems for elementary mathematics. In *Systems, Man and Cybernetics, 2004 IEEE International Conference on* (2004), vol. 1, IEEE, pp. 13–18.

[250] Sarrafzadeh, A., Fourie, L., Alexander, S. T., Overmyer, S., and Shanbehzadeh, J. Eve: An affect-sensitive pedagogical agent. In *Managing in an Interconnected World: Pioneering Bussiness and Technology Excellence* (2014), Global Business and Technology Association, pp. 573–579.

[251] Schmidhuber, J. Deep learning in neural networks: An overview. *Neural networks 61* (2015), 85–117.

[252] Schroeder, N. L., Adesope, O. O., and Gilbert, R. B. How effective are pedagogical agents for learning? A meta-analytic review. *Journal of Educational Computing Research 49*, 1 (2013), 1–39.

[253] Schultz, J. E. Area models–spanning the mathematics of grades 3-9. *The Arithmetic Teacher 39*, 2 (1991), 42.

[254] Seffrin, H. M., Rubi, G., Carlotto, T., Mello, G., and Jaques, P. A. Um resolvedor de equações algébricas como ferramenta de apoio à sala de aula no ensino de equações algébricas. In *Anais do Workshop de Informática na Escola* (2009), vol. 1, pp. 1791–1800.

[255] Selden, A., and Selden, J. Reflections on foundations for success: The final report of the national mathematics advisory panel. *The College Mathematics Journal 40,* 4 (2009), 318.

[256] Self, J. The role of student models in learning environments. *IEICE TRANSACTIONS on Information and Systems 77*, 1 (1994), 3–8.

[257] Sformo, T. *Practical problems in mathematics: For automotive technicians*. Cengage Learning, 2008.

[258] Shanty, N. O., Hartono, Y., Puri, R. I. I., and de Haan, D. Design research on mathematics education: Investigating the progress of indonesian fifth grade students? Learning on multiplication of fractions with natural numbers. *IndoMS. JME 2,* 2 (2011), 147–162.

[259] Shirvani, H. Pre-service elementary teachers' mathematics content knowledge: A predictor of sixth graders' mathematics performance. *International Journal of Instruction 8*, 1 (2015), 133–142.

[260] Shirvani, H. Pre-service teachers' procedural and conceptual knowledge of fractions. *International Journal of Scientific and Research Publications 6,* 5 (2016), 324–328.

[261] Siegler, R. S., Duncan, G. J., Davis-Kean, P. E., Duckworth, K., Claessens, A., Engel, M., Susperreguy, M. I., and Chen, M. Early predictors of high school mathematics achievement. *Psychological science 23*, 7 (2012), 691–697.

[262] Siegler, R. S., Fazio, L. K., Bailey, D. H., and Zhou, X. Fractions: The new frontier for theories of numerical development. *Trends in cognitive sciences 17*, 1 (2013), 13–19.

[263] Siegler, R. S., and Lortie-Forgues, H. Conceptual knowledge of fraction arithmetic. *Journal of Educational Psychology 107,* 3 (2015), 909.

[264] Silver, E. Probing young adults' thinking about rational numbers. *Focus on learning problems in mathematics 5* (1983), 105–117.

[265] Simpson, E. *The classification of educational objectives in the psychomotor domain: The psychomotor domain,* Vol. 3. Gryphon house, 1972.

[266] Simpson, E. J. The classification of educational objectives, psychomotor domain. *Illinois Teacher of Home Economics 10*, 4 (1966), 110–144.

[267] Soylu, Y., and Soylu, C. Learning difficulties of 5fh class in primary education at fraction: Ordering, adding, subtraction, multiplication in fraction and problems related to fraction. *Journal of Erzincan Education Faculty 7*, 2 (2005), 101–117.

[268] Stafylidou, S., and Vosniadou, S. The development of students' understanding of the numerical value of fractions. *Learning and instruction 14*, 5 (2004), 503–518.

[269] Stols, G., Ono, Y., and Rogan, J. What constitutes effective mathematics teaching? Perceptions of teachers. *African Journal of Research in Mathematics, Science and Technology Education 19*, 3 (2015), 225–236.

[270] Streefland, L. *How to teach fractions so as to be successful.* Utrecht, The Netherlands: Vakgroep Onderzoek Wiskundeonder-wijs en Onderwijscomptercentrum *(1984).*

[271] Streefland, L. Fractions in realistic mathematics education: A paradigm of developmental research*, vol. 8. Springer Science & Business Media, 1991.*

[272] Suh, J., Moyer, P. S., and Heo, H.-J. Examining technology uses in the classroom: Developing fraction sense using virtual manipulative concept tutorials. *Journal of Interactive Online Learning 3*, 4 (2005), 1–21.

[273] Τύπας, Γ., and Ντάφου, Ε. *Μεθοδολογικές προσεγγίσεις και αξιολόγηση στα Μαθηματικά*. In Τα Μαθηματικά του Δημοτικού μέσα από τα νέα διδακτικά εγχειρίδια. Παιδαγωγικό Ινστιτούτο, 2006, pp. 1–7.

