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Model of Pareto financial investments optimal portfolio

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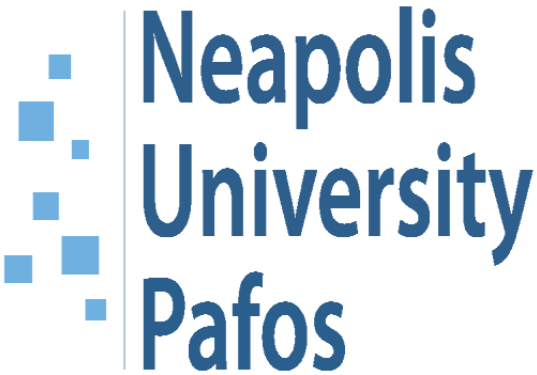

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Abstract

Investment portfolio issues take a leading position in modern economic science due to their relevance in a developed market. However, the conditions of the Russian economy does not allow fully apply the general principles of the portfolio investment theory and formed by foreign researchers arsenal of investment strategies.

In the conditions of increasing complexity of investment mechanisms, development of the securities market, improvement of financial technologies, as well as increased uncertainty inherent in the domestic stock market is especially important the development of modern methods of analysis and the formation of Pareto-optimal financial investments portfolio as a factor in reducing the overall risk. In this case, the classical theory of the investment portfolio formation is not always adequately reflects the full range of factors: the instruments liquidity, the value of the transaction costs, the analytical component of operations, etc.

In modern conditions, a serious problem in the decision-making process on the financial investments portfolio formation is the search for effective options portfolio, which consists in determining the composition of the financial assets in the portfolio. Theoretically, all portfolios resulting in is an increase of its economic value to be taken for implementation. In practice, portfolio investment is carried out in a strictly limited ability to attract funds. Based on the above, we have developed an algorithm for deciding to build a financial investments portfolio, which is based on the mechanism of directed search Pareto optimal variants of a financial investments portfolio. The proposed algorithm is used to determine the effective range of financial investment ratio "return / risk" and the volume of investments (for well correlated portfolios).

The developed algorithm allows finding all the Pareto optimal variants of the portfolio on the basis of available observations on the dynamics of profitability and risk of the financial assets included in portfolio. The main advantage of the algorithm is incorporated in its base directed search mechanism, which allows omitting the bulk of the known variants of inefficient portfolios.

Acknowledgment

I am using this opportunity to express my gratitude to everyone who supported me throughout the process of dissertation writing. I am thankful for their aspiring guidance, invaluably constructive criticism and friendy advice during the project work. I am sincerely grateful to them for sharing their truthful and illuminating views on a number of issues related to the project.

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INTRODUCTION

The relevance of the topic. Investment portfolio issues take a leading position in modern economic science due to their relevance in a developed market. However, the conditions of the Russian economy does not allow fully apply the general principles of the portfolio investment theory and formed by foreign researchers arsenal of investment strategies. Therefore, determining the basis for the formation of the fund portfolio it is inevitably have to be limited the using only those aspects of portfolio theory, which may be to some extent adapted to Russian conditions and address the specific manifestations of the various factors affecting the choice of securities for portfolio investment in the Russian economy. Conditions of the domestic stock market characterized by unstable market conditions, an abrupt change in share price, the high level of risk, as well as the lack of quality securities, require different approaches to the formation of investment portfolios compared with developed market economies countries.

In the conditions of increasing complexity of investment mechanisms, development of the securities market, improvement of financial technologies, as well as increased uncertainty inherent in the domestic stock market is especially important the development of modern methods of analysis and the formation of Pareto-optimal financial investments portfolio as a factor in reducing the overall risk. In this case, the classical theory of the investment portfolio formation is not always adequately reflects the full range of factors: the instruments liquidity, the value of the transaction costs, the analytical component of operations, etc.

Thus, the relevance of dissertation research topic determined by the lack of development of Pareto-optimal portfolio formation issue of financial investments under conditions of high uncertainty, incompleteness and inaccuracy of statistical information specific to the Russian stock market.

State of knowledge of the problem. To issues of financial investments portfolio formation and management have dedicated their work many domestic and foreign authors.

In the field of investment communication and the growth in economic theory was the work by R.Harrod[1], E.Domar[2], E.Hansen[3], J.Hiks[4], R.Solou[5], V.Leontief[6], Ya.Tinbergen[7], J.Robinson[8], N.Kaldor[9], N. Kondratev[10], Dzh.Tobin[11], I.Shumpeter[12] et al.

In the works of foreign authors such as V.Berens, G.Birman, R.Breyli, M.Bromvich, Yu.Brighem, J. Van Horn, L.Gitman, Dzh.Rozenberg, R.Holt, A. Shapiro have been summarized the experience of the investment activity in the developed financial markets countries.

The most outstanding foreign scientists who addressed many theoretical and practical aspects of the external investment flows management have been: G. Alexander, M. Albert, J. Bailey, T.R. DeMark, R.V. Colby, T.A. Meyers, G. Markowitz, J.J. Murphy, M.H. Meskon, F. Hedouri, W. Sharpe, A. Elder and others.

Among the studies on the role of financial markets investment in the Russian economy, should be highlight the work by M. Alekseyev, B.Alehina, N.Berzon, V.Galanova, A. Zakharov, A.Kilyachkova, I.Kon`kov, V.Kolesnikov, A. Kraev, B.Koltynyuk, Ya. Mirkin, B.Rubtsov, E.Rudneva, Yu.Sizov, Razumov I.V. et al.

At the same, many theoretical and practical issues related to the problem of modifying the basic provisions of the classical financial investments theories and their adaptation to the Russian terms of portfolio investment as a way of investing for

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1 Harrod R. (1938). Meeting a Trade Recession. Case for Monetary Reflation. Cooperation by the Banks. ASIN: B002LFYXX2. Publisher: London: The Times.

2 Domar E., Richard Musgrave.(1944) Proportional Income Taxation and Risk-Taking. Quarterly Journal of Economics.

3 Craig, J., C. Dibrell, and E. Hansen. (2011) Natural environment, market orientation and firm innovativeness: the life cycle of Perspective Magazine small business management. ASIN: 467-489.

4 Hiks Dzh.(1969) Theory of Economic History. London. Oxford University Press.

5 Solow, Robert M (1956). "A Contribution to the Theory of Economic Growth". Quarterly Journal of Economics.

6 Leontief W. (1966). Input-Output Economics. New York: Oxford University Press.

7 Tinbergen J. (1980). Revision of the international order / J. Tinbergen; under the total. Ed. A.A. Ryvkina; lane. I.A. Bonk; foreword. DM Gvishiani. - Moscow: Progress.

8 Robinson J (1948). The Economics Of Imperfect Competition. London: Macmillan and Co.

9 Kaldor N. (1941). The white paper of national income and expenditure. Economic journal, 51:181-91.// Kaldor N. (1940) A model of the trade cycle. Economic journal, 51:458-73.

10 Kondratiev N.D. (2002) Large cycles of conditions and the theory of foresight - selected works. – Moscow: "Economics".

11 Tobin James, Hester Donald D. (1967) Risk Aversion and Portfolio Choice. New York. John Wiley and Sons.

12 I.Shumpeter. (1918). The Crisis of the Tax State.Leipzig. // (1934) The Nature and Necessity of a Price System, Economic Reconstruction.// (1935) The Analysis of Economic Change. REStat.

long-term technical and innovative development of the industrial enterprises are still poorly developed and determined the purposes and objectives of scientific research.

The purpose of this thesis is the improvement of the portfolio investment theoretical model and the development of formating methods the financial investments portfolio in the Russian financial market conditions.

The stated aim of the study determined the need to address the following **objectives**:

• to conduct a comparative analysis of the basic provisions of the classical financial investments optimal portfolio theory;

• to provide a theoretical substantiation the model of financial investment ratio "return / risk";

• to verify the classical portfolio theory with the Russian practice of portfolio investment;

• to analyse the behavior of financial investments portfolio by the ratio of "return / risk";

• to conduct an empirical test of the Markowitz classical theory on the data of the Russian financial market and offer the basic elements of a proactive strategy of the formation a portfolio financial investments;

• to propose a decision-making algorithm for building a financial investments portfolio on the principle of Pareto optimality;

• to justify the introduction of a VaR technique mechanism in the investment portfolio optimization.

The **subject of research** is the basic theoretical principles of the classical financial investments theories and methodological bases of portfolio investment optimization.

The object of study are interconnection and interdependence arising in the process of portfolio investment in the Russian financial market.

Theoretical and methodological basis of research.

The theoretical basis of the study were research and development the theory of the market portfolio formation of G.Markowitz, J. Tobin, W. Sharpe as well as the theory of optimal portfolio of F. Black, Modigliani F., M. Miller, M. Scholes.

To solve the problems in the study we used the formal-logical methods which allow through formal logical ordering of material to reduce the real factors to abstract theoretical schemes and to establish specific quantitative proportion between the phenomena and processes taking place in the developing financial market.

In the study also widely used statistical techniques and methods of economic-mathematical modeling allowing for systematization and processing the huge reservoir of statistical dynamic situational materials of the Russian stock market and give a formalized description of economic phenomena and their properties, as well as to assess the cause-and-effect relationships and aside from minor details, concentrating attention to the main linkages.

Reliability of the results and conclusions of inputs based on statistical data on the dynamics of the Russian financial markets presented in the bulletins of the Federal State Statistics Service of the Russian Federation, the Bank of Russia, the Ministry of Finance of the Russian Federation, the Federal Service for Financial Markets, reports and analysis of research institutes, specialized scientific publications as well as original material collected by the author during the research. This has ensured a representativeness of the underlying data, the scientific validity and reliability of the main findings and conceptual positions of dissertation research.

Scientific novelty of the dissertation research is the modification of the theoretical positions and methodological foundations of classical portfolio theory, based on new identified relationships and interdependencies arising in the process of financial investment in developing stock market.

The most significant results obtained by the author possessing the features of scientific novelty and for the defense:

• Have been conducted the verification of the Markowitz model for the Russian portfolio investment conditions through introducing a model of external constraints that characterize the high volatility of the domestic stock market in terms of "return / risk"; that allowed to come to the conclusion that the partial applicability of the model for the Russian stock market. The audit showed that the Markowitz model gives the correct distribution of financial assets in the portfolio only if the amount of assets is less than eight and a level of return given by the investment is low.

• the author supplemented the portfolio theory instrument by the inclusion of technical analysis methods in the mechanism of formation a financial investments portfolio that will improve the diversification of investments taking into account the ratio of "return / risk."

• have been supplemented the basic provisions of portfolio theory by introducing new conditions characterizing heterogeneity, information asymmetry and high volatility in developing the Russian financial market, which enabled us to formulate theoretical propositions to minimize market risk, based on the principles of Pareto optimality.

• the author proposes an algorithm for building an effective decision-making portfolio of financial investments, based on the theory of boundary production functions. The main advantage of the proposed algorithm is incorporated in a directed search mechanism Pareto-optimal choices of investments, which allows to omit the bulk of the known inefficient sets of financial investment;

• have been proposed the mechanism of Pareto optimization financial investments portfolio based on the inclusion of VaR – is a technology assessment of ratio "return / risk", which increases the efficiency of portfolio investment comparative with investment portfolio optimization criterion of W. Sharpe.

The practical significance of the research. Practical importance has the scientific developments containing Pareto-optimal investment portfolio formation algorithm, that allows to omit the deliberately inefficient investment portfolios. Application of this algorithm can significantly improve the efficiency of investment activity in the Russian financial market. The practical significance also have research and development containing the algorithm of constructing the investment portfolio on the basis of fundamental analysis techniques complement by the tools of technical analysis. Application of these methods can significantly increase the effectiveness of the portfolio policy in the securities market of the Russian Federation.

Testing results of the study. Developed by the author recommendations have been discussed at the All-Russian scientific-practical conference dedicated to the 90th anniversary of Doctor of Economics Professor A. Kashchenko “Eeconomic theory, applied economics and economic practice: problems of interaction" **(Yaroslavl, Yaroslavl State University. Demidova P.G., 2006)**

Publication. In general, the topic of the PhD thesis of the author published 1 monograph (16 pp); 6 articles and 1 abstract (totaling 2.5 pp). Among them, three publications in journals recommended by Higher Attestation Commission of the Russian Federation.

The structure and scope of the thesis. The dissertation includes an introduction, three chapters, including 7 paragraphs, conclusion, bibliography of references that determined by the logic of research and reflects tasks. The main content of the work is set out on 115pages of computer text Times New Roman, font 14, 1.5 spacing, illustrated with 7 graphs, 18 tables. The list of references includes 110 sources.

1. THEORETICAL APPROACHES OF FINANCIAL INVESTMENT PORTFOLIO FORMATION

1.1.Formation theory of the optimal financial investments portfolio

There are some of the most attractive areas of capital investment. Attractiveness criteria for each investor may be different, but there are two main parameters on which the final decision is making. They are risk and investment returns. The potential investor make conclusions about the attractiveness of a particular sector of the economy, the scope of business, financial and credit institutions that offer their services in this area, by using the comparing and evaluating methology of these figures. In case with banking system the situacion is quate simple, the figures include: credit rating, the amount of the authorized capital, the list of major corporate customers, deposit interest rates and other indicators characterizing the stability and the credibility of the present financial and credit institution. Quite another matter for the Russian investor is investment portfolio, a new type of capital investment, which is associated with many questions arise during the evaluation of these two fundamentally important indicators.

Speaking about the relative novelty of portfolio investment concept, we mean its Russian interpretation. Despite more than a decade-long history, we could talk about the real development only with the end of 1998. System of short-term liabilities and federal loans that brought enormous profits with minimum risk in the market helped to mobilize nearly all financial resources. The unstable political situation in the country in the 90 years of the twentieth century, high inflation, imperfect, and sometimes a lack of certain provisions of the law regulating investment activity had have most negative impact on the Russian investment market.

The broad development of scientific and quasi-scientific research in the field of portfolio investment has been developed by more than a century of Western investment market history, especially American. A huge number of studies conducted by scientists and financial companies have identified certain patterns in the dynamics of the investment market and helped to create many theories that describe and explain

the mechanism of mutual relations, as well as the philosophy of investor behavior on the market. All these theories and models are designed to serve as a support for decision-making. The aim of every investor is the maximum profit with minimum risk, so it could be assumed that empirically assess the applicability (or inapplicability) of Western models in the domestic market investments, we can identify features and possibly the "nature" of the Russian investment market, and thereby reducing the risk of capital loss.

The interest in formulating and solving problems in the theory of investment has significantly increased in recent years in our country. Among these tasks a significant place occupy the optimization problems of portfolio investments. Indeed, selecting different options for capital allocation between financial instruments in which capital is invested, we will have different results. It will be so if as a result we consider the magnitude of income, received within a predetermined period. Obviously, in some sense, the optimal allocation of investment capital should ensure the best possible result[13].

At the same time, the decision on capital allocation is often taken in conditions of uncertainty, when the return on capital invested in financial instruments is random. Thereby emerge a capital investments risk, so the investment portfolio optimization problem should be solved and put in the presence of risk.

It should be emphasized that the uncertainty of situational dynamics of investment markets and, particularly, the domestic financial market, generated, above all, by more important than in Europe the influence of random factors, wearing a political and economic nature, and, consequently, a large level of capital investing risk in various segments of the investment market. The latter circumstance necessitates to consider the peculiarities of Russian investment markets, including the level of optimization performances investment portfolios and developing stable numerical methods for solving them.

Classical portfolio theory has three stages of development. The first stage - the original - was the development of a mathematical foundations of the portfolio theory.

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[13] Kurelenkova Y. *Comparison of optimal investment portfolios compiled using different measures of risk.* Available from: http://www.sbras.ru/ws/YM2005/9378/index.html

The two subsequent are a modern theory of portfolio investments: the second is a theory of the market portfolio described in the works of Markowitz[14], J. Tobin[15] and W. Sharpe[16]; the third is the formation of the theory of optimal portfolio created on the basis of the theory of the portfolio investments in the works of F. Modigliani[17], M. Miller[18], F. Black[19], M. Scholes[20] and Robert Merton[21].

The traditional approach to investing prevailed before the advent of the modern theory of portfolio investments had two major drawbacks. First, it focused on the analysis of the behavior of individual assets (stocks and bonds). Second, the main characteristic of the assets in it was exceptionally yield, whereas another factor - the risk - did not get a clear assessment in investment decisions. The modern level of the theory of portfolio investments development overcomes these disadvantages. The formation of this new approach actually ended a long period (from the end of 20-ies of XX century.) known in financial theory as "initial stage of the theory of portfolio investments development."

The modern theory of portfolio investment (portfolio investment theory) were laid in the fifties in the work of the American mathematician and economist G.Markowitz. The main merit of the Markowitz is the bringing of the stochastic approach into this theory, according to which the yield of the invested capital is treated as a random variable characteristics determine the expected value of risk and return. Consequently, for the first time in the theory of investment, starting with the H. Markowitz work, the risk of investing got a precise mathematical numeric

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[14] Harry M. Markowitz, (1959) *Portfolio Selection*: *Efficient Diversification of Investments,* New York, *John Wiley and Sons*, Available from: - http://cowles.econ.yele.edu

[15] Donald D. Hester, James Tobin. (1967). *Risk Aversion and Portfolio Choice*. New York, *John Wiley and Sons*, Available from: http://cowles.econ.yele.edu.

[16] William F. Sharpe. (1970) *Portfolio Theory and Capital Markets*. New York, *McGraw-Hill Book Company.* Available from: http:// www.wsarpe.com

[17] F. Modigliani, Merton H. Miller (June 1958). *The Cost of Capital, Corporation Finance and the Theory of Investment*" ,American Economic Review. Available from: http:// www.nodelprize.org

[18] F. Modigliani, Merton H. Miller (June 1958). *The Cost of Capital, Corporation Finance and the Theory of Investment*" American Economic Review. Available from: http:// www.nodelprize.org

[19] Black F., Scholes M. (1973) *The pricing of options and corporate liabilities*.// Journal of Political Economy № 81. Available from: http://www.gobi.stanford.edu

[20] Black F., Scholes M. (1973) *The pricing of options and corporate liabilities*.// Journal of Political Economy № 81. Available from: http://www.gobi.stanford.edu

[21] Zvi Bodie, Robert C Merton, William F. Samuelson(1992). *Labor supply flexibility and portfolio choice in a life cycle*
Model. North-Holland *Journal of Economic and Control* p.p. 427-449. Available from: http:// www.people.hbs.edu

expression, which allowed to construct mathematical models of optimization problems investment portfolios[22].

According to G. Markowitz, profitability of investing in financial assets is a random variable which expectation is taken as the expected value of return from investing in financial instruments. As an indicator of risk, Markowitz offered to take the measure of deviation from the expected value that is the standard deviation of return as a random variable. Such a choice of a mathematical risk expression allowed to implement in portfolio investment scheme the known economy principle, which reads as diversification among several investment targets reduces the risk compared to the risk of investing into individual objects.[23]

Mathematical model of Markowitz portfolio optimization issue of financial investments belongs to a class of quadratic programming problems. The theory of the numerical solution was developed in 50-60 years in wellknown Rand Corporation, where Markowitz had been made his research, along with one of the creators of linear and nonlinear programming Dantsing.

However, his work did not attract much attention from theoreticians and practitioners economists. For the 50-ies of 20th century the application of probability theory in financial theory and practice was itself a very unusual case. Besides, the undeveloped computing and the complexity of proposed by Markowitz algorithms, procedures and formulas are not allowed to carry out the actual implementation of his ideas.

The impact of G.Markowitz portfolio theory significantly increased after introduction the works on similar issues by J. Tobin[24] in the early 60-ies of XX century. It should be noted some differences between G.Markovic and J. Tobin approaches. The first of these approaches lies in the area of microeconomic analysis and focuses on the behavior of the individual investor, who generates optimal portfolio, from his point of view, based on his own evaluation of risk and return of

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[22] Harry M. Markowitz (1959*). Portfolio Selection: Efficient Diversification of Investments*. New York *John Wiley and Sons*. Available from: http://cowles.econ.yele.edu

[23] Harry M. Markowitz (1959) *Portfolio Selection: Efficient Diversification of Investments*. New York *John Wiley and Sons* Available from: http://cowles.econ.yele.edu

[24] Donald D. Hester, James Tobin. (1967) *Risk Aversion and Portfolio Choice*. New York *John Wiley and Sons.* Available from: http://cowles.econ.yele.edu

selected assets. In addition, initially this model concerned mainly equity portfolio, ie risky assets. James Tobin is also proposed to include the risk-free assets in the analysis (e.g. government bonds). In fact, his approach is a macroeconomic, cos` in this case the main object of study is the distribution of the total capital in the economy in two forms: cash (money) and non-cash (in the form of securities).

Markowitz in his works emphasized the mathematical analysis of effects and the development of algorithms for solving optimization problems, but on the economic analysis of initial postulates of the theory. J. Tobin approach mainly accentuate on the analysis of factors that have forced investors to create a portfolio of assets, but keep capital in any certain one (eg, cash) form. In addition, James Tobin analyzed the adequacy of the quantitative characteristics of the portfolio, which are the source of data in the G.Markovic theory.

Nowadays G.Markowitz model is mainly used on the first stage of the assets portfolio formation during the distribution of capital invested in their different types (stocks, bonds, real estate, etc.). Univariate model of W. Sharpe[25] is used in the second stage, when the capital is invested in a particular market segment and distributed among the individual specific assets which constitute the selected segment (ie on specific stocks, bonds, etc.).

The basic idea of the Markowitz model is statistically consider the future income generated by financial instrument as a random variable, ie income from financial instruments vary somehow randomly[26]. Then, if somehow determine the definite probability of occurrence for each financial instrument, we can get the probability of income distribution for each alternative investment. To simplify the Markowitz model assumes that income from investment alternatives are normally distributed.

Using Markowitz model should be determine the parameters, which describe the amount of investment and risk; so it allows to compare the different alternatives of capital investment from the standpoint of goals and thus create a scale for evaluation various combinations. As the scale of the expected income amount a

[25] William F. Sharpe. (1970) *Portfolio Theory and Capital Markets.* New York *McGraw-Hill Book Company.* Available from: http:// www.wsarpe.com

[26] Markowitz H.M. (1987) *Mean-Variance Analysis in Portfolio Choice and Capital Markets*. Blackwcll Publishers. Available from: http://cowles.econ.yele.edu

number of potential revenue actually use the most probable value, which in the case of a normal distribution the same as expectation[27].

Let formed a portfolio of *n* financial instruments. The expected value of income on the E-th financial asset (E_i) is calculated as the arithmetic mean of the individual potential revenue R_i with weights P_{ij}, the probability of occurrence assigned to them:

$$E = \sum_{i=1}^{n} R1 \cdot Py \qquad (1.1)$$

where $P_{ij} = 1$;

n - specifies the number of earnings estimates for each financial asset.

Dispersion indicators used to measure the risk, so the more scatter of possible income, the greater probibility of risk that the expected income will not be received. Thus, the risk is expressed by values rejection (lower!) of income from the most probable value. Scattering measure is the standard deviation (σ_i), and the greater the number the greater the risk:

$$\sigma_i = \sqrt{\sum Py \left(Ry - Ey\right)^2} \qquad (1.2)$$

In Markowitz model for measuring risk standard deviation is used instead of the dispersion D_i, which is equal to the σ_i in the square, since this indicator is advantageous over the art calculations[28].