[274] Thompson, N., and McGill, T. J. Genetics with jean: the design, development and evaluation of an affective tutoring system. *Educational Technology Research and Development 65*, 2 (2017), 279–299.

[275] Tichá, M., and Hošpesová, A. Problem posing and development of pedagogical content knowledge in pre-service teacher training. In *Meeting of CERME* (2009), vol. 6.

[276] Tunç-Pekkan, Z. An analysis of elementary school children's fractional knowledge depicted with circle, rectangle, and number line representations. *Educational Studies in Mathematics 89,* 3 (2015), 419–441.

[277] Τζελέπη, Σ., and Κοτίνη, Ι. Δραστηριότητες ενίσχυσης της υπολογιστικής σκέψης. In *4ο Πανελλήνιο Εκπαιδευτικό Συνέδριο Κεντρικής Μακεδονίας «Αξιοποίηση των Τ.Π.Ε. στη Διδακτική Πράξη»* (2016), pp. 100–108.

[278] Vamvakoussi, X., Christou, K. P., Mertens, L., and Van Dooren, W. What fills the gap between discrete and dense? Greek and flemish students' understanding of density. *Learning and Instruction 21*, 5 (2011), 676–685.

[279] Vamvakoussi, X., Van Dooren, W., and Verschaffel, L. Naturally biased? In search for reaction time evidence for a natural number bias in adults. *The Journal of Mathematical Behavior 31,* 3 (2012), 344–355.

[280] Vamvakoussi, X., and Vosniadou, S. How many decimals are there between two fractions? Aspects of secondary school students' understanding of rational numbers and their notation. *Cognition and instruction 28*, 2 (2010), 181–209.

[281] Van de Walle, J. A., Karp, K. S., and Bay-Williams, J. M. *Elementary and Middle School Mathematics: Teaching developmentally*, 8th ed. New Jersey: Pearson, 2012.

[282] Van der Meij, H., Van der Meij, J., and Harmsen, R. Animated pedagogical agents effects on enhancing student motivation and learning in a science inquiry learning environment. *Educational technology research and development 63*, 3 (2015), 381–403.

[283] Van Dooren, W., Lehtinen, E., and Verschaffel, L. Unraveling the gap between natural and rational numbers. *Learning and Instruction 37* (2015), 1–4.

[284] Vanlehn, K. The behavior of tutoring systems. *International journal of artificial intelligence in education 16*, 3 (2006), 227–265.

[285] Viola, P. A., and Jones, M. J. Rapid object detection using a boosted cascade of simple features. In *2001 IEEE Computer Society Conference on Computer Vision and Pattern Recognition (CVPR 2001), with CD-ROM*, 8-14 December 2001, Kauai, HI, USA (2001), IEEE Computer Society, pp. 511–518.

[286] Vrachnos, E., and Jimoyiannis, A. Design and evaluation of a webbased dynamic algorithm visualization environment for novices. *Procedia Computer Science 27* (2014), 229–239.

[287] Wagner, G. Simurena-a web portal for open educational simulation. In *Proceedings of the 2012 Winter Simulation Conference (WSC)* (2012), IEEE, pp. 1–12.

[288] Wang, C.-H., and Lin, H.-C. K. Constructing an affective tutoring system for designing course learning and evaluation. *Journal of Educational Computing Research 55*, 8 (2018), 1111–1128.

[289] Wiggins, J. B., Boyer, K. E., Baikadi, A., Ezen-Can, A., Grafsgaard, J. F., Ha, E. Y., Lester, J. C., Mitchell, C. M., and Wiebe, E. N. Javatutor: an intelligent tutoring system that adapts to cognitive and affective states during computer programming. In *Proceedings of the 46th acm technical symposium on computer science education* (2015), pp. 599–599.

[290] Wijaya, A. The relationships between indonesian fourth graders' difficulties in fractions and the opportunity to learn fractions: A snapshot of TIMSS results. International Journal of Instruction 10, 4 (2017), 221–236.

[291] Wilkerson-Jerde, M. H., and Wilensky, U. J. How do mathematicians learn math?: Resources and acts for constructing and understanding mathematics. *Educational Studies in Mathematics 78,* 1 (2011), 21.

[292] Wilson, A. J., Dehaene, S., Pinel, P., Revkin, S. K., Cohen, L., and Cohen, D. Principles underlying the design of" the number race", an adaptive computer game for remediation of dyscalculia. *Behavioral and brain functions 2*, 1 (2006), 19.

[293] Wilson, V. Focus groups: a useful qualitative method for educational research? *British Educational Research Journal 23,* 2 (1997), 209–224.

[294] Woodard, T. The effects of math anxiety on post-secondary developmental students as related to achievement, gender, and age. *Inquiry 9*, 1 (2004), 1–16.