Investors wishing to invest optimally, interested not in comparison of certain types of financial instruments, as in comparison of all possible portfolios, cos` it allows to use the effect of risk dispersion, ie determined by the expected value and variance of portfolio income. The expected income value E of financial assets portfolio is determined as the sum of the most probable income E_i different financial assets n. The income weighted by the relative shares x_i (i = 1, ..., n) corresponding to capital investments in each financial instrument, such as a bond or share:

$$E = \sum_{i=1}^{n} X_1 \cdot E_1 \qquad (1.3.)$$

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[27] Markowitz H.M. (1987) *Mean-Variance Analysis in Portfolio Choice and Capital Markets*. Blackwcll Publishers. Available from: http://cowles.econ.yele.edu

[28] Markowitz H.M. (1987) *Mean-Variance Analysis in Portfolio Choice and Capital Markets*. Blackwcll Publishers. Available from: http://cowles.econ.yele.edu

The variance (dispercion) of this amount is applicable with certain limitations, cos` the change in the share price in the market does not take place in isolation from each other and covers the entire market as a whole. Therefore, the variance depends on the degree of scattering of individual financial assets, as well as all the financial assets in the aggregate at the same time to increase or decrease the rate, ie depends on the correlation between changes in rates of certain financial assets. With a strong correlation between the individual courses (ie, if all assets simultaneously raise or lower) risk due to deposits in various financial assets can not fail to reduce or enlarge. If rates of financial assets is not correlated, in the extreme case (portfolio contains an infinite number of assets), the risk could be eliminated completely, as the fluctuations in the average would be zero. In practice, the number of financial instruments in the portfolio is always finite, so the distribution of investment in various assets can only reduce risk, but it can not be excluded completely.

In determining the risk of an identified portfolio of financial investments is necessary to consider the correlation rate assets. As an indicator of correlation Markowitz uses the covariance C_{ik} between courses changes of individual financial assets.

Thus, the total variance of the portfolio can be calculated by the following formula:

$$\upsilon = \sum_{i-1}^{n} \sum_{k=1}^{n} x_1 \cdot x_1 \cdot C_{ik} \qquad (1.4.)$$

By definition, for i = k • C_{ik} is equal dispersion of the financial asset.

This means that the dispersion and the risk of the portfolio depends on the risk of the asset or correlation between individual assets (ie, the systematic risk of the market) and the proportion of x_i individual financial assets in the portfolio as a whole.

Considering theoretically limiting case in which the portfolio can include an infinite amount of financial assets the dispersion asymptotically will be approaches the average value of the covariance C.

Markowitz has developed a very important provision for modern portfolio theory, which states: the total risk of the portfolio can be decomposed into two

components. On the one hand, its so-called systematic risk; it can not be excluded and affects all financial assets almost equally. On the other hadn, specific risk for each financial asset could be avoided by managing a portfolio of financial investments. Thus the sum of money stacked on all objects must be equal to the total volume of investments (eg, part of the funds in the bank account entered into the model as an investment with zero risk), ie sum of the relative shares of x_i in total must be equal to unity:

$$\sum_{i-1}^{n} x_1 = 1 \qquad (1.5.)$$

The problem lies in the numerical determination of the relative proportion of assets in the portfolio (values Xi), which are the most profitable for the owner. Markowitz restricts model decision by the necessity of distinguishing from the entire set of "admissible" portfolio only few which riskier than others. It refers the portfolio containing at the same income level the greater risk (dispersion) compared with others, or portfolios, less profitable at the same level of risk.

Using the method of critical lines developed by Markowitz could be distinguished unpromising portfolios that do not satisfy constraints. Thus remain only efficient portfolios, ie, portfolios that contain minimal risk for a given income or bring the highest returns for a given maximum level of risk to which the investor can go.

This fact is very important in the modern theory of portfolio investments. Selected in this way portfolios are combined in a list containing information about the percentage of the portfolio of certain financial instruments, as well as the income and risk portfolios. The selection of a particular portfolio depends on the maximum risk to which the investor is ready to go.

From a methodological point of view Markowitz model can be defined as the practical-normative, that certainly does not mean the imposition of a certain style of investor behavior in the investment market. The main task of the model is to show how the objectives are achievable in practice.

As follows from the Markowitz model it is not necessary to set the income distribution of certain financial instruments. It suffices to define only the values that

characterize this distribution: the expectation E_i, dispersion D_i and covariance C_{ik} between income of certain financial instruments. It should be analyzed before drawing up the portfolio. In practice, a relatively small number of financial instruments to make such calculations to determine the expected return and variance are possible. The determining a correlation coefficient is very high complexity. For example, analyzing the 100 shares required the rate of more then 500 covariances.

Avoiding such a high complexity W. Sharpe[29] proposed **index model (Capital Asset Pricing Model - CAPM).** He has not developed a new method of compiling the portfolio, he just simplified the problem so that an approximate solution can be found with much less effort. Sharp introduced the so-called ß-factor, which plays an important role in modern portfolio theory.

This simplification made portfolio optimization techniques applicable in practice. To the 70-th years of programming development and improvement of statistical estimation techniques indicators "alpha" and "beta" of certain financial instruments and yield index of the overall market, led to the emergence of the first software packages to meet the challenges of portfolio management, securities. This allowed to pay mathematicians attention to the seriousness of these problems, which eventually led to the formation of modern portfolio theory.

In Sharpe index model is used close (and itself undesirable dispersion due to reduction of risk) correlation between changes in foreign individual assets. It is assumed that the necessary input data can be approximated using only one basic factor and the relationship linking it to changes in foreign individual stocks. Assuming the existence of a linear relationship between the rate of a financial asset and a certain index could be forecasts evaluation using the index value to determine the expected rate of the asset. Besides it is possible to calculate the cumulative risk of each asset in the form of total dispersion.

The model is based on the following assumptions:

- all investors maximize their expected utility what`s why should investe in assets;

[29] William F. Sharpe. (1970). *Portfolio Theory and Capital Markets.* New York *McGraw-Hill Book Company.* Available from: http:// www.wsarpe.com

- does not consider the dynamics of decision-making, the process cover only one period is the same for all investors;
- selection of alternative investment options provided by the ratio of expected return and risk (measured by standard deviation);
- investors are opponents of risk;
- all investors have the same information about the probable distribution of the expected return on assets, and the probable distribution is normal;
- in the market are unlimited opportunities for investment and borrowing money at a single risk-free interest rate;
- there are no transaction costs for circulation of assets;
- dividends and capital gains are taxes equally;
- in the market is not possible to influence the market price of the asset by individual investors through a large volume of transactions;
- all assets are highly liquid and infinitely divisible, ie the investor can buy part of the asset.

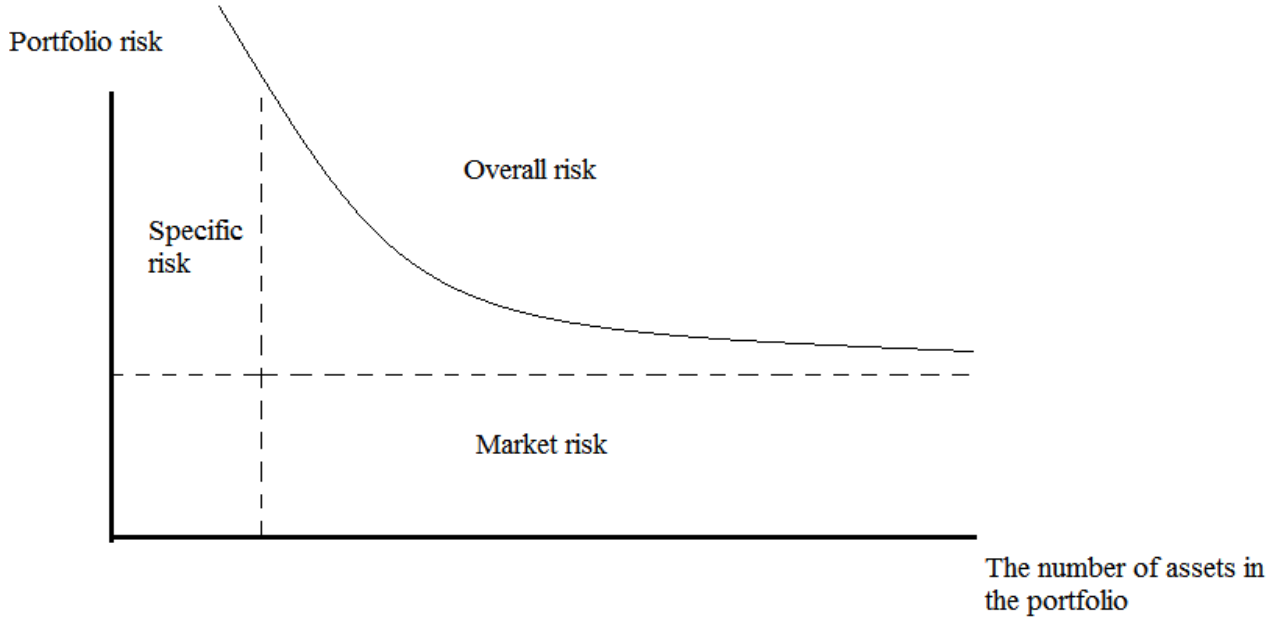
However, the CAPM model is useful in practice, since for large financial institutions, considering their scope of activities, operating costs compared with the volume of transactions is not significant, the possibility of borrowing and short selling are sufficiently large, access to market information is almost unlimited. Some people compare the CAPM with mechanics models without friction, which, nevertheless, determine the main features of the phenomena being studied. Thus, the model base is the assumption of rational behavior of market participants, with equal financial and information opportunities and identical targets with homogenous expectations, leads to the formation of a new asset prices mechanism in the market, at which the equilibrium state. In a perfect market, all investors acting rationally and using the same prediction coeficients of yield values K and the risk V, regardless of their wealth (capital) and preference relations, constitute one and the same portfolio of risky assets, which corresponds to point M (the market portfolio) with a yield to K_m and risk of σ_m.

W. Sharpe identified two components of the overall risk of any asset (this risk can be quantitatively measured be dispercia from the expected value):

1. Specific corporation risk (issuer risk), ie diversifiable risk, (total risk), which eliminates the combination of financial instruments in the portfolio;
2. Non-diversifiable (systematic or market) risk (market risk).

Diversifiable risk associated with the financial situation of the issuer or a financial instrument with its inherent commercial and financial risk. Market risk arises from circumstances beyond the issuer's reasons, ie is not peculiar to the corporation as the issuer of financial instruments. The risk of purchasing power and interest rate risk are components of market risk. Since forming a portfolio the investor can eliminate the specific risk (picking assets, the correlation coefficient which is not equal to +1), the risk of a well-diversified portfolio will depend on market risk and included in this portfolio financial assets. Combining the assets in the portfolio, investor reducing risk, ie reduces the dispersion of the portfolio. With increasing number of assets in the portfolio, the risk of the portfolio decreases very quickly. Decreasing the number of asset reducing the risk slows down as more assets are positively correlated with each other. This effect is shown in the graph.

Graph 1.1.



Source: Author, 2014

A significant reduction in specific risk can be achieved even with forming a small portfolio (about 30 different investment instruments). Absolute leveling of specific

risk requires the inclusion in the portfolio of all assets traded in the market, ie the formation of the market portfolio.

W. Sharpe introduced the concept of beta-coefficient to measure the market risk of the asset. To evaluation the inclusion of new assets in a diversified portfolio will affect his risk is not so important to know the overall risk of the asset $\sigma^2_{gen.}$ Enough to know the market risk σ_m^2 and to determine the sensitivness of the asset to the market movement ($\sigma_p^2 = \sigma_m^2$). This sensitivity is measured by ß-factor. Let β_i indicates the level of instrument deviation *j* in relation to the market portfolio. Coefficients β_i can be defined as $\beta_i = \frac{V_{jm}}{\sigma_m^2}$,

where V_{jm} – is covariance between the return on assets and market yield, determined by the dynamics of the market index (stock market index, which is quoted on the asset);

σ_m^2 - is the dispercion of the market return.

Meaning of the coefficient β_i is simple, it shows the effect of market conditions on the behavior of asset j.

Based on the concept of beta-factor model was developed the valuation model of financial assets (CAPM). This model suggests that in competitive capital markets in equilibrium (when all assets are equally estamates on the market) premium risk for investing in asset *j* is directly dependent on the sensitivity of the asset to the movement of the market (ie, from β_i):

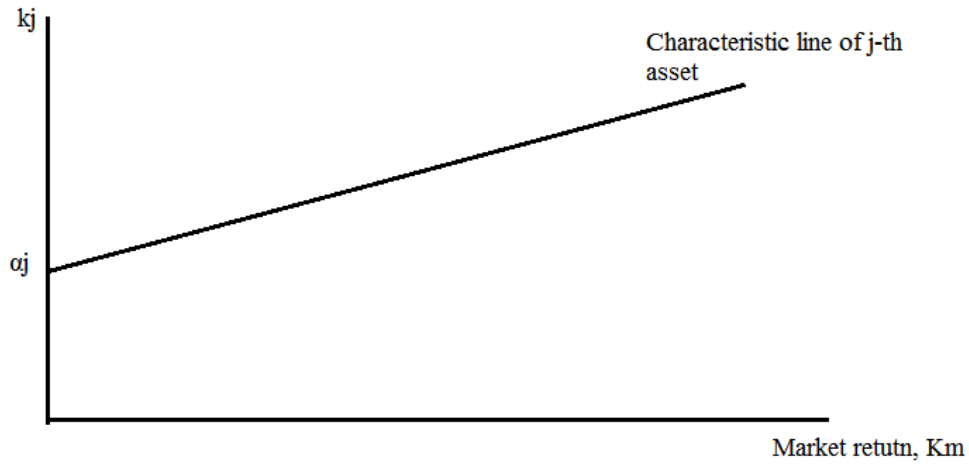
$k_j - k_a = \beta_i (k_m - k_0)$, (1.6.)

$k_j = k_0 + \beta (k_m - k_0)$,

where $(k_m - k_0)$ – is market premium for the risk.

This relationship of risk and return for a particular asset j represented graphically, is called the characteristic line of a financial asset (*Security Market Line - SML*).

Graph 1.2.



Along the abscissa axis is postponed the risk assessment (ß- coefficient), the vertical axis is the expected return on risk-based tool. For the risk-free asset ß-coefficient is zero.

Thus, ß- coefficient is the slope line reflecting the dependence of the asset yield on the market yield. Equation (1.6.) leads to an important conclusion: the premium risk for any financial asset included into the optimal portfolio is proportional to the premium risk associated with the portfolio as a whole. Assets with ß less than one and greater than zero are moving in the same direction as the market, but more slowly. The market can be considered as the portfolio of all assets, and consequently, the average activity in the market (medium risk) has a coefficient of ß = 1.

$ß_j$ coefficient allows predicting the future price of an asset j (increase or decrease) with the knowledge of the market forecast behavior. The behavior forecast of price asset (through ß) allows evaluating investment risk and expected return. The more $ß_j$ coefficient the greater expected return than the market return.

ß portfolio coefficient is a weighted average coefficient ß of financial instruments included in the portfolio. The greater the portfolio risk, the greater must be the payment in the form of higher yields. If diversifiable portfolio risk of assets included in it effectively compiled, it could be omit. This risk does not provide for compensation in the form of higher yields. Only market risk (ß-risk or systematic risk) must be compensated.

So, the price movement of a financial asset is determined by systematic and unsystematic risk. Systematic risk due to macro factors inherent in all market assets.

Characteristic line reflects the relationship of the expected (equilibrium) yields k_j of shares j and market expected return k_m. The degree of dependence graphically represented slope characteristic line (Graph. 1.2.), the numerical value equal to the tangent of the angle formed by a straight line with the horizontal axis.

However, the state of equilibrium, even if we assume that all the assumptions of CAPM perform, its just the yield in the market to which seeks asset j. Actual yield average may deviate.

In contrast to the expected values lying on the characteristic line on the graph (1.2.), most of the values somehow deviate due to the influence of random, non-systematic factors. Each point represents the excess over the yield (or decrease) associated with the movement of the market, the amount of non-market component of profitability.

Actual values of yield explained as follows. K_j changes over time in accordance with the characteristic line reflect the influence of market factors. These factors have a systematic impact on the financial asset j. Deviation from the characteristic line reflects the impact of specific for corporations j factors.

The difference between the actual yield of the asset j and the equilibrium yield rate prescribed by coefficient $ß_j$ is called an alpha coefficient (a_j). Bandwidth a_j could be both positive and negative characterizes undervalued or overvalued of asset j.

It should be noted, that the notion of return change for an asset reflects the change of the market value and dividends. Considering the market portfolio the dividend yield component can be measured as the average dividend yield of the market index.

The straight *SML* reflects the choice of average investor in equilibrium (the investor does not need to buy or sell assets), where the required return on an asset (or risky assets) equal to the expected return. If the expected return on an asset j exceeds the yield required by the investor he prefers to purchase larger amounts of asset j.

If the huge number of investors find that assets j is undervalued (low price and high expected return kj), the increase of demand for assets j will increase prices, which leads to a decrease of expected profitability. Purchase of assets j will stop when the expected return be equal to the required and equilibrium is established.

Thus, the equilibrium state of the market develops as a result of adjusting the structure of the individual investor portfolios and pressure through the supply and demand for courses on financial assets. Having the information about the courses of financial instruments, investors are able to calculate expected returns and correlation coefficients. CAPM approach allows to calculate the required return. As long as the expected return is equal to the desired overall demand for a given financial instrument does not coincide with a proposal, it will affect the price movement.

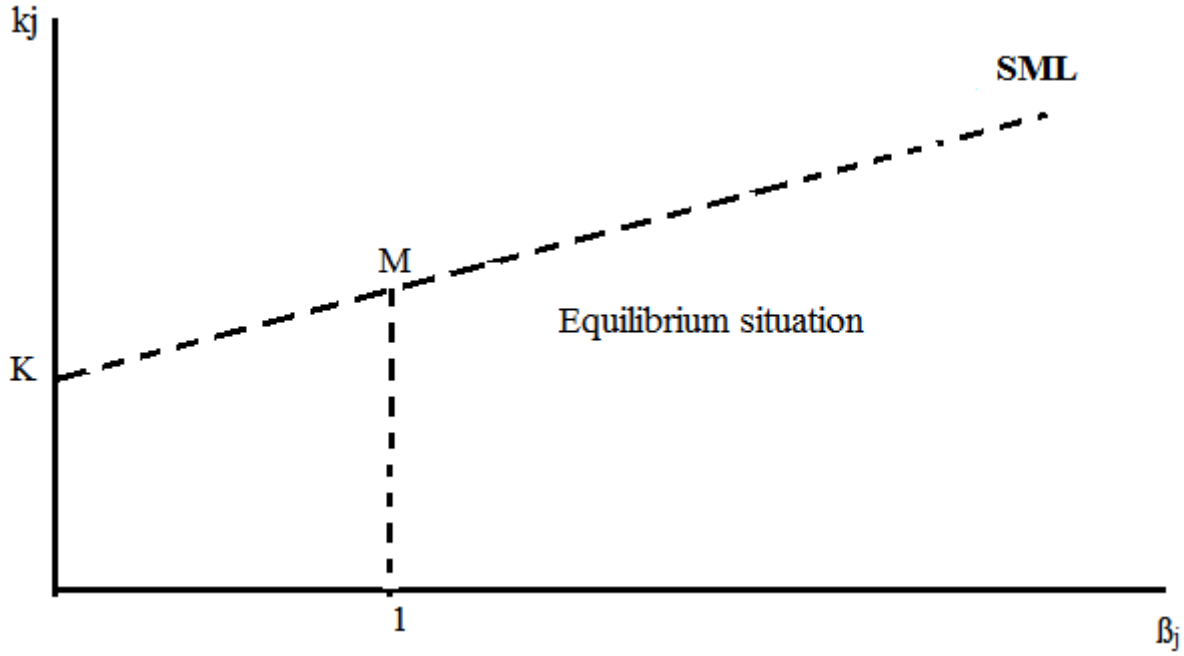
The slope of the SML reflects the attitude to risk in the market (in this economy), because it shows the range of the average investor. The less than the average market risk-averse investor, the greater the angle of SML straight, in order:

1. the premium for risk on any risky assets is bigger (including all assets);

2. the required return on all risky assets is higher.

The greater the risk, the more premium for risk in general case. At a lower risk appetite investor requires a large premium for a fixed level of risk ß ($k_2 > k_1$).

Graph 1.3.



Factors that influence the position of the SML line.

1. *Influence of inflation*. Risk-free rate of CAPM return is the nominal, ie includes two components: the real risk-free rate of return (interest rate) and expected inflation. With the growth of expected inflation increases the

nominal risk-free rate of return, which leads to a change in the position of direct SML, as shown in Graph 1.3. (direct shift SML ordinate).

2. *Changing risk appetite.* Factor of changing attitudes to the risk changes the slope of the SML. The graph shows the change in the position SML with increasing of risk aversion. Increases market premium risk (km-k0), which leads to an increase in the required market yield with k_{m1} to K_{m2}. Required yield on other risky financial instruments and portfolios are also growing, but considering the degree of influence of systematic risk (less than systematic risk assessment is the value of ß-coefficient, the smaller the increase in yield).

3. *Changing the ß- coefficient*. Investor can change the measure of systematic risk (ß-coefficient) through:

• a combination of physical assets;

• changes in the proportion of debt capital in the total capital.

In addition, the ß-coefficient could be affected by external factors such as changes in the competitive state of the industry or state restrictions on production. All of these changes are reflected in the yield required changes.

Equation SML of the asset $j \perp k_j = k_0 + ß\ (k_m - k_{0)}$ claims, that the required (and therefore in equilibrium expected) return of an asset j includes two components: the risk-free asset yields and premium for risk. Premium for risk investing in the assets Y depends on:

1) premium for risk of the market portfolio (on this portfolio, consisting of all the assets market, $ß = 1$ and the premium is equal to $k_m - k_0$);
2) the values of ß-coefficient for the assets j. If $ß_j = l$, then the required rate of return on asset j coincides with the average yield on all assets, ie equal to the yield of the market portfolio. If $ß\ j>1$, then the risk premium on asset j higher then market premium risk by a factor $ß$ j, respectively above the overall required rate of return.

In practice, there is no possibility of estimating the expected values of yields both a specific financial instrument and on the market portfolio. Theoretical premise of estimation of coefficient $ß$ on expectation of future changes is replaced by the

estimate on past observations of the behavior of the yield k_j and k_m. How past changes could be the guarantee of future development (often investors expectations are based on the probable distribution of past results), so β j can be an indicator of k_j changes depending on k_m changes.

The model considers the dependence of the premium risk on an asset j from the premium risk on the market index: α_1 is return of an asset j at zero profitability of the market, ie the impact of market risk is absent. Accordingly, α_j shows which yield a financial instrument will provide to the owner for diversifiable (specific) risk. In some sense, this is an additional premium to the risk-free asset in case of zero market premium risk. At zero market premium risk in each asset market equilibrium condition will be zero α-coefficients. And average coefficients of all α-assets is equal to zero, ie at the same time some assets value can be positive, and in some cases be negative. Graphically, α-factor is equal to the segment intercepts the ordinate axis.

The line reflecting the dependence of yield on the asset j from the stock index is based on regression analysis (minimizing the sum of squared deviations of the observed values of the data points and the corresponding points on the line). Summary measure of the degree of coupling of return on assets and the index is the coefficient of determination or the correlation coefficient R^2. For example, the numerical value of $R^2 = 0,8$ it shows that 80% of the return variation of the asset can be explained by changes in yield index. With a large number of observations and closeness of points to the proximity of characteristic line $R^2 \longrightarrow 1$;.

Natural to wonder about the relationship of the linear market model and the CAPM. Market model is single-factor model in which risk is a function of the coefficient, ie the unifactor model is based on past data:

$$k_j = \alpha_j + \beta_j k_m + u \qquad (1.7.)$$

where C is income from asset j for a certain period;

k_m is income at market index,

α_j is ordinate of intersection with the vertical axis;

Pj is slope value of the line; and U is the value of the random error.

First of all it should be noted that in both models the amount of inclination is called "beta" and both in some way are linked to the market. However, they have significant differences.

Market model described by equation (1.7).) and CAPM described by equation (1.6.) are unifactor models of estimating the required return on the financial asset. The differences between these models are as follows:

1. In the market model is the market factor (stock) index, and in the CAPM is market portfolio, which covers more risky financial assets than those included in the stock index;
2. Market model, in contrast to the CAPM is not in equilibrium;
3. Theoretically ß-coefficient in market model does not coincide with ß-coefficient in CAPM mode (in market model it reflects sensitivity to the market index, and in the CAPM is change in the market portfolio). However, due to the inability to estimate the coefficient with respect to the market portfolio in the CAPM model used in practice of the ß- coefficient market model.

Another model which takes into account not only one but many factors is the arbitration pricing model. (Arbitrage Prising Teory - Model APT), proposed by Ross.[30]

Arbitration theory asserts that earnings per share depend on many factors: partly on macroeconomic factors and partly on factors affecting the specific (diversifiable) risk. The return to the market portfolio (as CAMP) can be only one factor.

The purpose of arbitrage strategies is to use the differences in the price of a financial asset or a related type in different markets or segments of markets for profit (usually without risk). Thereby using arbitration aloows to avoid disequilibrium between cash markets and in relations on cash markets. Thus, arbitration is a leveling element to form the most efficient capital markets.

Arbitration model - an alternative to the CAPM, it does not define a specific number of factors and their significance for the asset as well as for each asset will be its significant factors. As a factor may be the stock index, the gross national product,

[30] Stephen A. Ross(December 1976). *The Arbitrage Theory of Capital Asset Pricing*. Journal of Economic Theory # 3. Available from: http:// www.sul.stenford.edu

energy prices, interest rates, etc. For example, studies on the U.S. market showed a significant number of macroeconomic factors such as changes in industry production, inflation, personal consumption, money supply and interest rate. Salomon Brother Agency estimating multifactor model includes consideration of five factors: inflation, the growth rate of gross domestic product, interest rate, index of changes in oil prices, the growth in defense spending.

Collectively we can distinguish three groups of factors necessarily involves into arbitration model:

1. Indicators of overall economic activity (it may be the growth rate of industrial production growth rate srednennyh sales growth rate of GNP);

2. Indicators reflecting inflation;

3. Indicators of interest rate (the difference between the long and short term interest rates, the rate of return of the stock (market) index).

There are a lot of special studies on the particular financial instrument rate reacted to the change of similar risk factors in the past. Using these proportions is assumed that it is possible to calculate the behavior of financial instruments in the future. Naturally this is done using forecasts of risk factors. If the calculated rate of financial asset is higher than the current rate, it indicates the profitability of asset purchases.

The idea of compensation for greater risk compared to the risk-free asset in the APT model remains unchanged. If there is a risk-free investment and loan option (this option provides a yield or cost of capital when borrowing money at a rate k_0), then:

• greater risk for investors require higher returns;

• obtaining increased yield means the presence of risk factors.

Investors tend to increase yield without increasing the risk portfolio in the market. This possibility can be realized through the arbitration portfolio, ie a portfolio formation by simultaneously selling shares at a relatively high price and the purchase of the shares in another place at relatively low cost. This operation will allow the investor without investing get risk-free income.

Arbitrage opportunities appear if on assets or portfolios with equal sensitivity to various factors expected profitability. Investors going to obtain riskless income so the

arbitrage opportunity is exhausted. Thus, in equilibrium, assets and portfolios with the same sensitivity to the factors have the same value of expected return (adjusted for specific risk).

ARM advantage is fewer assumptions about the behavior of investors in the market compared to the CAPM. It is assumed that the actual return of any share is a linear function of τ factors:

$$k_1 = k_1 + b_{j1}F_1 + b_{j2}F_2 + ... + b_{j\tau}F_\tau + u_j \qquad (1.8.)$$

where F is actual return on assets j;

k_j is expected return of an asset j;

b_{j1} is yield on the asset sensitivity to factors j and i (sometimes used the term "factor loadings"); Fi is the value of factor i; u j - a random variable (with zero mean value) as part of a specific risk of the asset j.

The model considers portfolios available on the market of financial instruments. It is assumed that the number of included assets in the consideration far exceeds the number of factors r. Theoretically is possible to create such a portfolio that it will be risk-free and net investments in it be zero. This portfolio should have a zero expected return, because otherwise there will arbitrage operations, which resulted in asset prices will change as long as the expected return of the portfolio is reduced to zero.

Consider the construction of the arbitration in the absence of additional portfolio investments (money to buy financial instruments generated through the sale of other financial instruments). For example, suppose we have a portfolio of assets and the opportunity to earn arbitrage. The investor does not imply a change to invest in shares in an existing portfolio of assets. Change is achieved by changing the value of the portfolio shares in the portfolio. This change is denoted by wj. The assets wj shows the shares from the portfolio in the arbitration. Zero investment means that the $\sum w_1 = 0$. The riskless of portfolio requires the absence of systematic and unsystematic risk.

Portfolio yield of n shares equal to the weighted sum of return on individual stocks included in the portfolio:

$$k_p = \sum w_j k_j = \sum w_j k_j + \sum w_j b_{jr} F_1 + ... + \sum w_j b_{jr} F_r + \sum w_j u_j \qquad (1.9.)$$

Elimination of systematic risk is achieved through the selection of w_j so that for each factor j of weighted sum of multipliers br was zero (systematic risk multipliers for each factor gives the weighted average value of zero) to each factor from 1 to portfolio r. Sensitivity of factor j equal to the average sensitivity of the assets included in the portfolio. Consideration of a large number of assets in the portfolio eliminates specific risk and for large values of n weighted sum $\sum w_j u_j = 0$

Thus, the diversification of the portfolio allows us to write the expression for the portfolio return last term without specific risk. Final expression of portfolio returns:

$$k_p = \sum w_j k_j = \sum w_j k_j + \sum w_j b_{jr} F_1 + ... + \sum w_j b_{jr} F_r \qquad (10.1)$$

Actually built a portfolio with zero ß index for each factor, it does not require additional investments (some of the values wj are positive, which means that the purchase of assets, any - negative, which means the sale). Systematic risk is eliminated. If the yield cr is positive, then the portfolio is an arbitration, and the investor will seek to build it. Purchase and sale of certain assets on the market a large number of investors will lead to changes in price and affect the expected return.

In a situation of equilibrium yield constructed portfolio (and all other arbitrage portfolios) should be zero $k_p = 0$.

Then from linear algebra that the vector of expected return k_j can be represented as a linear combination of the vector of constant values (λ coefficients) and a vector of multipliers. There should be r+1 the constant coefficients λ 0, λ 1, λ 2, ..., λ r such that will expand the expected earnings per share i.

$$k_j = \lambda_0 + \lambda_1 \beta_{j1} + ... + \lambda_r \beta_{jr} \qquad (1.11.)$$

where bji is the sensitivity of yield stocks i to factor j.

To interpret the coefficients λ look riskless asset j with the yield $k_j$ko which is constant, and sensitivity to the factors in her zero $b_{0i} = 0$ for all i = l, ..., n. Consequently, $k_0 = \lambda_0$. Now, the expression for k_j can be represented as a premium to the risk-free asset:

$$k_j - k_0 = \lambda_1 b_{j1} + .. + \lambda_r b_{jr} \qquad (1.12.)$$

Obtain economic sense for the coefficients λ_j is the premium risk (price risk) in equilibrium for factor i. Let σ_i is expected return on the portfolio with unit sensitivity

to other factors. Such a portfolio is called pure factor portfolio. The expression rates of risk takes the form $\lambda_1 = \sigma_1 - k_0$.

A coefficient λ shows the excess return (compared to without risk yield) in net factor portfolio. This is a premium for the risk factor. As a result, for submission to arbitration model to obtain a version of factorial risk premia:

$$k_j = k_0 + (\sigma_1 - k_0)b_{j1} + (\sigma_2 - k_0)b_{j2} + ... + (\sigma_r - k_0)b_{jr} \qquad (1.13.)$$

Note that the resulting equation similar to the equation and the SML is its multidimensional analogue.

In general, any model of the investment portfolio are open systems and accordingly may be supplemented and corrected with changes in financial market conditions. Investment portfolio model allows to obtain analytical data required for optimal decision making in investing activities.

Getting a mathematical evaluation of the portfolio at different stages of investment, taking into account the influence of various factors makes it possible to continuously control the structure of the portfolio at each decision point, ie in fact manage risk.

Using computer models of implementation significantly increases the efficiency of obtaining analytical data for decision making. Therefore, carry out such basic properties as management: efficiency, continuity and efficiency.

Having considered all the classical portfolio theory, one could argue that the current level of development of portfolio theory takes into account the diversity of financial instruments and risk and return are interrelated investment decisions. The main issue is the choice of the theory of optimal portfolio, ie the definition of a set of assets with the highest rate of return with the least or a given level of investment risk and consideration of mutual correlation between income assets, which allows for effective portfolio diversification, significantly reducing the role of the portfolio relative to the risk assets included therein.

In recent decades, the use of portfolio theory has expanded considerably. Although the process of creating the modern theory of investment is far from over and continue active discussion and debate about its basic principles and results, the impact of this theory in the modern financial world is constantly growing.

We next consider the problem of distribution of investment capital in the presence of uncertainty.

1.2. The optimization ratio "return / risk" of financial investments portfolio in uncertainty.

Investors are often faced with a situation of uncertainty in the stock market. This situation occurs when the direction of future market movements is not obvious and is not understandable further market reaction to these or other events. As a rule, the market reaction is anticipating events in nature, and often draw a particular news about some event ends before its actual occurrence. Uncertainty means that the criterion of efficiency, capital investment in the selected financial instrument has a random character. Particularly important in the face of uncertainty is to find the optimal solution of multi-criteria problems based on the Pareto solution set. If financial instruments are numbered index i = l, ..., n., and is denoted by Ri; the effectiveness of capital investments in the i-th financial assets, as the efficiency depending on the specific problem can be selected different characteristics, such as:

1. $R_1 = S_1 (T) /S_1 (0)$ is the coefficient of capital building for a fixed period T;
2. $R_i = (S_i(T)-S_1(0)/S_i0)$ is rate during the period under review T;
3. R_i = TRR is the i-th investment project if the portfolio is formed (package) of investment projects;
4. $R_i = (S_{inp}(T)-S_{inok}(0)+di(T)/S_{inok}(0)$

S_{inok} (0) the purchase price of the asset i-ro issuer at the initial date t = 0, S_{inok} (T) is selling price of the asset i-ro issuer on the date of the final t=T, $d_r(T)$ dividends i-th of the issuer attributable to the period t=[0, T] if formed portfolio of assets in the investment market.

Mathematically, the assumption of a uncertainties mean that R_1 is random variables.

Full knowledge of the system of random variables $\vec{R} - (R_{1....} R_2)$ (random vectors $\vec{R}$) is determined by its distribution law. However dhsgz: the problem of optimal allocation of capital between these investment targets under the scheme Markowitz

enough to know only two characteristics of the random vector efficiencies is vector expectations $E(\vec{R}) = \vec{m} = (m_1....m_n)^1$

$m_1 = E(R_1)$ - the expected average value of the efficiency i-ro financial asset and the covariance matrix $W_1 W_y = cov(R_p R_j)$, i, j = 1, ..., n.

Ratings of the vector $\vec{m}$ and the matrix W can be obtained depending from the specific content of the economical and financial asset.

Assuming that the efficiency criterion R_i has an internal rate, if flow that investment payments r_{eft} determined by financial instrument, then, knowing the distribution of the random parameter values[31] NPVn determining the internal rate, you can use the Monte Carlo method. Monte Carlo experiment built a sample of random realizations of each efficiency R_i as a variable random

$$R_{rt,....}R_{iN,}i = 1...n \qquad (1.14.)$$

Estimates of Wф and m_1 the average values of the expected efficiencies m_1 and elements W_y of the covariance matrix W may be obtained based on a sample (1.14.) using the equations

$$m = \frac{1}{N} \cdot \sum_{i-1}^{N} R_d, \qquad \overrightarrow{W_n} = \frac{1}{N-1} \cdot \sum_{i=1}^{N} (R_{it} - \vec{m}_i) \cdot (R_{it} - \overrightarrow{m_1}) \qquad (1.15.)$$

If investment is made in risky financial assets such as stocks, the estimates of the elements of the vector m and matrix W can be carried out on the basis of historical data prices realized considered assets.

Received the assessment m_1 and Wij we can formulate and solve the problem of optimizing portfolios of financial investments. Denoted by xi share of capital invested in the asset with the number i. Obviously share Xi satisfy the natural conditions [32]

$$\sum_{i=1}^{n} x_1 \qquad \vec{x} = (x_1....x_n)^T \; 0 \le x_i \le 1, \; \text{i=1....n} \qquad (1.16.)$$

Under a portfolio of financial investments will be the vector of shares of capital invested in assets i = 1, ..., n. The task of choosing a portfolio of financial

[31] Distribution of random values (S, t) can be obtained as a result of preliminary calculations and expert estimates.
[32] We exclude the case of the so-called short sale (short sale), when allowed negative values xi, which means taking the i-ro asset in debt.

investments is equivalent to the choice of the vector $\vec{x}$ shares. For a fixed value of x efficiency portfolio of financial investments R_p given by:

$$R_p = \sum_{i=1}^{n} x_1 \cdot R_1 = (\vec{x}, \vec{R}) \qquad (1.17.)$$

Consequently, the expected average value of the efficiency and effectiveness of the portfolio σ_p^2 are given by:

$$m_p = \sum x_1 m_1 = (\vec{x}, \vec{R}). \qquad (1.18.)$$

$$\sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} x_1 \cdot x_y W_y = (W\vec{x}, \vec{x}) . \qquad (1.19.)$$

According to the theory of Markowitz portfolio risk is σ_p (sometimes selected as a risk σ_p^2).

Thus, the portfolio is characterized by two criteria are the expected average value of portfolio performance $m_p = m_p (\vec{x})$ and portfolio risk, both criteria depend on the composition of the portfolios $\vec{x}$. Criterion $m_p (\vec{x})$ should be increased, while the criterion $\sigma_p (\vec{x})$ is reduced, changing the portfolio (vector $\vec{x}$).

It is known that the optimal solution lies multicriteria problems of **Pareto solution set**, with the property that any improvement in the Pareto criterion for solutions leads to deterioration of other criteria. **Pareto set of solutions** for two-criteria tasks can be found by fixing one of the criteria and optimizing the other[33].

According to Markowitz scheme discussed in section 1.2., to find solutions to the Pareto two-criteria optimization problem portfolio of financial investments are made to minimize the risk of the portfolio $\sigma_p (\vec{x})$ at fixed values of the expected average value of $m_p (\vec{x})$.

Thus, the mathematical model of optimization of the portfolio according to the Markowitz scheme is as follows:

$$\sigma_p^2 = (W\vec{x}, \vec{x}) - \min$$

$$x \geq 0, i = 1....n, \sum_{i=1}^{n} x_1 = 1, \sum_{i=1}^{n} x_1 \cdot m_1 = m_p \qquad (1.20.)$$

[33] In this case, in addition to making Pareto solutions may appear, do not possess the property of Pareto, but such "extra" solutions are easy to identify and eliminate.

Obviously m_p belongs to the interval [m_{min} - m_{max}], where m_{min} = min {m_1 ..., m_n}, m_{max} = max {m_i ..., m_n}.

Tssk (1.20) belongs to a class of quadratic programming tasks as minimizing the quadratic form $(W\vec{x},\vec{x})$ with linear equality constraints and inequalities.

It is known that regular quadratic programming problem has a unique solution for each fixed m_p € [m_{min} - m_{max}] (task (1.20).) will be regular in case the matrix is positive definite). Positive definiteness of the covariance matrix W is, in particular, the lack of complete correlation between the effectiveness of capital investments in investment objects forming the payroll of the portfolio.

To avoid unauthorized neparetovskih decisions, you must first solve the problem of building a portfolio with the lowest possible risk value, a mathematical model which takes the form:

$$\sigma_p^2 = (W\vec{x},\vec{x}) - \min$$

$$x \geq 0, i = 1....n, \sum_{i=1}^{n} x_1 = 1, \qquad (1.21.)$$

Solution of the problem of formation of the portfolio with the lowest possible risk value (1.21.) is given by

$$\vec{x} = W^{-1}\vec{1}/(W^{-1}\vec{1},\vec{1}), \qquad (1.22.)$$

Where the vector $\vec{1}$ = (1, ..., 1)^r , W-1 is the inverse matrix of the covariance matrix W.

If the efficiency of capital investment in various investment objects are not correlated, the covariance matrix is diagonal W= diag ($\sigma_1^2 ....\sigma_n^2$), where σ_1^2 and σ_n^2 are dispersion efficiencies of capital investment for the considered investment objects. In this case the formula (1.22). Gives an idea solution because:

$$i = 1...n, x_i = x_i = \frac{\frac{1}{\sigma_1^2}}{\sum_{j=1}^{n}\frac{1}{\sigma_1^2}} \qquad (1.23.)$$

That in order to achieve the lowest possible risk of investing in assets numbered index i = l, ..., n, it is necessary to invest in the assets in question so that the share

capital for the i-ro asset was inversely proportional to the variance the effectiveness of capital investments in this asset.

Let $x_1 = x_i$, i = 1, ..., n is the solution of (1.21.), ie investment portfolio $\vec{x} = (x_1 .... x_n)^T$ provides minimal risk of capital investment. Reducing the risk for the portfolio $\vec{x}$ compared with the risk in capital investment individual assets is due to the diversification of capital across multiple asset.

We denote $m_p = (\vec{x}, \vec{m})$ as the expected value of the financial investment portfolio performance with the lowest risk investments. Then the solutions of the following tasks and they are the only two-criteria Markowitz task (1.20.)

$$\sigma_p^2 = (W\vec{x}, \vec{x}) - \min$$

$$x_1 \geq 0, i = 1...n, \sum_{i=1}^{n} x_1 = 1. \sum_{i=1}^{n} x_1 \cdot m_i = m_p \qquad (1.24.)$$

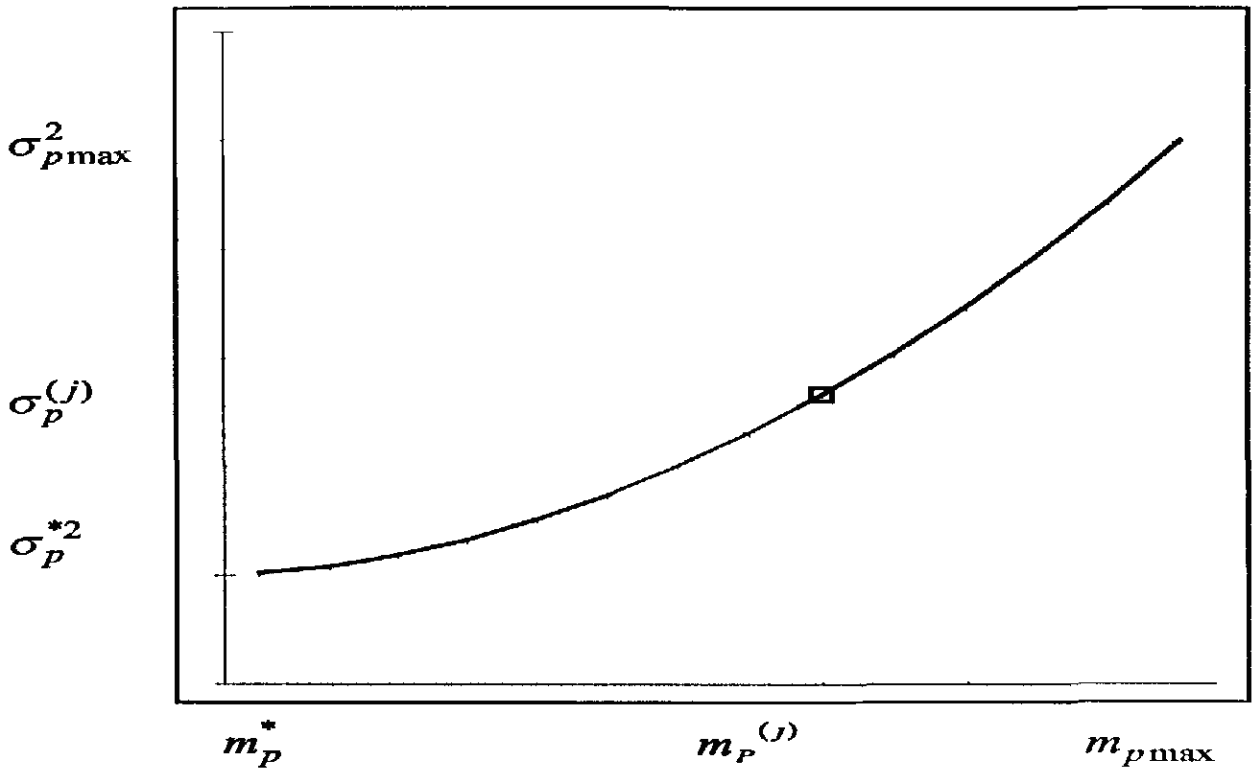
m_p € [m_{min} - m_{max}]

To the actual location of the problem (1.24) is necessary in the interval m_p € [m_{min} - m_{max}] select the discrete set of points $m_p = m^{(j)}$ for example, distributing them equally $m^{(j)} = m_p + j \cdot h, h = (m_{p\max} - m_p)/L$. Then problem (1.24.) is solved for each fixed value of $m_p = m^{(j)}$, j=1..., L-1.

The resulting set of portfolios and $\vec{x}^{(j)}$ with an expected value the effectiveness of capital investments and the risk of $\sigma^{(j)2} = (W\vec{x}^{(j)}, \vec{x}^{(j)}), j = 1...., L-1$ belongs to the Pareto set of portfolios of financial investments (such portfolios accepted also called effective or efficient portfolio solutions of the problem (1.24.)).

It can be proved that the plane (m_p, σ_p^2) curve Pareto solutions of Markowitz (1.24) is increasing and convex downward (sm.grafik.1.4.).

The curve of the Pareto decisions



The graph shows as many Pareto points $m^{(j)}, \sigma^{(j)^2}, j = 1...., L-1.$

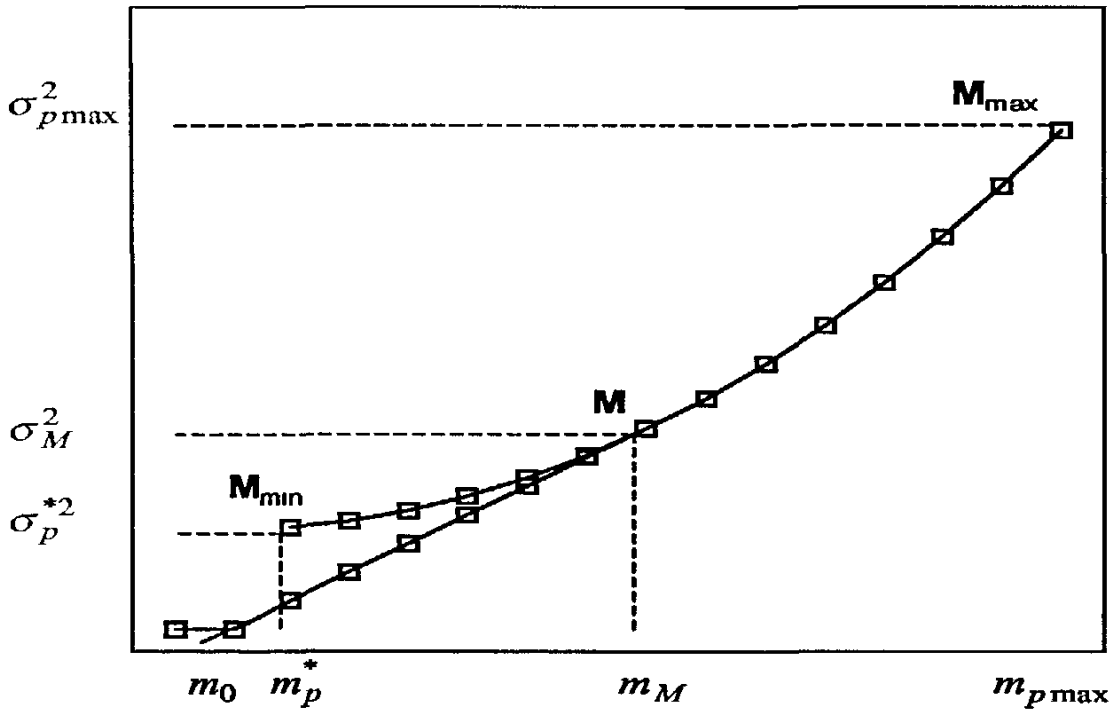
Thus, the set of efficient portfolios has the property that for any pair of efficient portfolios with higher expected value of the efficiency of capital investment will be more important risk.