[295] Wu, F. S., Cheng, H. N., and Reynolds, B. L. Effects of virtual opponents in video game on students with different fluency. In *Proceedings of the 16th International Conference on Computers in Education* (2008), Asia-Pacific Society for Computers in Education, pp. 761–766.

[296] Wu, W. C., Yang, Y. C., and Chuang, C. P. The analysis of simpson's psychomotor domain educational objectives and its application on the skill evaluation for the department of computer engineering at vocational school. In *Seventh IEEE International Conference on Advanced Learning Technologies (ICALT 2007)* (2007), IEEE, pp. 723–725.

[297] Wyberg, T., Whitney, S. R., Cramer, K. A., Monson, D. S., and Leavitt, S. Unfolding fraction multiplication. *Mathematics teaching in the Middle School 17*, 5 (2012), 288–294.

[298] Xu, L., Fei, M., Zhou, W., and Yang, A. Face expression recognition based on convolutional neural network. In *2018 Australian & New Zealand Control Conference (ANZCC)* (2018), IEEE, pp. 115–118.

[299] Yang, D.-C., and Sianturi, I. A. J. Sixth grade students' performance, misconception, and confidence on a three-tier number sense test. *International Journal of Science and Mathematics Education 19* (2021), 355–375.

[300] Yang, D. C., and Tsai, Y. F. Promoting sixth graders' number sense and learning attitudes via technology-based environment. *Journal of Educational Technology & Society 13*, 4 (2010), 112–125.

[301] Yang, E. F., Chang, B., Cheng, H. N., and Chan, T.-W. Improving pupils' mathematical communication abilities through computer-supported reciprocal peer tutoring. *Journal of Educational Technology & Society 19*, 3 (2016), 157.

[302] Yang, Z., Zhu, M., Qu, Z., and Zhang, Y. Research on organization of mathematics knowledge in good mathematical cognitive structure. *EURASIA Journal of Mathematics, Science and Technology Education 14*, 1 (2017), 291–302.

[303] Yeh, A., and Nason, R. A. Mixing colours: An ict tool based on a semiotic framework for mathematical meaning-making about ratio and fractions. In *Navigating Currents and Charting Directions: Proceedings of the 31st Annual Conference of the Mathematics Education Research Group of Australasia* (2008), vol. 2, MERGA Inc, pp. 605–611.

[304] Yeh, C. Y., Cheng, H. N., Zhi-Hong, C., Calvin, C., and Tak-Wai, C. A tetris game to support students' mental computation: Design and evaluation. In *18th International Conference on Computers in Education* (2010), Asia-Pacific Society for Computers in Education, pp. 432–436.

[305] Yenilmez, K., and Yasa, E. Primary school students' misconceptions about geometry. *Journal of Uludag University Education Faculty 21*, 2 (2008), 461–483.

[306] Zakaria, E., and Nordin, N. M. The effects of mathematics anxiety on matriculation students as related to motivation and achievement. *Eurasia Journal of Mathematics, Science & Technology Education 4*, 1 (2008).

[307] Zatarain-Cabada, R., Barrón-Estrada, M. L., AlorHernández, G., and Reyes-García, C. A. Emotion recognition in intelligent tutoring systems for android-based mobile devices. In *Mexican International Conference on Artificial Intelligence* (2014), Springer, pp. 494–504.

[308] Zatarain-Cabada, R., Barrón-Estrada, M. L., Camacho, J. L. O., and Reyes-García, C. A. Affective tutoring system for android mobiles. In *International Conference on Intelligent Computing* (2014), Springer, pp. 1–10.

[309] Zatarain-Cabada, R., Barrón-Estrada, M. L., Camacho, J. O., and Reyes-García, C. A. Integrating learning styles and affect with an intelligent tutoring system. In *12th Mexican International Conference on Artificial Intelligence (MICAI)* (2013), IEEE, pp. 247–253.

[310] Zatarain-Cabada, R., Barrón-Estrada, M. L., and Zatarain-Cabada, R. Learning styles and emotion recognition in a fuzzy expert system. *Research in Computing Science 65* (2013), 87–94.

[311] Zhou, B., Lapedriza, A., Khosla, A., Oliva, A., and Torralba,A. Places: A 10 million image database for scene recognition. *IEEE transactions on pattern analysis and machine intelligence 40*, 6 (2018), 1452–1464.

[312] Zhou, X., and Conati, C. Inferring user goals from personality and behavior in a causal model of user affect. In *Proceedings of the 8th international conference on intelligent user interfaces* (2003), ACM, pp. 211–218.

[313] Zucker, A., Kay, R., and Staudt, C. Helping students make sense of graphs: an experimental trial of smartgraphs software. *Journal of Science Education and Technology 23*, 3 (2014), 441–457.

[314] Zylka, J., and Nutzinger, H. P. Educational games in formal education - results of an explorative study using an educational game in school. In *International Conference on Computer-aided Learning (ICL 2010)* (2010).