The final choice of the portfolio should be among the set of efficient portfolios, establishing acceptable compromise between the level of investor expected portfolio performance and risk. Investors are risk-averse, choose portfolios that are close to $\vec{x}$ the lowest risk σ_p^2, but with the lowest expected efficiency of Pareto portfolios. Investors are risk-averse in the presence of greater expected efficiency, choose a portfolio close to the portfolio, defined point ($m_{pmax,}$ $\sigma^2_{p\,\max}$).

In fact, in any investment decision problem for investors in most cases, there is another alternative - putting capital rB to risk-free government securities with the bank rate. In this case, the pay-roll of financial assets and the asset should be included with an efficiency rate defined guards and zero risk.

Optimization problem portfolios of financial investment in the presence of a financial asset with zero risk were addressed and resolved by the American economist D.Tobin.[34]

Effective solutions of Tobin`s portfolio optimization problem is most easily represented graphically (look graph 1.5.).



For plotting the set of points (m_p, σ_p^2) relevant portfolio performance in the presence of an asset with zero risk, you must first solve the problem of Markowitz (1.24.), excluding asset with zero risk. Point (m_p, σ_p^2) for the set of efficient portfolios, excluding asset with zero risk are plotted by curve $M_{min}M_{max}$. Then on the axis fixed M_o with the efficiencies m_o which corresponding to the risk-free investment. Finally, from the point M_0 tangent to the Markowitz curve $M_{min}M_{max}$.

Since the Markowitz curve $M_{min}M_{max}$ increasing and convex downward, there is a unique point of tangency M family of direct rays emanating from the point M_0.

Points M of segment M_o-M correspond to the distribution of efficient portfolios invested capital in proportion: $x_0\% = \dfrac{(m_M - m_p^1)\cdot 100\%}{m_M - m_0}$ of capital invested in risk-free

[34] Donald D. Hester, James Tobin. (1967) *Risk Aversion and Portfolio Choice*. New York *John Wiley and Sons* Available from: http://cowles.econ.yele.edu

portfolio, and the rest part (100-x_0)% of capital is in risky assets in proportions determined by the point M, ie (100-x_0)-x_M%, i=1, ...n. of the capital invested in the i-th asset, where = $\vec{x}_M = (x_{iM} ....., x_{nM})^r$ is efficient portfolio corresponding to the point M.

We outline the portfolio formation in circumstances where permitted short sale (short sale). Note that the short sale operation makes sense especially in the market of financial instruments such as stocks. Mathematically permit a short sale means that the proportion of x_i can take negative values, the negative value of x; means that the i-th asset should take investor in debt for the transaction short sale[35]. Mathematical model of the portfolio in the presence of short fat has the form:

$$\min(W\vec{x},\vec{x}), \sum_{i=1}^{n} x_1 = 1, \sum_{i=1}^{n} m_1 x_1 = m_p . \qquad (1.25.)$$

Consider the matrix A with scale (2_M) the first row of which consists of one (1, ..., 1) and the second of (m_1, ..., m_n), and the size of the vector $\vec{f}$ with scale $2\ \vec{f} = (1, m_p)^1$.

$$\vec{x}_{opt} = W^{-1}A^1(AW^{-1}A^r)^{-1}\vec{f}, \qquad (1.26.)$$

It is possible to shown that the solution of $\vec{x}_{opt}$ problem (1.25.) is given by the equation: where A^T is a two-column matrix transpose to A, the first column consisting of the elements of the first row of A (they are equal to 1) and the second column is formed by the second row of numbers A (m_1 ..., m_n).

Fixing in vector $\vec{f}$ different values m_p € [m_{min} - m_{max}] using formula (1.26), we obtain efficient portfolios with minimum risk with fixing the expected average value of portfolio performance m_p.

In the next section we consider some modifications of the Markowitz model and their application to problems of capital investment subject to Russian conditions.

[35] Kryanev A.V. (2001). *Basics of financial analysis and portfolio investment in a market economy*. Moscow: MiFi.

§ 1.3. Verification of classical portfolio theory taking into account the Russian practice of portfolio investment

Mathematical Markowitz model (1.20.) does not impose any restrictions on the proportion of x_1, i=1,...n, except natural (1.16.). However, in solving practical problems on the share of investment x_1 often impose a priori restrictions.

Suppose that capital is invested in the stock market in financial instruments such as stocks. Then, based on preliminary estimates, and often the real possibilities of restricting top honors amount of shares that can be purchased on the stock market entered a priori restrictions on the number of shares in the portfolio, and thus formed into shares portfolio. Such restrictions have the form of group:

$$a_1 \le x_{j1} + ... + x_{jmi} \le b_j, \; j = 1....m, m_1 + ... + m_m = n, \qquad (1.27.)$$

where m is the number of a priori groups, m_j is the number of financial assets in the j-th group, a_j, b_j ($0 < a_j < b_j < 1$) are the lower and upper limits of the total share of the j-th group.

It is assumed that each financial asset (and each index i= l, ... n) is included in one and only one of the groups (1.27.).

For the system of a priori constraints (1.27.) will be consistent is necessary and sufficient that $\sum_{i=1}^{n} a_1 \langle 1 \sum_{j=1}^{m} b_1 \ge 1.$

In particular, if $a_j = b_j$, j = l, ... n, it means being formed in the fixing of the total share portfolio invested capital assets belonging to the j-th group.

Often the system of a priori constraints (1.27.) has the form $a_1 \le x_1 \le b_1, \; j = 1,...n,$ (1.28.)

ie restrictions are imposed separately on the share of investment capital for each asset.

With a priori restrictions for finding Pareto efficient portfolios have the problem

$$\sigma_p^2 = (W\vec{x}, \vec{x}) - \min \qquad (1.29.)$$

$$x_1 \ge 0, i = 1,...n, \sum_{i=1}^{n} x_1 = 1,$$

$$a_1 \le x_1 \le b_1, \; j = 1,...m,$$

$$(\vec{m}, \vec{x}) \in | \, m_p \cdot m_{\max} \, |,$$

Where (m_p, σ_p^2) and m_p **€** [m_{min} - m_{max}] determined by the decisions of the following tasks:

$$(W\vec{x}, \vec{x}) - \min \qquad (1.30.)$$

$\vec{x}$ is the solution of task (1.30.),

$$(\vec{m}, \vec{x}) - \max$$

$$x_1 \geq 0, i = 1...n, \sum x_1 = 1, a_1 \leq x_{ji} + ... + x_{jm_1} \leq b_1, j = 1...m,$$

$$m_p = (\vec{m}, \vec{x}), \sigma_p^2 = (W\vec{x}, \vec{x}),$$

$$x_1 \geq 0, i = 1...n, \sum x_1 = 1, a_1 \leq x_{ji} + ... + x_{jm_1} \leq b_1, j = 1...m, \qquad (1.31.)$$

$$m_{p\max} = (\vec{m}, \vec{x}_{\max}), \sigma_{p\max}^2 = (W\vec{x}_{\max}, \overrightarrow{x_{\max}}),$$

$\vec{x}_{\max}$ is the solution of task (1.31.).

In the task (1.30) efficient portfolio is the lowest risk, satisfying a priori constraints (1.27.) And in (1.31.) - Efficient portfolio with the highest expected return.

(1.30.) is the quadratic programming problem which is solved on the computer one of the well developed in computational mathematics numerical methods.

(1.31.) is linear programming problem, which can be effectively solved on a computer using a special algorithm that takes into account the specifics of the problem (1.31). (Of course, the problem (1.31.) Can be solved by well-known simplex method, but in this case the use of the simplex the method is impractical.)

As in the Markowitz task (1.20.) let`s set the points (m_p, σ_p^2) corresponding Pareto efficient solutions of the problem (1.29.) On the plane (m_p, σ_p^2) is an increasing convex curve down.

The following modification of the Markowitz model is associated with a situation where there is a strong correlated efficiencies considered investing in financial assets. In this case, the covariance matrix W of the vector R is efficiencies ill-conditioned or degenerate and, hence, Markowitz task (1.20.) And the problem of optimizing the portfolio of financial investments in the presence of a priori constraints (1-29.) Belong to the class of ill-posed problems, causing instability in solving them.

Economic and financial instability cause the problem of optimizing the portfolio of financial investments is the fact that the risk of the portfolio may vary slightly in the allocation of capital between assets whose efficiency is strongly correlated.

Instability of the optimization problem portfolio of financial investments from a computational point of view it requires regularization[36].

However, as is often the case in life, realized rule "a blessing in disguise" the degeneracy of the covariance matrix can be used to without significantly increasing the risk of the portfolio optimization to achieve an additional criterion. The fact is that in addition to the two considered hitherto criteria - effectiveness and expected value of portfolio risk is often desirable to optimize at least one criterion. The most important criterion is the amount of additional reinvestment associated s.restrukturizatsiey old structure to the new portfolio. This additional criterion is particularly important when large volumes of reinvestment (restructuring "large" portfolio).

Mathematically, the criterion volume of reinvestments $J_{dix}(\vec{x}, \overrightarrow{x_o})$ describes the "distance" between the old and the new portfolio composition

$$\overrightarrow{x_0} = (x_{10}....x_{n0}) and \vec{x} = (x_1....x_n)^r$$

$$J_{dir}(\vec{x}, \overrightarrow{x_o}) = \sum_{i=1}^{n} d_i \cdot (x_1 \cdot S_p - x_{i0} \cdot S_0)^2 \qquad (1.32.)$$

where S_o (S_p) are the capital invested in the old (new) of the portfolio, d_i - positive numbers determined by the financial content of financial assets. For example, if the investment is made in the securities market, then $d_i = 1 / P_i^2$, where P_i is a value of one bond of i-th issuer.

Note that in the general case So≠Sp, as at the date of portfolio restructuring it (or him) can be displayed (enter) certain capital.

Given the volume of reinvestment criterion regularized optimization problem portfolios of financial investments is as follows:

$$\sigma_p^2 = (W\vec{x}, \vec{x}) + a \cdot J_{drt}(\vec{x}, \overrightarrow{x_0}) - \min, \qquad (1.33.)$$

$$x_1 \geq 0, i = 1...n, \sum x_1 = 1, a_1 \leq x_{ji} + ... + x_{jm_1} \leq b_1, j = 1...m,$$

[36] Kryanev A.V. (2001). *Basics of financial analysis and portfolio investment in a market economy*. Moscow: MiFi.

where a> 0 is regularization parameter that sets the level of compromise between portfolio risk and criterion of reinvestment volume. $(\vec{m},\vec{x}) \in | m_p \cdot m_{\max} |$,

Mathematically the task (1.33.) also belongs to the class of quadratic programming tasks and solved on a computer using a special algorithm that allows including choosing an appropriate value of the regularization parameter a> 0.

Currently, mathematical optimization model portfolios of financial investments taking into account the volume of reinvestment criterion implemented in the software package used by various organizations of the Russian Federation securities markets.

Previously noted that the implementation of the scheme or its Markowitz modifications require knowledge of the estimates of the efficiencies of expected values vector and $\vec{m}$ elements of the covariance matrix W.

Assume that the capital investment made in the stock market. Then the efficiency of investments in shares of R i-ro issuer is calculated according to the formula:

$$R_1(t,T) = (P^{t\arg}(1+T) - P_1^{t\arg}(t) + D_1(t,T)) / P_1^{t\arg}(t), \qquad (1.34.)$$

where $(P_1^{t\arg}(t,T), P_1^{t\arg}(t)$ is the purchase price, sales (by exchange) of one share of i-th issuer to the relevant date; T is the period of investment; D_1 (t, T) is dividends per share i-ro issuer attributable to the time interval [t, t + T].

If the formation of the investment portfolio is made in the stock market shares, and the number included in the payroll of the financial instruments is low, the desired characteristics of an effective $\vec{R}$ can be estimated from realized (historical) values of $\vec{R}$ for the time prior to the current date, which is produced to create the optimal composition portfolio, according to the equations:

$$\vec{m}_1 = \frac{1}{N} \cdot \sum_{i=1}^{n} R_1(t_1,T), \qquad (1.35.)$$

$$\vec{W}_y = \frac{1}{N-1} \cdot \sum_{i=1}^{N} R_i(t_i,T) - \vec{m_i}) \cdot (R_j(t_j,T) - \vec{m_j}),$$

Where $t_N \langle t_{N-1} ... \langle t_1 \langle t$ is the current date, which is produced to create the optimal portfolio composition.

However if number of issuers to be considered priformirovanii portfolio exceeds several tens (and such order of the number of issuers discussed happens quite often),

the reliability of the estimates $\vec{m}$ and $\vec{W}$ directly on realized historical values $R_1(t,T)$ according to formulas (1.35.) becomes unacceptable mismatch requirements of a large number of realizations N>>n2, capacities use in practice this number of realizations bez.suschestvennogo distortion estimates for the current date.

Exit from the situation described in the stock market was found U.Sharp who offered a simple but as it turned out later, constructive Univariate regression model, the effective individual investment objects and efficiency of the overall market[37].

Sharpe mathematical model has the form:

$$R_1 = a_i + \beta_i \cdot R_M + \varepsilon_i \ , i = 1....n,$$

where R_M is the efficiency of the overall market, the implementation of which are counted as srednevzveshannye implementations R_i with weights proportional to the total volume of capital investments market shares placed in i-ro issuer.

Parameters of the regression model (1.36.) α_1 and $ß_1$ called ‚alpha' and ‚beta' are coefficients and their significance for the shares of leading corporations and firms are regularly published in the press and passed through various communication channels[38].

The Sharpe model (1.36.) assumes that εl, ..., εn is independent, i.e. cross-correlation of the efficiencies of various issuers of shares is due only to their relationship through the efficiency of the overall market.

Elements of the covariance matrix W are defined through the beta coefficients of the model (1.36.)

$$W_y = \beta_i \cdot \beta_j \cdot \sigma_M^2, i, j = 1,...n, i \neq j,$$

$$W_y = \sigma_i^2 = \beta_i^2 \cdot \sigma_M^2 + \sigma^2(\varepsilon_i)$$

Thus, the risk of investing in shares of the issuer i-ro σ_i^2 consists of two risks: $\beta_i^2 \cdot \sigma_M^2$ is the risk arising from random fluctuations of the effectiveness of capital investments in the shares of the issuer i-ro due to random fluctuations of the efficiency of the overall market (risk factor),

[37] Subsequently, the model is generalized to the case of Sharpe's consideration of many factors.
[38] Sometimes the alpha and beta coefficients are understood specially modified coefficients ai and ßi formula (1.36.).

$\sigma^2(\varepsilon_i)$ is the risk caused by its own independent random fluctuations efficiency i-ro shares of the issuer (nonfactor risk).

It can be shown that the diversification of investment capital leads to an averaging factor (market) risk reduction and non-factor (equity) risk.

Currently, besides the one-factor in Sharpe model is used multivariate models of the form:

$$R_i = a_i + \sum_{k=1}^{M} \beta_{ik} \cdot F_k + \varepsilon_i, i = 1....n, \qquad (1.37.)$$

where F_1, ..., F_M are factors affecting the change in the effective actions.

The main factors considered are usually taken: the level of bank interest rates; inflation; Prices for basic raw materials (oil, gas, etc.); industry factors.

In countries with developed market economies is widely used multifactor model BARRA, which contains 68 basic and industry factors.

In addition to traditional single-level multifactor models, including model BARRA, you can use the two-level multivariate models. For example, taking into account three factors - the effectiveness of the overall market R_M; industry efficiency $R_{(l)}$ and the region $R_{[r],}$ which belongs to the enterprise in the issuing two-level model has the form

$$R_i = a_i + b_{i,M} \cdot R_M + b_{i1} \cdot R_{(l)} + b_{ir} \cdot R_{[r]} + \varepsilon, \qquad (1.38.)$$

$$R_{[r]} = a_{[r]} + b_{[r]M} \cdot R_M + \varepsilon_{[r]},$$

$$R_{(l)} = a_{(l)} + b_{(l)M} R_M + \varepsilon_{(l),}$$

In the model (1.38.) ε_i $\varepsilon_{(l)}$ $\varepsilon_{[r]}$ are independent random fluctuations efficiencies i-ro shares of the issuer, the first branch, r-th region, respectively.

According to the two-level model (1.38) stochastic dependency R_i of the market does not only directly, but also through stoasticheskuv R_i dependence on the effectiveness of the industry "and the region, which belongs to the enterprise in issuing.

Duplex model allows, inter alia, to alpha and beta coefficients, as well as risk for shares of each issuer additive components relating to the market in general, and to the industry in the region, which makes it possible to analyze the degree of influence on

the alpha and beta factors and the risk of the overall market and industry in the region.

The possibility of such allocation is of particular significance for portfolio, investing in the Russian Federation, where the influence of industries and regions on the financial and economic characteristics of enterprises is of paramount importance.

According to the original model (1.38.) We obtain:

$$R_i = a_i + \beta_i R_M + r_{1,} \qquad (1.39.)$$

Where

$$a_i = a_{i0} + a_{i(l)} + a_{i[r]},$$

$$\beta_i = \beta_{i0} + \beta_{i(l)} + \beta_{i[r]},$$

$$r_i = \varepsilon_{i0} + \varepsilon_{i(l)} + \varepsilon_{i[r]}, \qquad (1.40.)$$

$$a_{i0} = a_i, a_{i(l)} = b_{i,l} a_{(l)}, a_{i[r]} = b_{i,l} a_{[r]},$$
$$\beta_{i0} = b_{i,M} \cdot \beta_{i(l)} = b_{i,l} b_{(l)M} \beta_{i[r]} = b_{ir} b_{[r]M},$$

$$\varepsilon_{i0} = \varepsilon_i, \varepsilon_{i(l)} = b_{il} \varepsilon_{(l)} \cdot \varepsilon_{i[r]} = b_{i[r]} \varepsilon_{[r]}$$

Hence the quantities in (1.40), (1.41) have the following meanings:

α_i - is a total-factor of i-th issuer; α_{i0} -contribution to the α-coefficient through direct communication i-ro issuer to the market; $\alpha_{i(1)}$ is a contribution to the α-coefficient by linking i-th issuer markets through industry to which the issuer; $\alpha_{i[r]}$ is a contribution to α-coefficient due to i-th issuer to the market through the region to which the reporting issuer. Similar values are components of the ß-coefficient.

From formulas (1.40), (1.41) implies the representation for the variance (risk in Markowitz scheme) i-ro issuer: $\sigma^2 R_i = \sigma_i^2 + \sigma_{n(i)}^2 + \sigma_{i[r]}^2 + \sigma_{i,M}^2$

where σ_i^2 is a component of risk due to fluctuations in the efficiency of their own independent of the issuer; $\sigma_{n(i)}^2 = b_{ij}^2 \sigma^2(\varepsilon_{(i)})$ is the component of risk due to fluctuations in the efficiency of their own industry, which is worn by the issuer; $\sigma_{i[r]}^2 = b_{i,r}^2 \sigma^2(\varepsilon_{[r]})$ is component of risk due to fluctuations n the efficiency of their own region to which the issuer belongs; $\sigma_{i,M}^2 = \beta_i^2 \sigma^2(R_M)$ is component of risk due to fluctuations in the efficiency of the overall market.

The resulting estimates of the coefficients of the model (1.40.) Allow us to estimate the covariance between different sectors and regions, which makes it possible to carry out a correlation analysis for industries and regions.

We have examined the problem of optimizing portfolios of financial investment in productions modifying classical Markowitz optimization scheme, including cases of instability optimization problems. Often, however, and especially with respect to Russian investment markets, it is necessary to consider tasking optimize portfolios of financial investments, are fundamentally different from the circuit Markowitz. First of all, the fundamental difference between new productions from Markowitz scheme relates to the need to consider a new definition of risk of financial investments associated with the formation of a portfolio of financial investments.

The fact that the definition of the calculated values of efficiency and risk Markowitz optimization scheme, respectively, as the mean value of the efficiency and standard deviation of the mean efficiency, treated as a random variable, often does not correspond to the real situation in the investment market.

Assume that the effectiveness of the considered financial instrument has been steadily growing over time. In this case, taking the decision to invest in the asset, we must as a predicted value of its effectiveness not take the average of the values already implemented, and that the value of which is determined by a growth trend for the next period of time, as designated by the term investor building a portfolio of financial investments . With this choice of the estimated values of the efficiency as a risk should take measure of the deviation of the predicted value.

In accordance with the above, we adopt the following definition of the calculated values of efficiency and risk.

As reference value, unlike schemes Markowitz will take the predicted value of the efficiency at a future date, the length of this period a future date to the current date is equal to the length of the period for which the determined efficiency. As the risk of investing in financial instruments will take the measure of deviation from the predicted values of the efficiency.

Specification of the baseline efficiency and calculated values in the Markowitz scheme is a special case of the scheme. In the Markowitz scheme calculated values

effectiveness and risks are the same for all investors (up to errors ochenok expectation and standard deviation of efficiency that are treated as a random variable). These values are a kind of internal characteristics of the financial asset. In essence, the scheme Markowitz-Sharpe-Tobin laid generally equal to all participants investichionnogo informative market. This scheme is often not adequately describe the real situation in any investment market. To some extent, you can still agree with the opinion that the majority of market participants are equally informed and has no additional data processing on the future state of the market, other than the data processing, which has already been implemented in the values of the characteristics of the market (based on which, in particular, the calculated efficiency). But it is absolutely wrong hypothesis that the calculated values of efficiency and risk of most investors (and especially all) will take estimates of mathematical expectation and standard deviation of efficiencies calculated in already realized values. And it would be a completely unqualified rekomendachiya application in all cases, the calculated values of efficiency and risk assessments of the expectations and standard deviation of efficiencies.

The above definition of the calculated values of efficiency and risk management allows for implementation of the proposed optimization schemes to beat the best results by investing in the best conditions, a forecast value of the efficiency. Forecast values calculated efficiencies and risk becomes for each individual investor and you can take as the calculated values of the efficiencies and risk optimization problem in a portfolio of financial investments those values according to the individual investor forecast obtained, possibly with additional a priori information is the most appropriate.

Therefore, each investor (which solves the problem of optimization) for any before it, the object of investment should deal with the random variable-predictive value for future efficiency R_{pre} date that is the current date on the time distance of the interval for which the determined efficiency. Take for specifying one of the options calculated values and risks.

We define the calculated values of efficiency and risk considered a financial asset as part of the investor expectation and standard deviation of a random variable R_{pre}.

When compiling a portfolio of financial investments, which includes assets, numbered index i = l, ..., n, it is necessary to estimate the covariance matrix of random variables

$$R_{pre} = (R_{ipre}....R_{npre})^i,$$

where R_{ipre} is likely effectiveness of i-ro for the asset in question of the investor.

Let the predicted values for effective investor uses the best of its ability (diagram) forecast, which can be applied to the dates already implemented in the past (maybe not all). Thus, it is assumed that for the last set of dates $t = t_i ..., t_N$ forecast algorithm allows to predict the future value of the interval length T efficiencies

$$R_{pre}(t_i), i = 1...n, k = 1...N$$

Let $R_{i\ (tk)}$ be a realized values of efficiencies. Then, under the assumption of stationarity of the covariance matrix W for vector predictive efficiencies R_{pre} we obtain an estimate of its elements W_{ij}:

$$W_{ij} = \frac{1}{N}\sum_{k=1}^{N}(R_{ipre}(t_k) - R_i(t_k))(R_{jpre}(t_k) - R_j(t_k)), i, j = 1...n.$$

Suppose, for example, the predicted values of efficiencies R_{ipre} for the current date belong to a finite intervals

$$R_{i\min} \le R_{ipre} \le R_{i\max}, i = 1...n$$

and random variables R_{ipre}, i = l, ..., n pairwise uncorrelated. Then, if the postulated uniform distribution law for each R_{ipre}, we have the equality:

$W = diag(\sigma^2(R_{ipre})...\sigma^2(R_{npre}))$ - is diagonal matrix,

$$\sigma^2(R_{ipre}) = (R_{i\max} - R_{i\min})^2/12, i = 1....n.$$

According to the introduced definition of predictive performance and risk for the portfolio optimization problem, as the calculated values of efficiencies taken score vector of expected average values $MR_{pre} = (MR_{ipre}, ..., MR_{npre})$, as well as the matrix W-covariance matrix of vector R_{pre}.

Let`s give another constructive scheme to determine the risk of the formation of portfolios of financial investments.

We denote by pi the probability of event ($R_{ipre} > R_j$) for all j = l, 2, ..., i-l, i + l, ..., n.

Assume that $p_i = 0$. This means that at any outcome realized efficiency the i-ro asset will never exceed the value realized efficiencies of all other assets. Consequently, in terms of optimizing the investment portfolio taken as a criterion of its expected value and effectiveness of the risk is completely useless to invest in the i-th asset, capital should be distributed only among those assets for which $p_i > 0$, in this case we have deliberately greater efficiency of realized value. Thus, the i-th asset, for which $p_i = 0$, should be excluded from the scheduling of assets included in the portfolio of financial investments.

Therefore, when solving the problem of optimizing the portfolio risk of investing in the i-th asset should increase indefinitely with decreasing pi down to zero.

Conversely, if $p_i = 1$, then the risk of investing in the financial asset portfolio in the formation of zero, even if his own risk Markowitz σ_{ipre} is nonzero and possibly exceed their risks of all other investment objects.

The above provisions are reflected in the empirical testing of the Markowitz model based on the data of the Russian securities market.

The main objectives are try to determine the optimal structure of the investment portfolio, its expected using the Markowitz model for risk and return. It should be noted that this model may solve the problem of short-term portfolio investments (t <1 year), because of the possibility that job any period of time allows for a model experiment on real databases, thereby obtaining the necessary statistical data.

A full analysis of the Markowitz model applicability is considered all the options, but it is difficult. There are plenty of external parameters defined, such as quotes about 50 issuers for 5 years, the interval defined by the portfolio return ($E|r_p|:|0;1|$), the date on which it is necessary to conduct research intervals to calculate the required parameters (days hours, minutes, depending on the format in which time information is received from the exchange).

In this regard, present only a few, the most typical, statistical data obtained from experiments.

Number of days of analysis: 120., Date of analysis: 30/05/2013.

Table 1.1.

Investment portfolio, consisting of 2 securities (SurgutNG [a] Gazprom [b])

Yield	0.00004	0.00058	0.0006	0.0009	0.0011	0.0013
a	0.57321	0.62845	0.43671	0.39567	0.37854	0.44981
b	0.54674	0.57954	0.65632	0.72089	0.76856	0.89091
Risk	0.00267	0.00364	0.00467	0.00498	0.00523	0.00511

Table 1.2.

Number of days of analysis: 35., Date of analysis: 30/06/2013.

Investment portfolio, consisting of 3 securities (***Tatneft [a], Mosenergo [b] Lukoil [c])***

Yield	0.0046	0.0062	0.0074	0.0079	0.0086	0.0097
a	0.4567	0.4326	0.3956	0.3217	0.2576	0.2265
b	0.5798	0.5275	0.4356	0.3654	0.2659	0.1546
c	0.2576	0.2876	0.3546	0.4768	0.5764	0.6872
Risk	0.00056	0.00063	0.00072	0.00083	0.00089	0.00093

Table 1.3.

Number of days of analysis: 70., Date of analysis: 30/10/2013.

Investment portfolio, consisting of 4 securities (***SurgutNG [a], Gazprom [b]), Norilsk Nickel [c] IrkutskEnergo [d]***

Yield	0.00019	0.00023	0.00027	0.00029	0.00031	0.00033
a	0.79541	0.82364	0.83567	0.85123	0.87631	0.88532
b	0.00785	0.01632	0.02654	0.34563	0.39764	0.43215
c	0.12356	0.11203	0.96543	0.91654	0.85432	0.81234
d	0.07985	0.05764	0.04637	0.03215	0.02543	0.01985
Risk	0.00243	0.00267	0.00251	0.00273	0.00278	0.00286

Table1.4.

Number of days of analysis: 95., Date of analysis: 15/11/2013.

Investment portfolio, consisting of 5 securities (***SurgutNG [a], Gazprom [b]), Norilsk Nickel [c], IrkutskEnergo [d], Tatneft [e]***

Yield	0.00084	0.00099	0.0014	0.0017	0.00196	0.0021
a	0.17543	0.14321	0.11487	0.10543	0.09876	0.09347
b	0.02435	0.03126	0.03896	0.44578	0.56743	0.58932
c	0.32876	0.39858	0.43766	0.47612	0.49788	0.48744
d	0.26547	0.30675	0.31546	0.27865	0.25345	0.23475
e	0.14326	0.16348	0.19764	0.23451	0.27841	0.29234
Risk	0.00156	0.00182	0.00192	0.00213	0.00195	0.00189

Table 1.5.

Number of days of analysis: 102., Date of analysis: 23/12/2013.

Investment portfolio, consisting of 6 securities (***SurgutNG [a], Gazprom [b]), Norilsk Nickel [c], IrkutskEnergo [d], Tatneft [e], Lukoil [f]***

Yield	*0.00013*	*0.00011*	*0.00014*	*0.00019*	*0.00021*	*0.00026*	*0.0003*
a	*0.12432*	*0.14325*	*0.13256*	*0.14675*	*0.15467*	*0.16543*	*0.17234*
b	*0.14654*	*0.16432*	*0.17123*	*0.17986*	*0.18321*	*0.18932*	*0.19431*
c	*0.23415*	*0.22543*	*0.21876*	*0.21415*	*0.19987*	*0.19421*	*0.20654*
d	*0.46532*	*0.54328*	*0.48765*	*0.43216*	*0.39156*	*0.37654*	*0.36854*
e	*0.08654*	*0.81235*	*0.76865*	*0.71234*	*0.6543*	*0.65987*	*0.69874*
f	*0.07654*	*0.79765*	*0.92765*	*0.10543*	*0.10876*	*0.11543*	*0.12586*
Risk	*0.00287*	*0.00268*	*0.00254*	*0.00242*	*0.0023*	*0.00246*	*0.00258*

Analyzing obtained during the experiment results, we can conclude that:

1. Listed pretty clear trend of decreasing risk with increasing amount of investment securities (Tables 1 - 5).

2. Markowitz model gives the correct distribution (0 <(*wl* ... *wj*) <1) only at low investment returns given by (E|r˜p|),, which was confirmed by low values of yield expectations(E|r˜p|:|0;1|).

3. For a portfolio consisting of more than 8-financial instruments the choice of the parameters for the ratio "return / risk" is not possible to get the correct distribution.

Thus, the Markowitz model is partially applicable to the Russian stock market.

In conclusion, it should be noted that the studies carried out ourself can not clearly give answers to all questions. Assessing the risk and return of the Russian securities market we can conclude, but only with some degree of probability, that the future situation in the stock market will be in the framework of the statistical data. The large number of factors that directly or indirectly affect the dynamics of listed issuers does not allow us to predict and reduce the risk of loss of investment to zero, but to reduce it to a minimum with the best possible yield.

The findings of the potential Markowitz model optimization the yield-risk relationship in the Russian context can be used to analyze the behavior of portfolio investments on the stock market.

2. THE BEHAVIOR OF A FINANCIAL INVESTMENTS PORTFOLIO ON THE STOCK MARKET

2.1. The behavior of a financial investments portfolio: structure and dynamics ratio "return / risk"

Investor forming an investment portfolio seeks to reach the most appropriate return. But this goal can not be achieved by simple selection the most profitable financial instruments. Such seemingly correct technique is not always-true, because as usual leads to an increased risk of the total investment portfolio. The desire to have the most profitable portfolio always controvercial to the desire to provide the lowest risk investments. The risk counts through using diversification of different constraints. Obviously, the optimum portfolio risk increases with the expected desired efficiency. In the presence of borrowed capital, we have an opportunity to build a portfolio with any expected efficiency, but the risk will grow indefinitely at the same time[39].

The yield and the risk of a particular financial asset can not be considered in isolation. Each investment should be analyzed from the perspective of its impact on the change in profitability and risk of the whole portfolio, in the case of the addition of the asset to portfolio and withdrawals from the portfolio as well.

Investor wishing to invest optimally is interested in comparison of various portfolios, but a comparison of certain types of financial instruments between them. The point of creating an investment portfolio and its management is to make the entire set of financial instruments with investment characteristics (risk and return) that are unattainable with the position of a single financial asset and are only possible as a combination.

Portfolio investments are the instrument using which the investor provides the optimal ratio for a investment return and risk of. Since all financial investments differ in the level of return and risk, the possible combinations of these in the portfolio

[39] Kirzhner L.A. (2009) ISBN: 966-3730-2-69.*Management of organizations*. CST.

changing characteristics as in the case of optimal combinations can achieve a significant reduction in the risk of the investment portfolio.

Changing courses of financial assets in the market does not take place in isolation from each other, but cover the entire market as a whole. Risk of a portfolio depends on the way the financial instruments, included in the portfolio, simultaneously decrease or increase at a rate, that is the correlation between courses changes of certain financial instruments. In situation of strong correlation between the individual courses (when all of the shares at the same time increase or decrease) the risk by investing in different securities can neither reduce nor increase. If rates of financial instruments is absolutely non correlative, then ideally, the risk could be removed completely.[40]

Making inclusion a large number of assets in the portfolio, the yield of which varies in different directions, we can obtain the following sequence whit the low profitability of some assets will be offset by other high-yield, which ultimately reduce the risk of the portfolio. This is the meaning of the portfolio to find a combination with a satisfactory risk / return ratio.

The challenge of constructing the investment portfolio is the wish of maximization the expected return on investment at a certain acceptable level of risk for the investor. Since changing market tools, you can create an infinite number of portfolios, so investors are interested only in efficient portfolios, each of which:

1. Provides maximum expected return for a certain level of risk.

2. Provides minimal risk to a certain value of expected return.

From this amount the investor chooses the optimal portfolio corresponding to its investment strategy with respect to risk.

Making the determination of the structure of efficient portfolio we will use the same tools that allows to calculate portfolios that maximize expected return for a fixed level of risk and minimal risk for a given level of expected return.

Let`s form some initial portfolio of several stocks traded on the MICEX.

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[40] Ivanov A. (2001). *Rationale for the structure of the investment portfolio.* Securities Market.№9. Maidenhead: *Open University Press.*

Table 2.1.

Structure of the initial portfolio	
Share	Percentage, %
GAZP (Gazprom)	12,38
LKOH (Lukoil)	8,95
RTKM (Rostelecom)	10,58
SBERP (Sberbank)	10,02
SNGS (SakhalinNeftegasServis)	12,44
SNGSP (SurgutNG)	11,63
TATN3 (Tatneft)	2,34
TATNP3 (Tatneft)	12,67
MSNG3 (Mosenergo)	12,83

Source: compiled by the author based on data from the MICEX, May 2014

Expected return and risk of the portfolio at time horizon of one day, are presented in Table:

Table 2.2.

Initial portfolio	
Value of the portfolio	2 214.0321
Expected return, %	0. 234651
The standard deviation of the expected return,%	0.011667

Efficient portfolio is a portfolio that at the same yield as the original has a lower risk. Finding it is done by minimising a function of the overall portfolio risk.

A change in the structure of the portfolio occurs providing a choice of the best variant the risk-return ratio. As a result is a portfolio with a minimum risk for a given level of return. Could be possible the optimization of finding the maximum return for

a given level of risk. But in the general case, the risk is negative and should be minimized.

Table 2.3.

Efficient portfolio	
Value of the portfolio	2 214.0321
Expected return, %	0. 234651
The standard deviation of the expected return,%	0.001957

Table 2.4.

Structure of efficient portfolio	
Share	Percentage, %
GAZP (Gazprom)	4,9
LKOH (Lukoil)	7,54
RTKM (Rostelecom)	8,36
SBERP (Sberbank)	38,32
SNGS (SakhalinNeftegasServis)	9,1
SNGSP (SurgutNG)	14,73
TATN3 (Tatneft)	10,32
TATNP3 (Tatneft)	3,53
MSNG3 (Mosenergo)	6,0

Source: Author, 2014

As can be seen comparing the two portfolios, efficient portfolio in this case has the same expected return but lower risk and the standard deviation of returns. The value of efficient portfolio is that the expected return can be obtained at a lower level of risk than the original portfolio.

However, over time the behavior of asset changes and results in the current portfolio may no longer be effective in terms of risk and return.

Selling some of the existing financial instruments and acquiring other investor can generate a new portfolio the best at any given time. However, the purchase of

new financial assets to the investor raises the question about the best ratio of the acquired financial assets.

Simplification of the process of optimal allocation of financial instruments in the portfolio is possible through supplementing fundamental analysis methods used in the classical portfolio theory, in particular the theory of Markowitz or methods of technical analysis.

As is known, there are two main methods of price analysis - fundamental and technical. Fundamental analysis allows to forecast the price in terms of financial and economic factors, and technical analysis reveals the psychological mood of stock trading, their willingness to buy or sell certain stocks, which means that signals the future price changes.

Fundamental analysis is concerned with the time-consuming research and requires a large funding or additional considerable costs if the results of fundamental analysis are purchased from a rating or analytical agencies. The costs of their own basic research in some cases may be commensurate with the income investors from trading in securities. Also, for example, if the results of fundamental analysis, it became known that the market is "wrong" evaluates any action now, there is no guarantee that it will "fix" this situation in the future.

Thus, fundamental analysis is capable of producing the forecast stock prices in the distant time perspective, however, the risk of such forecast can not be correct formalization and increases with the duration of the forecast.

Technical analysis is relatively inexpensive, allows you to quickly analyze a large number of shares, and does not require the creation of large databases. However, in contrast to the fundamental analysis, technical analysis does not allow to forecast stock prices in the longer term. As a general rule, short-term forecast made using technical analysis suggests only a trend (up or down) to change the price, but not the prices themselves.

At the same time, the risk of such a short-term forecast can be fairly well estimated by the previous changes of the asset.

Consequently, the application of the theory of Markowitz most correct to use the results of the technical analysis, assuming, for example, the same expected returns for all assets.

To verify this claim will be based on the results of the technical analysis of the MICEX index, assuming the same expected return, for all assets. This analysis will be carried out using the software package MetaStok Professional which represents a synthesis of the methods of modern portfolio theory and methods of technical analysis. This software package allows you to:

- Create a portfolio of investment assets based on the following optimization criteria: minimizing the risk of a portfolio at a given income; maximize revenue for a given risk;
- • Create and maintain a dedicated database on investment assets;
- • Create and edit the analyzed packages of investment assets;
- • Conduct a comparative analysis of investment projects for different values of parameters of the models of investment.

For clarity and simplicity of presentation, the results of the portfolio ormiruetsyaiz shares of three independent companies - "Lukoil", Gazprom and JSC "Rostelecom", traded on the MICEX. For the price of the purchase (sale) adopted the weighted average price on the MICEX at the transaction date. At the time of portfolio formation is assumed that the expected return on all assets of the same, ie portfolio is calculated using the criterion of minimum risk for the expected return.

To make a decision on the formation of an investment portfolio or selling it using the software package MetaStok Professional construct a trading system consisting of two parameters: MACD and Stochastits. This trading system will be used for all financial assets included in the portfolio. Using the built trading system, we use the following trading scheme:

1. Analyzes the dynamics of the MICEX index (Figure 2.1.). According to the analysis of the decision taken on the operations for the next trading day.

1.2. Signal "Buy (Sell)" is generated when the "Slow Stochastic % K crosses % D Slow Stochastic upwards (downwards) (Parameters Slow Stochastic: 14-3-5-10).

1.3. Signal "Buy" is formed when the two curves (% K and% D) are below the 20%, and the envelope MACD histogram has moved up (Settings MACD: 12-25-9).

1.4. Signal "Sell" is formed when the two curves (% K and% D) are above the 80%, and the envelope MACD histogram has moved down.

Taken together these signals makes it possible to identify moments of change in trend in the dynamics of the MICEX index. If you change the trend from negative to positive (index stopped falling and began to rise) system generates a signal "Buy", and, accordingly, if the trend changes from positive to negative (MICEX index began to fall), a signal is generated to "sell".

2. *After the signal "Buy" calculated structure of the investment portfolio. For this purpose, the time series of 14-day returns, calculated for each of the shares.*

Returns rows are form as follows. Rate of return calculated on a specific date that could be obtained by the share in case it was bought 14 days ago (price of buying / selling is adopted weighted average quotation of the MICEX).

$$D_t^i = \frac{C^i{}_t - C^i{}_{t-14}}{C^i{}_{t-14}} \cdot \frac{365}{14} \cdot 100\%, \qquad (2.1.)$$

where t is the settlement date; $C^i{}_t$ is i price of the asset on the date t.

It allows to calculate the profitability of certain shares and to predict its value in the future. So the investor will have the opportunity to make decision using the valuable analytical information for it.

Then the settlement date is shifted for one day ahead and calculation of yield repeated. The latest settlement date corresponds to the date of receipt of the signal "Buy."

Using the formed in that way rows calculates the current sample covariance matrix.

$$\sigma^2{}_n = \frac{1}{N-1} \sum_{k=0}^{N} (\overline{D^t} - D^i{}_{t-k})(\overline{D^j} - D^j{}_{t-k}), \qquad (2.2.)$$

where N is the time window for estimating the covariance matrix;

$\overline{D^t}$ is the average yield of the asset in the time interval [t - N, t];

$D^i{}_t$ is a return of an asset i at date t.

Source: compiled by the author based on data from the MICEX.

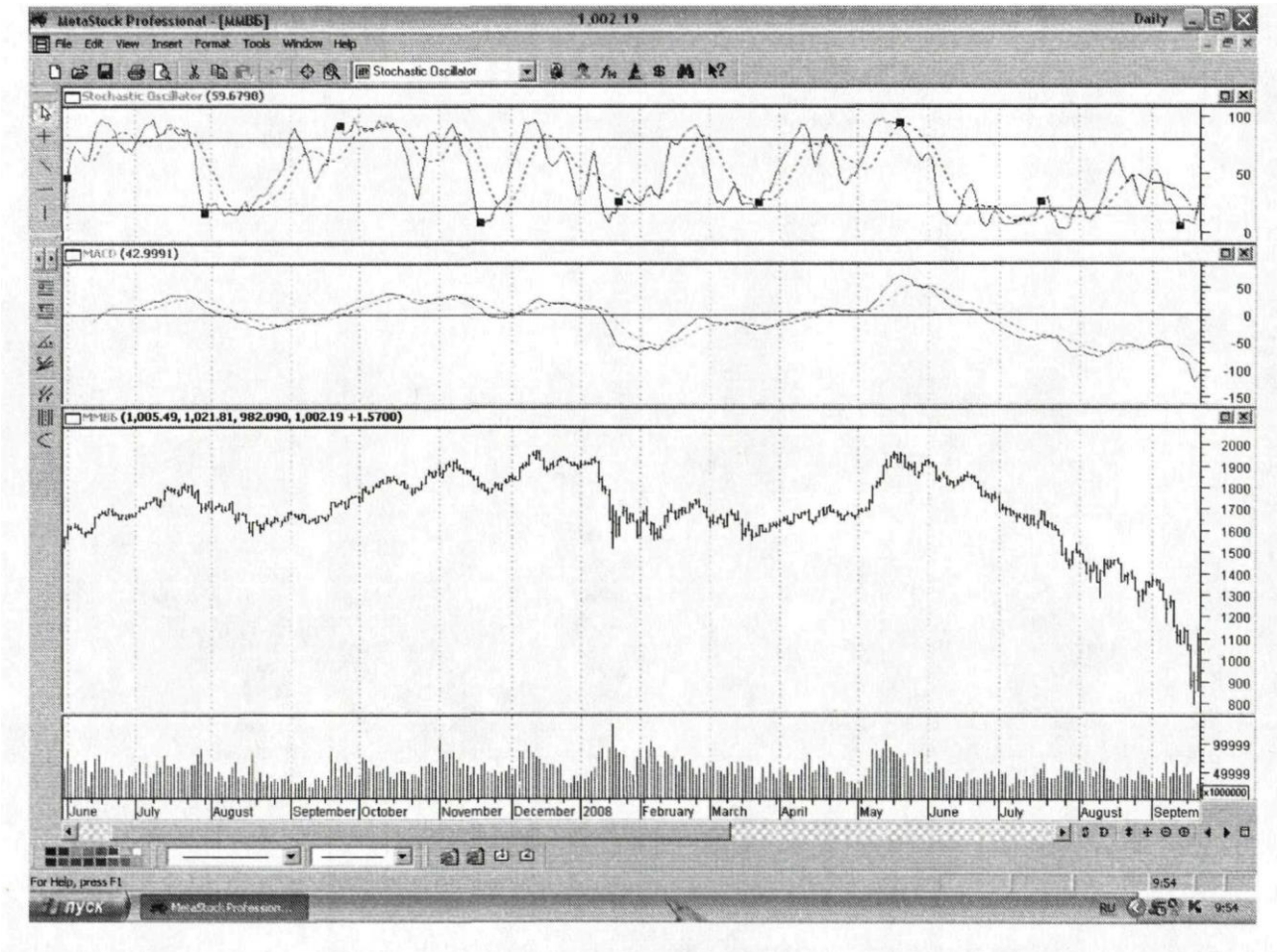
Next from the entire set of portfolios selectes portfolio ensuring the minimum variance (risk) of the expected return. On the next trading day based on the results of the calculation of the optimal portfolio, bought shares in the required proportions.

3 *After receiving the signal "Sell" all the shares are sold and the investor goes to wait for the next signal "Buy."*

Assume that on 10/05/13 the investor had one common unit of capital. Investment will be made in accordance with the above trading scheme.

Along with the best on the Markowitz portfolio by comparison will be form an alternative portfolio of the same stocks where the share of each share which will be the same and equal to 1/3.

Graph 2.1.



Source: Autor, 2014. Based on the MICEX data.

11/05/13 signal appeared "Buy" (label A graf.2.1,) (Both curves Slow Stochastic was below the level of 20%, and the envelope MACD histogram is directed upwards. Graph Stochastic% K denotes sploshnoyliniey and 0%-dotted) .

The rest calculations look at appendix I.

The analysis showed the following results (look appendix II):

The total yield from 11.05.08 to 01.08.13 on operations was as follows:

Portfolio formed by Markowitz - 64.8% per annum; alternative portfolio - 45.9% per annum. It is seen that even the idea of combining simple methods of technical analysis with diversification Markowitz yielded significant benefits with respect to alternative investments with equal shares of assets in the portfolio.

There is no doubt that the use of more sophisticated methods of technical analysis, allowing a good predictor not only trends, but also the price targets (target price) of assets, will increase the efficiency of the use of similar trading schemes.

Optimizing trading system for each financial asset in the portfolio can be optimized ratio of return / risk. To do this, create a portfolio of managed assets - the most liquid shares of common stock traded on the MICEX: "Lukoil", "Surgutneftegas", "Savings", "Gazprom".

Evaluation of each of the assets are small number of parameters for a simple trend-following system. The number of parameters of the system does not exceed three, including a stop-loss. In our view, the most reasonable theory, systems are built using moving average: the intersection with the moving average price or the intersection of the two moving average or minimum / maximum indicator TRIX (actually represents the first derivative of three smoothed moving average). A small number of parameters of the system makes it more reliable. It is obvious that, for example, the time series of 1500 bars on historical data, you can choose the system parameters from 1500, which will almost break even on retrospective testing, but it is unlikely that this system will be able to predict the direction of price change.

To smooth the curve of accumulation of profit, and hence reduce the risk to shareholders of OAO Gazprom, the most liquid, and the shares of "Savings Bank" as the least liquid, the game is played using a short sale. Using a short sale can reduce the overall risk of the portfolio is approximately 1.5 times.

Optimization of the parameters of trading systems is made on the entire set of historical data for each of the shares. For each of the stocks selected system providing enough stable income (curve accumulation of profits without reinvestment has a minimal deviation from the straight line expected profit). Selected optimal stop-loss for each asset.

Conducted parametric studies have shown that for each of the shares, regardless of the system used, there are some limits profitability. Moreover, these values are reached at a certain sensitivity of the system (a certain number of transactions carried out by the system in a certain period), the choice of which is determined by the average slippage and commission broker. Higher values can be obtained by increasing the number of system parameters, thus lowering its reliability. Risk, hereinafter understood as daytime rms or standard deviation of the expected return at the maximum yield is virtually independent of the choice of the system.

It follows:

1 When using a trend-following trading systems with a small number of parameters, the ratio of return / risk has a maximum value for each of the shares.

2 The maximum ratio of return / risk depends on the number of transactions generated by the system for a certain period of time, ie the sensitivity of the system. The optimum sensitivity of the system depends mainly on the average slip and from brokerage commissions.

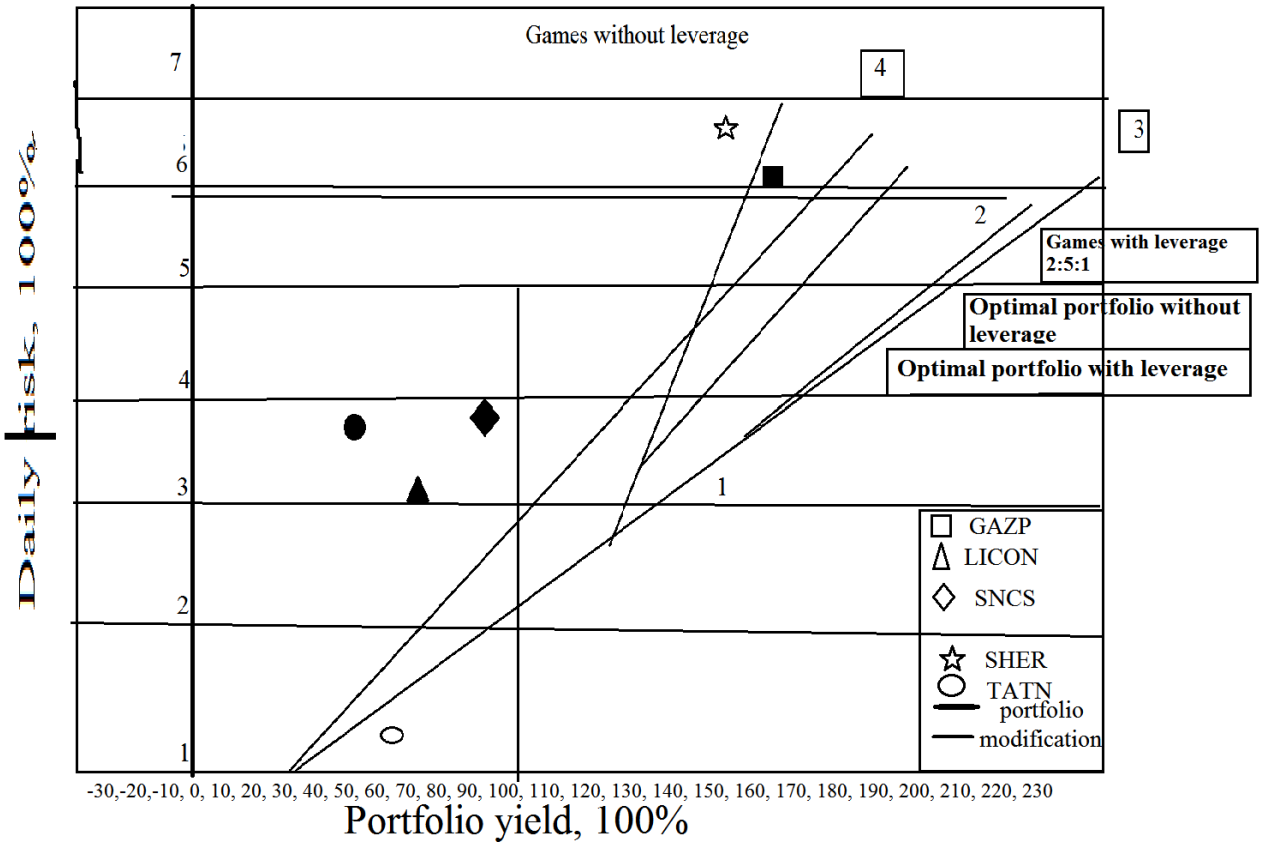
3 Increase the ratio of return / risk based on historical data can be achieved by increasing the options trading system, so at the expense of reliability.

The best results on the ratio of the yield / risk to the game for each of the shares in the portfolio are shown in Figure 2.2.

Calculated using the theory of Markowitz-Tobin portfolio front as shown in Figure 2.2. (a curve of parabolic shape). The curve of the parabolic shape - effective front corresponding optimal portfolios consisting only of shares "Lukoil", JSC "Sberbank", JSC "Surgutneftegaz", OAO "Gazprom", which shares matched by the theory of efficient portfolios Markowitz-Tobin so as to have the minimum standard deviation of the set of return (risk). Also noted the amendment effective front service short-sale, arising from the need of perevlozheniya borrowed less profitable assets

into more profitable with a yield higher than 130% per annum. The possibility of a risk-free placement of funds with a yield of 20% per annum determines the futility of optimal redistribution of assets in the portfolio, consisting only of shares in the direction of decreasing profitability (risk reduction) is lower than 105% per annum.

Graph 2.2.



Of the optimal portfolio having a yield of 105% and a risk-free deposit with a yield of 20% per annum, you can create a family of portfolios with the front in the form of interval 0 - 1, with much less risk.

If possible, the use of leverage to the cost of servicing 20% per annum, the optimal redistribution of assets in the portfolio of shares in the direction of increasing profitability (increased risk) than 105% per annum and is not justified. In this case, less risky to work with the best portfolio of stocks, which also has a yield of 105% per annum, using leverage, the lever of which depends on the desired yield: front 1-2.

At any annual period of time when using a system that provides the maximum value for the yield / risk, short fat is carried by about 40% of the portfolio. This forms 80% of the available funds. In short positions, we are about 40% of the time investment. Being on the outside beztrendovom market position in 25% of the time investment. During this process 100% of available funds.

Thus, the average daily balance of available funds - 57%. Under these conditions, the optimum is to work with a shoulder 12.3 (43% of funds invested) which corresponds to the expected return 216% per annum. With this shoulder average daily balance in the MICEX is reduced to zero. The ability to place output from the stock system balance risk assets slightly reduces exchange rate risk (from 4.5% to 4.3%) at the rate of 20%, but can significantly reduce the currency risk, if not used due to the possibility of shoulder 57% of the funds placed in the foreign currency assets. Sudden loss on the shoulder with a 2.3 probability of 95% can not be more than 4 standard deviation or 17% of the funds held in the MICEX. With the gradual losses are likely to beztrendovom market, profits decline below the average prevented the liquidation of all assets in the portfolio, but continues to maintain a virtual portfolio. Reverse entry into the market is carried out when the theoretical portfolio of the average income.

Analysis of the graph (2.2.) Shows that when working on the market the most liquid stocks of Russian companies need to focus on yield 216% annually, working with efficient portfolio having a yield of 105% and the risk of 2% from the shoulder 2.3. Required investor limit the risk (and the concomitant decline in yield restriction) effectively no reallocation of funds into less risky and therefore less profitable paper and does not decrease the shoulder, and a decrease in

limit of funds allocated in the MICEX.

Failure to use the arm to limit the possible return of the portfolio to 130% at the same time increase the risk by 10%. Getting higher yields require abandon diversification and work only with the shares of "Gazprom" and the risk-free contribution (segment 0 - 4), the risk increases by 43% compared to the optimal risk-diversified portfolio with a shoulder at the maximum possible in this case yields 175% per annum. Optimal portfolio using leverage can achieve profitability in the

240% per annum for the same risk as an asset as possible without the use of shoulder yield 175% per annum.

It should be noted that the term "arm" refers to the maximum possible leverage, ie limit on the value of funds used in the game to their own facilities within the system of the MICEX. In practice, due to the possible location outside positions on one or more financial instruments, the presence of unhatched arrived shoulder size smaller correspondingly less maintenance costs shoulder. Thus, the possibility of placing funds in safe (risk-free) return on assets and the opportunity to use leverage (borrow) can significantly reduce the risks of working with a briefcase. To summarize, it can be concluded that the expansion of the methodological device portfolio theory to include methods of technical analysis in the mechanism of formation of a portfolio of financial investments can help improve the process of diversification of investments, taking into account the ratio of "return / risk".

2.2. Active strategy of portfolio investment (the results of the empirical test of the Markowitz theory)

One of the most urgent problems of modern portfolio theory is the task of forming a portfolio of financial investments. Methodology for creating a portfolio of financial investments began to emerge in the twenties with the advent of the concept of the "true" price (Fair Prica) shares. Usually distinguish two types of strategies to achieve the portfolio: passive and active.

Separation of modern portfolio investment strategies for passive and active originates from the work of W. Sharpe[41] and J. Tobin[42]. Tobin noted that the market portfolio, ie. E. Aggregate of all existing at the moment investors of securities is effective. Moreover, any combination of the market portfolio with the risk-free asset yields again efficient portfolio that has less risk, although with less expected return.

[41] William F. Sharpe. (1970) *Portfolio Theory and Capital Markets*, New York: McGraw-Hill Book Company. Available from: http:// www.wsarpe.com

[42] Donald D. Hester, James Tobin. (1967) *Risk Aversion and Portfolio Choice.*(New York: John Wiley and Sons. Available from: http://cowles.econ.yele.edu.

Must be borne in mind that such output Tobin made on the basis of theoretical assumptions, which do not fully correspond to the situation prevailing in the domestic stock market. In particular J. Tobin suggested that the market is in equilibrium (ie,. E. All investors to complete their portfolios ended), operational and transaction costs (cost of intermediaries, staff, information services, taxes, etc..) Are insignificant, the investor has able to receive and make loans to the same risk-free rate, and so on.

Nevertheless, the idea that the market portfolio is probably the closest to an effective marked the beginning of a passive strategy of forming a financial investments portfolio. This strategy means that an investor in the preparation of the portfolio to determine its expected yield focused entirely on the market portfolio and a little on changing the composition of the portfolio after its formation. An investor who uses a passive strategy, minimizes costs market research and the formation of the portfolio in the presence of a sufficiently high guarantee of stable profitability.

Must be borne in mind that the investor following passive rate should perform periodic inspection and change the composition of its portfolio. The market is constantly changing, so the weight of the individual securities in connection with a change situation may change so the difference between the expected income from the income investor market portfolio can be significant. In addition, in view of the difficulties that arise in calculating the yield of the market portfolio (which, recall, must include all of the shares, traded in the market), in practice, the yield is calculated on the basis of a market index, which does not reflect the behavior of the entire market and introduces some error.

In fact, an investor in the preparation of their portfolio is focused on some benchmark portfolio (benchmark portfolio), ei such the yield of which is the reference compared with the real yield of the portfolio. And it does not necessarily regarded as a reference only the market portfolio. There are dozens of different investment funds, called "index" funds, which focus on the benchmark portfolio consisting of securities, selected according to certain parameters (for example, a part of any index). So, there are index funds holding securities in any one industry, such as energy, telecommunications and so on. D. Fairly widespread funds that invest only

in securities of young growing companies, or in fast-growing share of large industrial enterprises with large capitalization. Number of funds focused only on paper bringing high dividends.

Passive strategy is typical for conservative and moderately aggressive investors. As the main objectives of this strategy are the investment protection against inflation and a guaranteed income with minimal risk and low management costs.

The main objective of an active strategy is obtaining a yield higher than the benchmark portfolio yield. It is a pre-defined characteristics of the benchmark portfolio. Next is a comparison of the market portfolio yield with the reference. Since, according to the capital asset pricing model Sharpe-Litner, the expected return of the portfolio is completely determined by its market risk, ie its beta, so for adequate comparativeness we must assume that both portfolios have the same factor. Based on the fact that each security in the portfolio provides its share of the overall coefficient equal to the product of its beta coefficient on the part of the portfolio, this assumption is tested.

Active strategy involves careful monitoring of the market, rapid acquisition of financial instruments that meet the investment objectives, as well as a rapid change in the structure of the portfolio. The main feature of the active strategy is to seek an investor to beat the market and get a yield in excess of the market average.

This type of strategy requires significant costs associated with the preparation of the information and analytical solutions, acquisition or developing its own technical and methodological support. Significant costs that are typical for an active strategy, driven by the need to ensure market activity and access to the systems of exchange and OTC trading, transaction costs, creating a network for the "purchase of the shares, and so on. D. This type of strategy can be selected only participants who have sufficient own capital, professional staff, as well as significant experience in managing your own portfolio of securities and trust management portfolio of clients.

The basic requirement for an active strategy of building a portfolio of investments favor the elimination of problems associated with the use of the existing normative methodological approach. However, the main problem of the concept of portfolio diversification acts incorporated in them initially nonremovability affecting

all, without exception, assets market risk. Thus, the main objective of finding and developing the required models of investment portfolio should be a technique which allows to minimize the risk and market risk.

Basis for studying the possibility of establishing effective active strategies should make a qualitative study of the existing lack of effectiveness of passive methods, as well as a detailed analysis of the reasons for the existence of such inefficiencies. The basis of this research is the analysis of some critical assumptions of classical portfolio theory, the consequence of which is the principle is possible existence of non-diversifiable risk.

The main controversial assumption of modern portfolio theory seems the original formulation of the G.Markowitz problem discussed in the first chapter, according to which[43]:

• The market is efficient, and the price always corresponds to their fair value; due to the accessibility of information and the same interpretation of its investors.

- All investors have the information about the potential risk and yield securities, which is measured as a continuous random change in the value of the asset shall be distributed according to the normal law. All of the available set of assets estimated by the investor for making efficient portfolios, the total correlation of assets in which

cancel each other out, which fully protects the portfolio against the risk of individual securities. Correlation of the portfolio in an efficient market at are recognized constants.

• The investor positioning at the time T0 sum S puts it in the optimum for itself a diversified portfolio. The maximum available level of diversification and therefore the optimal combination of investments is the market portfolio (consisting of all potentially available in the market assets). Optimal for the investor is recognized according to his requirements to the risk / return combination of the market portfolio / risk-free lending or borrowing.

In fact, these conditions are clearly indicated on hold investor throughout the investment horizon of the so-called "long" position, or, in other words, using the

[43] Sharpe, Alexander G. and J. Bailey. (1997) *Investments*: Moscow: INFRA-M. Per. Translated from English.

strategy of "buy and hold" in relation to the market portfolio, which is the basis of passive index style. Getting to the end of a certain time period of return on such an investment portfolio of the index directly proportional to the increase in value of the assets constituting the index for the period. Thus, we can assume that, guided by a certain logic, the investor has a tendency to always rely on the fact that the value of the assets in the future will continue to grow, the risk is that the "entire market will fall" or market risk taken by the investor as the inevitable cost for participation in the investment process. In other words, the investor at each time interval consistently predicts the value of assets on the market, in the case if such a forecast was always negative even theoretical use passive index strategy of "buy and hold" would appear, at least, strange. Meanwhile, according to the initial assumptions stated above G.Markowitz problem, change in return on assets are distributed as random continuous quantities; whereby is meant the inability to predict their future values.

As can be seen, two essential conditions for the functioning of modern portfolio theory g somewhat at odds with each other, which confirms the need for empirical research on their compliance with the conditions of a real investment. As the object of the prototype used the Russian market portfolio of shares, a role which made the RTS index.

In the first stage of review reasonableness of investment expectations historical tendency to increase the market value of Russian stocks.

The test showed the following results:

• positive weekly return was observed in 201 cases, accounting for 52.34% of the total number of observations

• negative weekly return was observed in 183 cases, accounting for 47.66% of the total number of observations.

From a statistical perspective, a 5% difference in the observed outcomes, we can neglect, this distribution can indeed be described by the normal law underlying the approach G.Markovitsa. However, the next question is - how it is logical from the point of view of the rational investor to hold a "long" position on a fully diversified market portfolio, while about half of the cases, this portfolio has historically demonstrated a negative deviation value, thereby materializing an inherent market

risk diversification. But according to the same normal distribution positive and negative changes do not necessarily have to change whenever one another, so that may watch the whole series of independent changes of a positive nature that historically and lead to an increase in the value of assets in the long term - may argue adherents of the index model.

On the second stage of the analysis of the dynamics of the RTS index as the market portfolio, is the search for and evaluation of the impact of the general nature of exchange rate variations such unidirectional series. Under unidirectional series of changes weekly returns of the market portfolio will be understood at least 3 consecutive weeks unidirectional deviations. The test showed the following results:

• seen as a positive one-way series, and their negative counterparts, the total of all of the observed series was 55

• a series of them with a positive yield observed 30 cases, accounting for 54.54% of the total number of observations

• of those series with a negative yield were observed in 25 cases, accounting for 45.45% of the total number of observations.

Again, the results were shown historically almost equal, indicating practically equiprobable possibility of long periods of decline in the value of the market portfolio. From which we can conclude that the decline in asset prices in the whole market is quite a natural phenomenon, and continuously hold a long position in the market portfolio of assets in anticipation of its growth can not be considered rational behavior of the investor. When the value of the assets falls, so does the value of the portfolio, what an investor is hardly interested.

Consequently, the underlying in G.Markowitz task condition initial purchase and continued throughout the period of retention of efficient portfolio investment is not always the best investment decision on the Russian stock market, however due to lack of whatever meaningful basis for forecasting the exchange rate constant growth.

The next step in the analysis is to assess the conformity of the actual conditions of the following for the original problem G.Markowitz market efficiency, random changes in asset prices and their static correlations. To perform this analysis problem we calculated the coefficient of correlation of the dynamics of the current weekly

return RTS from its previous changes in the period 2000-2008. In fact, the results of this comparison were obtained by examining the correlation coefficient of the number series index changes with itself, shifted in time by one by one week, which allows to evaluate the degree of dependence of the current incremental yield from the previous. In the case, the effectiveness of the Russian stock market, the presence of a statistically significant relationship, in principle, it would be impossible because of the random nature of the distribution of increments.

The test showed the following results:

• for 63.71% of the time value of the correlation coefficient does not exceed 0.8, and did not fall below 0.8, thus showing no statistically significant relationship of current market changes from the previous, which fully complies with the terms of the efficiency of the market (value of 0.8 is used for clarity, ideally independence is the zero correlation, and the value of the coefficient generally observed in pure form rarely seen)

• for 36.29% of the time value of the correlation coefficient greater than significant level was 0.8 or below -0.8, indicating a right-dependence of the distribution, the increments in asset prices, which in turn points to the "inefficiency" of the Russian stock market these. periods.

• 83% of previously identified series of unidirectional change coincided with periods identified correlation index increment corresponding to the conditions of market inefficiencies.

• As you can see, the correlation coefficient itself is also not absolutely static value, demonstriruya- fluctuations of values from - 0.83 to 0.94, which, in principle, perhaps, should not occur on an "absolutely an efficient market."

Consequently, in certain periods of the Russian stock market meets the requirements of efficiency, and in; some was ineffective. Assessing the dynamics of the correlation coefficient series returns, it can be assumed that the degree of effectiveness as well as the correlation is not static and tends to change depending on market conditions. Thus, the application of modern portfolio theory, which is based on the condition of the effectiveness of 'the market as a constant value can not be considered' rational for a long period of investment, due to the fact that in practice,

the degree of efficiency of the market is changing dynamic quantity (the introduction of a number of foreign researchers the terms "low", "moderate" market efficiency confirms the results obtained). Moreover, periodic moments of coincidence connectivity incremental market rates, with the time series of their unidirectional movement suggests the non-randomness of major market movements and, as a consequence, the emergence of predictive capability in asset prices during such periods.

Results of the analysis of some critical prerequisites of modern portfolio theory, allow you to modify the essential terms of the functioning of a possible model for the formation of the active investment portfolio and to introduce its formalization new, more realistic look at the author's assumptions, then you can proceed to the very stage of modeling. New admitted assumptions will be based on the contradictions of practical conditions of the investment environment, the conditions of functioning G.Markowitz task. The basic assumptions of the active model will be the following assumptions:

• The investor operates in a dynamic market efficiency

• The investor does not always act rationally, relying on a passive strategy of modern portfolio due to the fact that the shares do not have a pronounced tendency to historical growth

• On the basis of the first and second assumptions, the investor has to predict the future changes in prices of assets, ie apply some active strategy

• Applying some active strategy, the investor does not always have all of the available market information, as well as by all available means of its interpretation (prediction) due to the fact that the efficiency of the market is a dynamic quantity

• It is possible now proceed to a qualitative modeling.

As already noted, the basis for the search for active strategies must address list management methodology in the theory of non-diversifiable G.Markowitz risk. Theoretical possibility to completely eliminate market risk arises when one of the following two conditions.

• Attachment of all capital invested in the investment in unrisky asset (the investor eliminates activity under conditions of uncertainty, which in practice is implemented in the form of disengagement from

investment in risky assets of the stock market)

• Simultaneous with the occupation of "long" positions of the sales of securities "uncoated", better known as the occupation of "short" positions.

And if the first condition does not imply any investment activities, in the second it is necessary to elaborate. It is logical that seeks to act rationally investor should be aware that a series of negative returns in the market are quite a natural phenomenon, as eto- was shown earlier, and therefore use the mechanism of sales without the "cover" in order to maximize the effectiveness of their investments. Let's see how the mechanism of the "short" eliminates both own and market risk: taking both "long" and "short" positions in the same price for the same asset, in, the absence of transaction costs, the investor does not bear the risk of absolutely nobody:

• Growth in the value of the asset by the amount of + X% leads to profitability on the "long" position + X% and loss on the short to X%

• Loss of value of an asset by an amount to X% leads to a loss on a long position to X%, which is fully offset by gains on short + X%

• As a result, whatever the direction of the price movement of the asset risk is assumed by the investor are zero.

This is an absolute diversification of investments, and there is no need to collect any portfolio in general, as a result of profitability on the occupied position will have full inverse correlation as one paper and the portfolio as a whole. In this case, in contrast to the diversification G.Markowitz no 'residual market risk component during these operations remains, which means that the market risk in its modern sense portfolio theory completely eliminate. The only question is, what its complete elimination is achieved due to the complete elimination of the potential yield that, as a logical outcome of the absolute degree of diversification, resulting in a loss of economic sense to invest in the stock market. Therefore, investors focused on the increase in the value of its assets in investment decisions in any case have to decide, "long" or it will be "short" position. Meanwhile, the main risk in the case of

employment korotkoy- positions in diversified index portfolio will be the overall market growth, not "drop the whole market," as the most commonly accepted interpretation of market risk in the theory G.Markowitz. Consequently, the term market risk in the classical sense is losing relevance, transforming into a category tion of potential profitability of investments, because the investor has the opportunity to earn extra income through short positions should it materialize.

Consider what category to perform the replacement used in modern portfolio theory market risk categories. With the potential to completely abandon the risk, the investor still, in an effort to obtain the required rate of return, will make his bid for a "long" or "short position".

As you know, in the end, the entire risk of the investor's equity will depend on the choice of type of bet (for "long" / "short" position) and its size, showing how a particular version of events in the market is the investor, as the most probable. The type and size of the tax rates are determined based on the algorithm used, decision-making, or, in other words, the prediction method used. Consequently, the overall risk of loss in active investment depends on the efficiency of the method used by the investor, and not from the abstract, as a result, and unmanageable category "market." Thus, the concept of ownership, market risk, as well as the potential return, modern portfolio strategy G.Markowitz modified into potential risk and return used in investing method for predicting market movements in the case of an active strategy of portfolio formation.

Indeed, in the case of dynamic efficiency, market investor has some capability to assess the likely future market movements in the form used by the forecasting methods and theory can properly evaluate things as growth, and decline of the entire market as a whole. In this case, the potential investor's income will directly depend only on his analytical skills in the form of the potential effectiveness of the algorithms used in the analysis of investment decisions, and the only thing that really runs the risk of the investor - the likelihood that the resulting forecast is not justified in view of the fact that the the current level of market efficiency, the method used to be unworkable. After all, as has been shown previously, fundamental and technical approaches to financial analysis repelled from vastly different assumptions and may

show varying degrees of effectiveness depending on the changing market conditions.[44] Nevertheless, in these circumstances, the investor has the right to decide which of his famous prediction methods will be most effective in the practical application, and thus to decide how it would be appropriate for the level of risk characteristic of this method.

Thus, the risk of the portfolio in terms of the active form becomes a function of the quality of the used method of forecasting, and not from the unpredictable behavior of assets on the market in the case of the formation of a passive portfolio. In this formulation of the problem all the risks faced by acting in conditions of uncertainty the investor, absolutely manageable, which was a prerequisite for the required model in this paper and will be clearly shown.

Highlighting the main idea of the model of active portfolio formation, we proceed to its quantitative description of a mathematical interpretation. In order to further correct comparison and evaluation of the effectiveness of the desired model with its passive analog, mathematical description of the model will be carried out entirely by analogy with the entered by G.Markowitz approach.

With the currently (T_0) some capital (S) the rational investor determines the purposes of its investment policy for a future point (T_i), for which the proceeds to the evaluation of historical effectiveness available to him at the moment of forecasting methods (M). The logic of the approach is that a rational investor would not be inclined to invest in the framework of the investment objectives, historically used unattainable M. In this case there are a number (N) for M, and N in the long run tends to infinity, due to the fact that the investor free to invent new or used absolutely any existing M and their infinite combinations, in their quest to find the best, the best. However, it should be noted that at the time T_0 investor has only 0 to N (finite in number T_0) M, due to the fact that the efficiency of the market is a dynamic quantity (at any given moment, each individual investor has a limited number of M to choose from, and not the fact that other investors use or even know about them.)

[44] Haertfeld M., E. Lozovskaya Hanusch E. (2005) *Fundamental and technical analysis of the stock market.* - St. Peter. Series "The Academy of Finance".

The evaluation of all available M investor is based on indicators of potential yield (R_m) and risk (V_M) associated with the use of each M. Rate these figures can only be based on actual historical results of the application of M in the form of cost dynamics managed by him S. Accordingly, the rate of return M perform ratio similar return on assets is the expectation of return method:

$E(R_m) = p_1 * r_1 + p_2 * r_2 + ... + p_n * r_n$

$E(R_m)$ is expected return M;

wherein $E(R_m) > 0$

It should be noted that all methods that have a negative expectation of return should be immediately excluded from the number used by the investor at the time T0, as a rational investor is not inclined to invest S according to a losing option, however, in the future the investor can again test your M on the performance, due to the fact that the expectation of efficacy in a dynamic market can also be a dynamic variable.

Meanwhile, as a measure of evaluating the risk M G.Markowitz embodiment proposed can not be used due to the fact that the dynamics of S applied to the standard deviation will show a mean value of both negative and positive value of deviations S, while real risk taken by the investor are the only negative deviations. But also to use the average value of negative deviations will also incorrect in view of the fact that at low average indicators there is likely to suffer significant largest single loss. For example, if the average negative deviation S to 10% per year in the early period, the deviation may occur -50%, whereupon the value S is not changed until the end of the year, bringing the investor loss, half of the capital.

Thus, as an indicator of the risk of M will make the implementation of the worst historical version of a maximum historical incidence of S on its maximum value during the period, because the only thing that really interests the investor, how much can decrease the maximum value of its investments:

$$V_M = MIN(S_{m1} - S_{m0}) / S_{m0} \qquad (2.4.)$$

V_M is risk S managed by M;

with $S_{mn} < S_{m0}$

S_{mn} is the cost of capital managed by T_0.

Knowing that in a dynamic market efficiency, due to heterogeneity of the assumptions underlying the concept of any M, there is a risk of instability practical effects of each individual M in period T_0-T_1 investor will seek to use several M, or, in other words, to form of M portfolio (MP). Continuing the approach by G.Markowitz, total return S, managed by MP will be expressed in terms of total expected return of each M, according to its share in the total set:

$$E(R_{portf.M}) = \sum W\, E(R_m) \qquad (2.5.)$$

$E(R_{portf.M})$ is yield of portfolio M_p

Now we define the cumulative risk of MR. It is logical that in these circumstances, the investor will seek to develop a portfolio of absolutely independent of M. Indeed, if in a given market situation one M show low efficiency, then a portfolio similar to him on the content of M will do the same. In this case, you must enter the following essential conditions for the share of each method used in the portfolio:

$$W_M > 0 \qquad (2.6.)$$

Indeed, the share of M is zero means that the method is not used at all, and the negative - that is to be used on the contrary, that in fact is already completely different M, which should be considered separately for inclusion as a positive share Mr.S purpose of selection independent of M, as the concept of G.Markowitz, the investor expects total correlations yield curves S managed on a case known to him M, with the aim of finding such combinations of M, overall correlation coefficient which will tend to 0:

$$PS_{Ml}S_{M2} = CovS_{m1}S_{m2} / (\text{б}S_{Ml} * \text{б}S_{M2}) => 0 \qquad (2.7.)$$

And although the concept of G.Markowitz is optimal to have a negative correlation in the case of M_p full negative correlation means that the profit of one that used M_1 will be offset by a loss on the M_2, in view of the fact that the estimated correlations yield curves Managed S. In the same time, a set of completely independent M with a positive mathematical expectation of profit will allow the investor under various conditions of market conditions to manage a portfolio with a set of operating independently of M and acting at the same time successfully. Theoretically, the performance of all the above conditions solves the problem of

instability of the practical application of the results of each individual M for varying degrees of efficiency of the market, while allowing N, tends to infinity actively manage S, completely independent of the risk of falling efficiency of one particular M.

In other words, it is possible to obtain a set of combined active strategies M_p, spread aggregate results of which is characterized by index dispersion curve of capital S:

$$б\ 2Mp = \sum W_2 S_m б2 S_m + 2\sum WS_{M1} WS_{M2} (pS_{M1} S_{M2} бS_{M1} бS_{M2}) \qquad (2.8.)$$

б 2Mp is dispersion S managed by Mp

In which, when the condition (2.7.) expression $2\sum WS_{M1} WS_{M2} (pS_{M1} S_{M2} бS_{M1} бS_{M2})$ will tend to zero. Thus, the dispersion of the scatter results M_p takes the following form:

$$б\ 2Mp = \sum W_2 S_m б2 S_m \qquad (2.9.)$$

At the same time, the share capital on one W M, with a potential increase to infinity as N tends to zero, so that б $2M_p$ and will tend to zero.

This conclusion can be interpreted as follows: the use of the investor, subject to an infinite number of prediction methods may pick up the use of such a portfolio of methods with zero total correlation, that the spread of possible returns around the expected average value will tend to zero, while the average value of the expected return is positive. In other words, the total return of the portfolio will seek methods to its mean, that in case of positive mathematical expectation of all samples in the portfolio means the following:

• The yield curve S managed by M_p is growing all the time without hesitation up or down (the variance is equal to zero), and hence S at T_1 can not be less than S at T_0 as required in (2.4.) Which means the complete elimination of the investment risk

• The yield curve S will strive to achieve the highest possible level of expected returns, which means that the complete elimination of the investment risk at the same time there is an additional to maximize the efficient use of the capacity of profitability M_p.

Thus, under certain assumptions, can be a model for the construction of the active portfolio formation, which can eliminate all investment risk.

Unfortunately, this result could theoretically be obtained only under ideal conditions, when the number of possible methods do all the time tends to infinity. However, the operating conditions of the active model was defined by a finite number of at the time. Then, by the dynamic efficiency of the market, and the purpose of the work is the search of a model as close to real investment environment.

Therefore, the assumption of an infinite number of methods to construct a working model can not be used. The second critical baseline condition complete removability of risk is a necessary total zero correlation coefficient methods in M_p, despite the fact that under these conditions the dynamic efficiency of the market is also a factor can not be static "value of zero. However, according to the idea of active portfolio formation, this assumption must remain working - the investor seeks to act rationally in the face of uncertainty is obliged to constantly check a set of methods used to conform the optimal criteria. That is realized in practice in the regular review of M_p, in order to achieve the required quality of the independence of its member M. Because it involves the active formation of active operations. Therefore more correct to keep the variance of S in the form б $2M_p$ = $\sum W_2 S_m б 2S_m + 2\sum WS_{M1} WS_{M2} (pS_{M1} S_{M2} бS_{M1} бS_{M2})$.

This shows that for a finite number N of M methods yield curve will be subject to certain fluctuations, including down, because of the problem (2.4.) Is an implementation risk. However, these fluctuations and, therefore, included in their risk is a function of the independent relevant indicators for each M, and do not depend on the volatility of market efficiency, which was probably due to the total zero correlation M_p.

Thus, the risk of the portfolio becomes a simple function of the risk of individual constituent M:

$$V_{\text{portf. M}} = \sum WMIN_M (S_{m1} - S_{m0}) / S_{m0} \qquad (2.10.)$$

$V_{\text{portf. M}}$ is risk S managed by Mp;

with $S_{mn} < S_{m0}$

In the case of the condition of general uncorrelated and positive profit expectations M in the portfolio, all available to the investor M_r having these characteristics make the working set M_p, which will be carried out to select the most effective of the available options.

All M_p, which is not part of the set, it should be recognized as non-working, since the increased risk of loss of efficiency in the case of changes in market characteristics. It should be noted that the resulting set is not working in terms of modern portfolio theory, efficient, due to the fact that all independent Mp may have a different balance between profitability and risk, ie different outcome measures, efficiency. Measured as the ratio of the actual yield and the risk of Mp will be the sum of the weights of the specific performance of each individual M, which stands meter modified factor V.Sharpa:

$$K_{eff} = R_{portf.M} / V_{portf.M} = \sum WR_m / \sum WMIN_M (S_{mn} S_{m0}) / S_{m0} \qquad (2.11.)$$

As can be seen, the indicator will show what the average yield Mp provides one unit of risk realization of the worst historical cumulative incidence of capital managed Mp. Investor evaluates the effectiveness of the available working set him Mp, with the result that all portfolios with maximum K_{eff} make optimal set of solutions.

$$M_p\text{-opt at MAX } (K_{eff}) \qquad (2.12.)$$

In view of the fact that all M_p characterized by maximum K_{eff}, can have different levels of R-and portf.m VIIopT (j, M, challenge the decision of the investor will be identical to that solved in G.Markowitz theory, the optimization problem of mathematical programming to find the version of M_p with maximum K_{eff}, ensuring maximum profitability for this at risk or the minimum risk for a given level of return.

However, the final similarity of problems solved in the final stage of active and passive models of portfolio-management does not mean equal results application. We show that the essential conditions the functioning of the active model, the current G.Markowitz model is not optimal.

Underlying in modern portfolio theory the need to buy and hold a diversified portfolio,[45] says that investors tend to anticipate the total growth in assets which make

[45] Sharpe, Alexander G. and J. Bailey. (1997). *Investments: Per. Translated from English*. - Moscow: INFRA-M.

up the portfolio throughout the investment period. The uniqueness of the expectations and actions of the investor speaks about the uniqueness of the method used, the prediction is realized in the irrational, at least for the Russian stock market, an effort to buy as a constant value. Consequently, in practice M_p generally not formed, or theoretically formed, but only the only possible identical M_x. Logically, created for, the conditions of absolute efficiency of the market, this will not be possible M_x is efficient in the case of market failure, that is exposed to the risk that the degree of efficiency of the market as dynamic values change.

In this, the problem statement, the condition (2.7.) Required to eliminate the risk of possible failure of one M in periods characterized by varying degrees of dynamic efficiency of the market, is not performed due to the fact that the correlation between identical M_x portfolio is equal to one. Consequently, this M_x is exposed to changing market conditions, and thus, not even in the working set is completely independent methods with positive expectation. Due to the fact that the optimal set of methods defined in the working set immunized inefficient market M_p and Ml there, as shown above, is not included, we can talk about that M_x, and hence the current model G.Markovitsa in a dynamic market efficiency 'is not optimal, as required.

In this case, you can talk about the fact that the basic requirement of the desired active "model is also satisfied: the portfolios managed by the best M_p exposed only to the risk, which are, in themselves constitute their M, regardless of the conditions in which the investment activity. Investor free yourself decide whether to accept it or not this risk. performance is unacceptably optimal M_p investor may withdraw its application, and try to find a more effective option through the development of new, or the use of old, unknown dosih him M. "In this case, the uncertainty and inherent market risk in the case of modern portfolio theory does not threaten him in view of the fact that the investor known increased risks of using this model in periods when the market is not completely efficient, and likely fall in the value of assets on the market is not unexpected, and more that can be used to recover additional yield. The presence of their risks for each individual M actively managed portfolio is a natural means of different underlying assumptions about the degree of market efficiency, made during its development. Acting in the active model, investors will constantly

strive to find new and more effective M, and their combinations uncorrelated M_p, potentially leading to an increase in the number N of M used, but as a consequence of the problem (2.10.), with potentially continuously increasing number of M in the portfolio, the entire investment risk is potentially continuously decrease.

In practice, this approach is used often in the form of the use of a portfolio of automated trading systems technical analysis. The number of such systems is increasing all the time (tends to infinity) due to the fact that investors are constantly optimize the old and invent new algorithms for obtaining the yield above the market average.

Thus, this study showed that the modification of the basic provisions of the portfolio theory by introducing more realistic assumptions, based on the practical modalities of the Russian stock market, to minimize the impact of non-diversifiable market risk in the formation of an investment portfolio.

3 METHODS OF IMPROVEMENT AND FORMATTING THE FINANCIAL INVESTMENT PORTFOLIO

3.1. The algorithm for generating a Pareto optimal portfolio of financial investments.

Formation of a financial investments portfolio is made in a certain logical sequence. In our opinion, this sequence can be represented by a seven steps (stages).

At the first stage, defines the priorities and criteria for the financial investments portfolio formation based on the objectives of technical and innovative company development, as well as specific tactical conditions of the market environment.

The second step is finding the investments variants in financial assets for possible implementation. This process should be carried out regardless of ability to mobilize financial resources, financial market conditions and other factors. Adverse external and internal conditions of portfolio investment should not interfere with the search and consideration of new options. Database for portfolio investment in financial assets must always greatly exceed the opportunity to complete their resource provision. The more actively organized the enterprise search of financial assets available for investment, the more it provides a financial investment alternatives. All of this increases the possibility of the formation effective financial investments portfolio on this basis.

At the third stage, is the pre-treatment of certain financial assets, in order to create an information base for subsequent careful examination of financial investment portfolios.

Further, in the fourth stage, conducts a comparative assessment of the possible financial investments portfolios and their primary selection for a more in-depth analysis. The selection is based on certain system important for enterprise performance. For a comparative analysis of the value of portfolios some individual indicators appropriately standardized. The most attractive and efficient portfolios identified in the analysis became a subject to further in-depth examination.

For portfolios that have passed initial screening, at the fifth stage, is occurs the final their forming. It requires a more thorough analysis and detailed evaluation of technical, economic and financial performance of financial investments.

In the sixth stage, based on the values of key parameters of financial investments portfolio, held the exploration of the options for effective investment portfolios. This is an important step in the whole decision-making process of building a financial investments portfolio, so it will be discussed in detail below.

At the final stage, the seventh, makes the final decision on building a financial investments portfolio based on variants of efficient portfolios, generated from alternative portfolios, as well as unformalized information.

The advantages of the proposed phased approach to the financial investments portfolio formation is the best organization of investment planning (organizational aspects) and the possibility of a gradual build-up of efforts and costs allocated to the implementation of this process (economic aspect).

The greatest difficulty in the process of a financial investments portfolio building is the sixth stage - the search for effective options portfolio, which consists in determining the composition of the financial assets in the portfolio. Theoretically, all portfolios of financial investments, leading to an increase its economic value (EVC) should be taken to implementation (if they are not mutually exclusive). But, in practice, the investor`s operation is occur in the conditions of strictly limited capacity to attract financial resources from various sources. Under real conditions, investors often face the challenge of rationing the use of available capital.

Thus, the process of finding effective options for a financial investments portfolio is to construct an algorithm portfolio formation, which would be described quantitatively by means of action and the corresponding results of the operation.

Based on the above, we have developed and propose an algorithm for the financial investments portfolio formation. Algorithm is based on the mechanism of directed search Pareto-optimal options of financial investments portfolio. The proposed algorithm is intended to determine the effective range of financial investment ratio "return / risk" and the volume of investments (for well correlated portfolios).

In developing the algorithm, we proceed from the assumption that the target criteria for the formation of a financial investments portfolio are: the integral effect (gain) on disposal (NPV_b and generally $SNPV_i$), the value at risk (RI_t) and the amount of available financial investments (PVI_t). As a measure of the latter can be used by the standard deviation of the portfolio effect, which will provide cost measurement of this parameter, as well as the first two characteristics of a portfolio. Using the same indicator $SNPV_t$ to assess the effect of portfolio allows taking into account the strategic importance of financial investment for the company (through inclusion in the analysis of the portfolio invested in the development of options). For all projects on the basis of these characteristics can be calculated profitability ratios ($SNPV_t$ / PVI_i) and variations (RI_i / $SNPV_i$).

Thus, the formation of a portfolio of financial investments can be represented as the following target selection criteria:

$$Д_p = \sum (x_i SNPV_i) \to \max, i = 1,...n; \quad (3.1.)$$

$$R_p = f(x_i RI_i) \to \min, i = 1,...n; \quad (3.2.)$$

$$I_p = \sum (x_i PVI_i) \to \min, i = 1,.., n, \quad (3.3.)$$

where D_p, I_p, R_p are target selection criteria (or income and risk of the portfolio, the amount of investments);

Xi = 1 if the project i is included in the portfolio, and Xi = 0 - otherwise;

n – is the total number of alternative financial investments portfolios.

In general, the risk of a portfolio of financial investments is equal to:

$$[\sum\sum (x_i x_j RI_i RI_j cor_{ij})]^{\frac{1}{2}}, \quad (3.4.)$$

where cor_{ij} - is correlation of income from assets i and j.

In the case of the independent sale of investment portfolio risk is equal to $[\sum (x_i RI_i)^2]^{\frac{1}{2}}$

However, in this setting, the proposed algorithm is focused only on the efficiency of financial investments without reference to the expected result and the boundaries of the production company.

Therefore, we believe that the proposed algorithm has to be supplemented by the provisions of the basic theory of boundary production functions. These functions

are intended to describe the relationship of the industry production functions and technological methods of production, potentially feasible at the enterprise level, ie .:

$F_0 (X) = \max\{F_k (X)\}$,

where F_0 – is boundary production functions,

F_k- is production functions of individual enterprises.

In other words, each resource value correspond some volume of production, which uses a number of resources in the most efficient manner. Empirical production border can be constructed on the basis of observations of the actual values of the output and costs of resources by allocating a subset of the observations set corresponding to the most efficient production methods[46].

From this definition, it follows that for option financial investments portfolio held (owned) in the framework of the empirical production frontier (in the sense of maximizing output and minimizing the cost of financial resources), there is another set of financial investments having a Pareto-optimal ratio "return / risk" [47] at acceptable volumes of financial investments.

However, this approach has a major drawback, which is the need to have a pre-set all observations (all possible variants of the portfolio). And that means, in relation to the search algorithm Pareto-optimal financial investment portfolios, even for a relatively small amount of alternatives portfolios this is not possible, since the number of "potential options sets of financial investment increases exponentially with a linear increase in the number of initial alternatives.

So, taking into account the expressed provisions can determine the sequence of formation of a Pareto-optimal financial investments portfolio for which we proposed, the algorithm:

• In the beginning, built reference sets of different options financial investments portfolios (call them sets portfolios first, second, third, etc., order);

• Further, by directional sorting options portfolios are arranged in descending order of their effectiveness (on the above proposed criteria);

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[46] Antonov M.V., Pomansky A.B. (1994). *Credit Rationing and the efficient allocation algorithm leveraged* // Economics and Mathematical Methods. T. 30. Vol. 1.

[47] Razumov I.V. (2001). *Economic relations and market dynamics in the Russian stock market*. - Yaroslavl: Yaroslavl State University. Razumov I.V. (2003). *Economic theory of "risk-rent": economic interests and "pentoorientirovannoe behavior of the stock market in terms of information differ between its market.* Yaroslavl.

• Then, remove from the set of first-order portfolios latter being the most inefficient in the corresponding set, and the first set is supplemented by the criteria of efficiency (ratio of "return / risk" and the amount of investment) of a set of portfolios of the second order;

• The procedure is repeated for the next set of efficiency and further to the point where either option selected portfolio will not dominate in the reference set of portfolios, or will not go through all the options for a portfolio of financial investments.

This procedure algorithm of a financial investments portfolio allows find all the Pareto optimal options investment portfolio on the basis of available observations on the dynamics of profitability and risk of the financial assets included in the portfolio. The main advantage of the algorithm is incorporated in its base directed search mechanism, which allows omitting the bulk of the known variants of inefficient portfolios (in other words, do not build all the sets of portfolios). This significantly reduces the time and simplifies the complexity of the calculations, which is especially important in financial investments decisions. As often a very wide variety of alternatives portfolios does not allow optimization modeling, even with the assistance of computer models. For example, even for the ten initial financial investment options being considered for implementation, will be received over a thousand possible portfolios (set in 1023) and 20 for the number of possible options portfolios exceed 1 million.

It is understood that the use of these conditions currently used empirical methods can not guarantee the achievement of the best results. The proposed algorithm is analytically allows to select the source (reference) set of portfolios to form a set of efficient portfolios, among which must make the choice of the best, a Pareto optimal financial investments portfolio.

However, it should be borne in mind that the efficient portfolios are often uneven in quality: on one of the particular criteria may be preferable to one efficient portfolio, and other criteria are very different. Therefore, to select one of the optimal portfolio of a set of efficient portfolios, in each situation it is necessary to use additional information about the purpose of financial investments, ie taking into

account the relative importance of individual criteria (remaining, in our work, formalized and therefore unused in reasoning).

3.2. Optimizing investment portfolio using VaR methodology

In the financial world, there are many techniques of risk assessment. Among them are the following: Value-at-Risk, beta analysis of the theory of CAPM, APT, Shortfall, Capital-at-Risk, Maximum Loss, and a number of other classical techniques. Some of these technologies are known for a long time, while others are just beginning to catch on in the banks, investment and insurance companies, pension funds.

To show how these techniques work in the risk assessment we will focus on technology of risk investment portfolio management, which, in recent times, is becoming increasingly popular among investors, - Value at Risk (VaR). For example, as noted in the study of New York University Stern School of Business, about 60% of pension funds, the United States used in its work VaR.

Essence Valueat Risk (VaR) is a clear and unambiguous answer to the question that arises on financial operations: what is the maximum loss incurred by the investor at risk for a certain period of time with a given probability? This implies that the value of Var is defined as the greatest expected loss, which is a given probability can get an investor for n days. The key parameters of VaR is the period for which the calculation of risk, and given the likelihood that losses will not exceed a certain value. For example, the standard for broker-dealer reports on transactions with OTC derivatives, transmitted to the Commission on Securities and Exchange Commission United States, are the 2-week period, and 99% of the probability. The Bank of International Settlements to assess banks' capital adequacy set the probability at 99% and a period of 10 days. JP Morgan publishes its daily VaR values at 95% confidence level.

Since the concept of value at risk (VaR, VaR) is designed to give a clear and unambiguous answer to the question raised in the operations of the financial markets: what is the maximum loss you can incur for a certain period of time with a given

probability for a given portfolio, it follows that value of VaR for 'portfolio of a given structure is defined as the greatest expected loss due to price fluctuations on the financial markets, and is calculated:

• For a period of time in the future (time horizon);

• With a given probability of not exceeding it (confidence interval);

• If this assumption about the behavior of the market (the method of calculation).

Confidence interval and time horizon are key parameters, without which there can be no payment, no interpretation of the value of VaR. For example, the values of VaR 10 million. Dollars.

for the time horizon of one day and a confidence interval of 99% would mean that:

• The probability that within the next 24 hours, the losses are less than 10 million. Dollars., Amount to 99%;

• The probability that losses will exceed 10 million. Dollars., Over the next days is equal to 1%;

• Losses in excess of 10 million. Dollars. Expected on average once a hundred trading days.

Thus, the cost of risk-taking is a monetary indicator that reflects the expected loss with a given probability.

Market Risk Management, in its modern sense, the most formalizable and regular problem. The first mathematical models of changes in market prices appeared in the early XX century. Samuelson suggested in the 60s and still actively used by the geometric Brownian motion model to describe the dynamic of stock prices.

Powerful impetus to the development of this theory was given the work of Black, Scholes and Merton in the 70s.

Suppose that we plan to create a portfolio of the following Russian companies:

OAO Gazprom ordinary (GAZP);

LUKoil, ordinary (LKOH);

Mosenergo ordinary (MSNG);

Rostelecom OJSC, ordinary (RTKM);

Surgutneftegas ordinary (SNGS);

Tatneft, common (TATN).

Portfolio Selection exclusively from the so-called "blue-chip" shall be illustrative only and not principally important.

We are faced with the problem of estimating, calculating the market risk Var (Valne-at-Risk), both for the entire portfolio and for individual companies with investments in the specified number of days.

First of all, pay attention to the high positive mutual correlation of changes in prices (Table 3.1.). This is very important, because it means that stock prices are more likely to change in synchronization which, naturally, must be considered when forecasting future values of the portfolio.

TABLE 3.1.

	GAZP	LKOH	RTKM	SNGS	TATN	MANG
GAZP	1	0.81321	0.76542	0.73447	0.78652	0.84832
LKOH	0.81321	1	0.74321	0.78495	0.72360	0.75476
RTKM	0.76543	0.74321	1	0.69437	0.71095	0.70876
SNGS	0.73447	0.78493	0.69437	1	0.74365	0.74298
TATN	0.78652	0.72361	0.71095	0.74365	1	0.80604
MANG	0.84832	0.75476	0.70876	0.74298	0.80604	1

Analysis of VaR estimates with uniform distribution (Table 3.2.) Capital shows that in the time period considered most vulnerable to market risk investments in shares of Tatneft and Rostelecom, and vice versa, the lowest market risk is inherent in LUKoil shares. Few are more risky investments, Surgutneftegas and Mosenergo. It follows that reducing the stake in Rostelecom and Tatneft in the portfolio, increasing their share of the expense of other issuers, can significantly reduce the VaR portfolio as a whole. Indeed, the initial capital of $ 100 000 and a uniform distribution of capital for issuers to-day. 95% Pareto VAR portfolio of nearly 149,25 dollars (see table 3.2.).

TABLE 3.2.

The allocation of capital in fractions

GAZP	LKOH	RTKM	SNGS	TATN	MANG	Sum
0.16666	0.16666	0.16666	0.16666	0.16666	0.16666	1
Pareto VaR						149,25

For additional details look appendix III.

Classical problem of investing is that the problems of temporarily placement free funds in a certain amount for a certain period - investment horizon. It is assumed that the market is an opportunity to invest available funds in several assets. The yield of each asset is a random variable characterized by mean and standard deviation.

The abovementioned problem was first formulated and solved by Markowitz is one of the founders of modern portfolio theory[48]. The algorithm for finding the optimal portfolio is to construct a set of efficient portfolios. However, the set of efficient portfolios can not determine the optimal portfolio. Selection of the optimal portfolio depends on the risk appetite, which is characterized by a utility function. In turn using a utility function to construct the curve of indifference. Point of contact of the indifference curve and the curve that defines the boundary of efficient portfolios is optimal only for a specific investor. The main difficulty of this algorithm consists in determining the indifference curve as an objective assessment of risk appetite.

An alternative is an approach to portfolio selection criterion allowable loss (drawdown criteria), developed by D. Marshall[49]. In the criteria for allowable losses is determined by a one-period portfolio, which, if it is repeated for each period over the life of the investment, maximizes the expected return subject to certain allowable losses allocated funds for the entire investment period. Parameters of the method is the length of the investment horizon, the maximum allowable losses and the level of trust. The advantage of this method is the rejection of "highly abstract criterion of utility[50]" and the choice of the optimal portfolio with a clear view of the acceptable

[48] Sharpe, Alexander G. and J. Bailey.(1997). *Investments: Per. Translated from English*. - Moscow: INFRA-M.
[49] Marshall. JF, Bansal VK (1998) *Financial Engineering: The Complete Guide to Financial Innovation: Per. Translated from English*. - Moscow: INFRA-M.
[50] Ibid.

level of losses of the portfolio. The disadvantage is the need for constructing a set of single-period and multiperiod efficient portfolios.

There is also another approach to the selection of an optimal portfolio, which does not require construction of the set of efficient portfolios in the definition of indifference curves[51]. Under this approach, the quality of the optimization criterion is selected ratio of the square of the expectation of portfolio returns to its dispersion, which is actually a square criterion Sharpe. The disadvantage of this approach is that it does not consider the level of acceptable losses in portfolio optimization. In addition, this method can not calculate the optimal portfolio of investor risk aversion, that is, an investor who maximizes both your income and risk.

Will rely on a constructive, in our opinion, the concept of which is that when searching for the optimal portfolio is very important to a direct account of allowable losses of the portfolio. This idea coincides with the above methodology given D. Marshall.[52] Besides, something like that was formulated and William Sharpe. He suggested as a measure of utility for a particular use of the assets expected return for expulsion risk fee. Fee for the risk W. Sharpe defined as the square of a measure of risk (variance) divided by the measure of tolerance investor risk appetite[53]. However, this approach again uses the concept is not entirely clear - a measure of tolerance investor risk appetite.

The aim of this study is to develop an approach to the selection of the optimal portfolio of assets, which would not need to construct the set of efficient portfolios and to determine the indifference curves. In addition, this approach should explicitly consider the level of investor losses directly in the optimization of the composition of the portfolio.

In our opinion, the goal most fully meets the following approach. It consists in the fact that the composition of the portfolio for the investor to avoid the risk, is chosen so as to maximize the worst value of the portfolio at the end of the period of ownership of a portfolio of assets. For the investor, risk-averse, the task is to

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[51] Ignatotkin I. (1998). *Is necessative effective set for portfolio optimization*? // Journal *Securities Market*. - №8 (119)
[52] Marshall. JF, Bansal VK (1998) *Financial Engineering: The Complete Guide to Financial Innovation: Per. Translated from English*. - Moscow: INFRA-M.
[53] Sharpe, Alexander G. and J. Bailey.(1997). *Investments: Per. Translated from English*. - Moscow: INFRA-M.

maximize the best value of the portfolio at the end of the period of ownership portfolio. For the investor, a risk neutral, the choice of the optimal portfolio is to maximize revenue.

Consider the most common case in practice - the case of an investor to avoid risks. Selection of the optimal portfolio is determined by the

conditions to maximize the yield of the portfolio assets of the worst:

$$ř = \mu - h_p \cdot \sigma \rightarrow \max \qquad (3.5.)$$

or taking into account the fact that VaR = $h_p \bullet \sigma$

$$ř = \mu \text{ - VaR} \rightarrow \max \qquad (3.6.)$$

where ř - is the worst return on a portfolio of assets, μ -is the expected return of the portfolio, h_p – is the number of standard deviations in the quantile of order p (confidence level), for example, the level of confidence p = 95% is equal to 1.65; and at the level of confidence p = 99% is equal to 2.33; σ is the standard deviation of portfolio returns, VaR - Value-at-risk - the level of acceptable losses.

Sharpe portfolio optimization criterion [54]can be written as:

$$SR = \frac{\mu}{h} \rightarrow \max \qquad (3.7.)$$

Profitability and the standard deviation of portfolio returns are determined by the well-known formulas:

$$\mu = \sum_{i=1}^{N} X_i \cdot r_i, \qquad (3.8.)$$

$$\sigma = \sqrt{\sum_{i=1}^{N}\sum_{i=1}^{N}(X_i \cdot X_j \cdot \sigma_y)} \qquad (3.9.)$$

where X_i and X_j – are percentage i -th and on j -th on the assets in the portfolio; $\sigma_n = p_y \bullet \sigma_i \bullet \sigma_j$ - is covariance of yield i-th and j-th the asset; σ_y - is the correlation coefficient of yield i-th and j-th asset; σ_i and σ_j is standard deviation of return i-th and j-th asset; r_i - is the expected return i-th and j-th the asset, N – is the number of possible assets in the portfolio. The indices i and j refer, respectively, to the i-th and j-th asset.

[54] Ignatotkin I. (1998). *Is necessative effective set for portfolio optimization*? // Journal *Securities Market*. - №8 (119)

In an optimization problem, there are natural limits. They are as follows: the proportion of assets in the portfolio must be a positive number Xi ≥ 0, and the sum of shares of all assets in the portfolio must be equal to unity:

$$\sum_{i=1}^{N} X_i = 1 \qquad (3.10.)$$

We compare the method based on the Sharpe ratio, and the approach of using VaR methodology. The initial data for the problem of optimizing the composition of the portfolio is a list of assets that may become part of the portfolio, their expected returns and standard deviations of returns from their mean values for each asset. For example, Table 1 shows initial settings on the 4 assets. To simplify the supposed return on assets is independent. Then all the coefficients of correlation in p_y i≠ j equal to zero, and $p_y = 1$.

TABLE 3.5.

Baseline data on assets

№	Annual rate of return	Standard deviation of return
1.	12%	1%
2.	13%	2%
3.	12%	1%
4.	14%	2%

The composition of the portfolios was carried out using a spreadsheet MS Excel. The calculation results are shown in Tables 3.6. and 3.7. (Look appendies III and IV).

At selected levels of confidence of 95% and 99% expected rate of return, 13.91% and 13.59% of the portfolio, calculated by the proposed method is higher than the expected return of 12.76%) portfolio, obtained by optimizing the Sharpe ratio. Standard deviation of 0.56% of the portfolio obtained by optimizing the Sharpe ratio is lower than the standard deviation of 0.93% (confidence level 95%) and 0.78% (confidence level 99%) portfolios obtained using VaR approach. Nevertheless, the ratio of expected income and standard deviations of the considered optimal portfolios

such that at 95% confidence level for the worst yield 12.38% of the portfolio obtained by the VaR approach higher than the worst yield 11.84% of the portfolio, with the resulting by optimizing the Sharpe ratio. At the level of confi dence 99% the situation is exactly the same: the worst yield 11.77% of portfolio, obtained by the VaR approach, higher than the worst yield 11.46% of the portfolio obtained by optimizing the Sharpe ratio.

Thus, in the present case, the proposed approach is more effective compared with the approach based on the optimization criterion Sharpe[55].

As well as the approach based on the optimization criterion Sharpe, the proposed method has the following advantages: no need a) to calculate the set of efficient portfolios and b) use the indifference curve. In addition, he has more dignity. The method allows you to explicitly control the market risks, namely the level of losses associated with the change in return on assets. Risk appetite is clearly defined in terms of VaR approach, namely, the level of trust. The greater the level of trust, the less risk appetite.

Note that due to the presence of the second term in equation (3.1.) Describes the effect of the proposed approach is diversification - to reduce the risk by increasing the number of assets contained in the portfolio. For example, for the case of four assets that have the same yield and risk optimal portfolio is comprised of those assets, taken in equal proportions (25%).

With the proposed method can be calculated and the investor's portfolio, risk-averse. In this case, it is necessary to maximize the best possible return on the portfolio, ie .:

$$r = \mu + h_p \cdot \sigma \to \max \qquad (3.11.)$$

Thus, the proposed approach to portfolio optimization based on VaR technology does not require the construction of efficient set of portfolios, as well as the use of the investor's indifference curve. Attitude of the investor to the risk determined by the level of trust, which has a clear interpretation than the tolerance of the investor's risk appetite. VaR calculation methodology is not only used by

[55] Ignatotkin I. (1998). *Is necessative effective set for portfolio optimization*? // Journal *Securities Market*. - №8 (119)

portfolio managers, but also regulators. So in the United States, regulators require banks to book a three-time 10-day 99% VAR for market risks.

Despite its popularity, VaR has a number of disadvantages:

• First, VaR does not account for the possibility of large losses that may occur with small probabilities (less than 1-0.01).

• Secondly, VaR can not distinguish between different types of tailings distribution losses and therefore underestimates the risk when the loss distribution has "fat tails" (ie, its density decreases slowly).

• Third, VaR is not a coherent measure, in particular, it does not have the property of subadditivity. There are examples where Var portfolio is more than the sum of the two VaR of subportfolios, of which it is composed. This is contrary to common sense. Indeed, if we consider the risk measure as the amount of capital in reserve for covering market risk, to cover the risk of the whole portfolio is no need to reserve more than the amount of the provision is subportfolios.

In addition, VaR encourages trading strategies that give good returns in most scenarios, but sometimes it can lead to catastrophic losses.

Unfortunately due to the limited amount of work to explore the entire volume of scientific material was not possible. Given the recent events that take place on the world stage, to which itself is the Russian Federation and the impact of these developments on world markets and prices conjunctural shares our advice research can be directed precisely on the scientific analysis of the causes, consequences of this particular critical period for our state. The author is not going into a detailed description of the current situation can only assume that the scientific interest in this period can be the subject for further research model for the formation of a Pareto optimal financial investments portfolio in the Russian market. Of particular interest in our sole discretion played sanctions taken by other states in relation to the Russian Federation and their direct impact on the quantity and price of domestic enterprises shares. Taking into account the sharp decline in stock prices and increase the risks of yield the formating model of a Pareto optimal financial investments portfolio is too discordant with the period studed by the author, and as a result can serve as material for a new research and a new approach making the financial investment portfolio.

APPENDIX I:

Portfolio calculated by Markowitz was (0, 0.27, 0.79), ie total capital must be invested in shares of JSC "Rostelecom" and Gazprom.

12/05/13 were bought shares of JSC "Rostelecom" for as low as 97.779 RUB and Gazprom at a price 116.05. At the same time was bought alternative portfolio with weights (1/3, 1/3, 1/3), respectively, at prices NK Lukoil 1880. 12 RUB. RAO Gazprom - 10.97 RUB and JSC "Rostelecom" -95.4 RUB. (Table. 2.5., and 2.6.).

04/10/13 signal appeared "Sell" (label graf.2.1) (% K crossed% D from top to bottom).

15/05/13 portfolios were sold at prices respectively (1890.63 RUB, 10.62 RUB, 110.67 RUB).

24/10/13 signal appeared "Buy" (label C graf.2.1.) (% K crossed% D from the bottom up).

Portfolio, calculated by Markowitz, was (0. 71, 0.073, 0.12).

15/10/13 shares were purchased at prices respectively (2111.82 RUB, 14.7 RUB, 105.90 RUB) (Table. 2.5., And 2.6.).

17/11/13 signal appeared "Sell" (label D graf.2.1) (% K and% D is above 80%, the envelope of the MACD histogram is facing down).

18/11/13 portfolios were sold at prices respectively (2087.01 RUB, 14. 7 RUB, 117.23 RUB).

10/02/14 signal appeared "Buy" (label E graf.2.1) (Both curves Slow Stochastic was below the level of 20%, and the envelope MACD histogram pointing up).

Portfolio, calculated by Markowitz, was (0.29, 0.71, 0.02).

10/02/14 shares were purchased at prices respectively (1901.81 RUB, 12.31RUB)

20/02/14 signal appeared "Sell" (mark F graf.2.1) (% K crossed% D downwards).

03/01/14 shares were sold at prices respectively (2012.1 RUB, 110.25 RUB).

17/03/14 signal appeared "Buy" (label G graf.2.1.) (Both curves Slow Stochastic was below the level of 20%, and the envelope MACD histogram pointing up).

Portfolio, calculated by Markowitz, was (0,03 0, 1,002).

13/03/14 shares were purchased at prices respectively (1897.02 RUB, 06.67 RUB and 84,500 RUB)

10/05/14 signal appeared "Sell" (label H graf.2.1) (% K and% D is above 80%, the envelope of the MACD histogram is facing down).

11/05/14 shares were sold at prices respectively (1866.24 RUB, 07.87 RUB, 75.5 RUB).

25/05/14 signal appeared "Buy" (label J graf.2.1) (Both curves words Stochastic was below the level of 20%, and the envelope MACD histogram is facing up).

Portfolio, calculated by Markowitz, was (0.31, 0.06, 0.49).

26/05/14 shares were purchased at prices respectively (1968.86 RUB, 06,21 RUB, 83,5 RUB).

6.06.14 signal appeared "Sell" (label K graf.2.1) (% K crossed% D from top to bottom).

09.06.14 shares were sold at prices respectively (2035.54 RUB, 8.42 RUB, 88.8 RUB).

19/06/14 signal appeared "Buy" (label L graf.2.1) (% K crossed% D from the bottom up).

Portfolio, calculated by Markowitz, was (0.43., 0.28, 0.09).

20/06/14 shares were purchased at prices respectively (2099.8 RUB, 10.53 RUB, 84.10 RUB).

12/07/14 signal appeared "Sell" (label graf.2.1 M) (% K and% D is above 80%, the envelope of the MACD histogram is facing down).

13/07/14 shares were sold at prices respectively (1940.19 RUB, 10.76 RUB, 90.7 RUB).

26/07/14 signal appeared "Buy" (label N graf.2.1.) (Both curves SlovStohastik were below 20%, and the MACD histogram envelope is facing up).

Portfolio, calculated by Markowitz, was (0.03, 0.01, 1.2).

27/07/14 shares were purchased at prices respectively (1957.32 RUB, 11.12 RUB, 80 RUB).

APPENDIX II:

Table 2.5.

Markowitz diversification

Date	**NK Lukoil**	**Portfolio share**	**Gazprom**	**Portfolio share**	**JSC Rostelecom**	**Portfolio share**	**Capital**
11.05.13	1880.12	0	10.97	0.27	95,4	0.79	1,16
04.10.13	1890.63	0	10.62	0.27	110.67	0.79	1,12
24.10.13	2111.82	0.71	14.7	0.073	105.9	0.12	0.903
18.11.13	2087.01	0.71	14.5	0.073	117.23	0.12	0.9
10.02.14	1901.81	0.29	12.31	0.71	112.7	0.02	1.02
17.03.14	2012.1	0.29	12.24	0.71	110.25	0.02	1.00
10.05.14	1866.24	0.03	07.87	0.00	75.5	0.97	1.18
25.05.14	1968.86	0.03	06.21	0.00	83.5	0.97	1.10
06.06.14	2035.54	0.31	8.42	0.06	88.8	0.49	0.89
19.06.14	2099.8	0.31	10.53	0.06	84.10	0.49	0.865
13.07.14	1940.19	0.43	10.76	0.28	90.7	0.09	0.81
26.07.14	1957.32	0.43	11.12	0.28	80	0.09	0.84

Table 2.6.

Alternative portfolio

Date	**NK Lukoil**	**Portfolio share**	**Gazprom**	**Portfolio share**	**JSC Rostelecom**	**Portfolio share**	**Capital**
11.05.13	1880.12	0.33	10.97	0.33	95,4	0.33	1.27
04.10.13	1890.63	0.33	10.62	0.33	110.67	0.33	1.21
24.10.13	2111.82	0.33	14.7	0.33	105.9	0.33	1.25
18.11.13	2087.01	0.33	14.5	0.33	117.23	0.33	1.24
10.02.14	1901.81	0.33	12.31	0.33	112.7	0.33	1.20
17.03.14	2012.1	0.33	12.24	0.33	110.25	0.33	1.21
10.05.14	1866.24	0.33	07.87	0.33	75.5	0.33	0.94
25.05.14	1968.86	0.33	06.21	0.33	83.5	0.33	0.96
06.06.14	2035.54	0.33	8.42	0.33	88.8	0.33	0.98
19.06.14	2099.8	0.33	10.53	0.33	84.10	0.33	0.99
13.07.14	1940.19	0.33	10.76	0.33	90.7	0.33	1.00
26.07.14	1957.32	0.33	11.12	0.33	80	0.33	1.10

APPENDIX III:

By increasing the proportion of Lukoil to 0.25 and a decrease in the share of Tatneft to 0.083 Pareto VaR will have only 132,74. It may be thought that it is necessary to complete all the capital put in Lukoil, and not split it across multiple issuers. Setting a stake in Lukoil to 1 and the rest 0, we really obtain the total portfolio VaR equal to 110,97 (Table 3.3.).

TABLE 3.3.

GAZP	LKOH	RTKM	SNGS	TATN	MANG	Sum
0	1	0	0	0	0	1
Pareto VaR						110,97

Optimization of capital allocation for the construction of the portfolio with the lowest Pareto VaR provides a solution to a non-trivial (Table 3.4.). 95% VaR of the Pareto optimal portfolio is 96, 34, which is less than the usual investment only in equities NK Lukoil.

TABLE 3.4.

GAZP	LKOH	RTKM	SNGS	TATN	MANG	Sum
0	0.87435	0	0.12321	0	0.00244	1
Pareto VaR						96,340

APPENDIX IV:

TABLE 3.6.

Optimal portfolio composition obtained by the optimization of the Sharpe ratio and using VaR approach

№	Optimization by the Sharpe ratio	Optimization using VaR approach	
		level of confidence 95%	level of confidence 99%
1.	29,1%	0,0%	7,5%
2.	7,9%	8,7%	10,3%
3.	29,1%	0,0%	7,7%
4.	33,9%	91,3%	74,5%

TABLE 3.7.

Comparison of parameters optimal portfolios obtained by optimizing the ratio Sharpe using VaR approach

№	Optimization methods	The expected return on a portfolio	The standard deviation of portfolio returns	Sharpe ratio	The worst return on the portfolio	
					level of confidence 95%	level of confidence 99%
1.	Optimization by the Sharpe ratio	12,76%	0,56%	22,94	11,84%	11,46%
2.	Optimization using VaR approach (level of confidence 95%)	13,91%	0,93%	14,97	12,38%	
3.	Optimization using VaR approach (level of confidence 99%)	13,59%	0,78%	17,43		11,77%

CONCLUSION

The most important problem of the financial market research is to find the theoretical models that adequately reflect the modern processes of portfolio investment in terms of information and great differ between high volatility of the domestic stock market in terms of "return / risk." The solution to this problem is possible on the basis of empirical testing the basic provisions of the classical portfolio theory, in particular the Markowitz theory. Carried out in the thesis empirical test of the basic provisions of the classical portfolio theory, in particular the Markowitz theory on the basis of the Russian financial market has shown that:

• There is a fairly clear tendency to reduce investment risk by increasing the number of securities.

• Markowitz model gives correct distribution only at low level of defined return on investment that is confirmed "small values of the expectations for profitability.“

• For a portfolio consisting of more than eight financial instruments to choose the parameters for the relation "return / risk" and get the correct distribution is not possible.

Thus, the Markowitz model is partially applicable to the Russian stock market.

Assessing the risk and return of the Russian securities market, we can only suggest, with a certain degree of probability, that the future situation in the stock market will be in the framework of the statistical data. The large number of factors that directly or indirectly affect the dynamics of the listed issuer does not allow predicting and reduce to zero the risk of investment loss, but to reduce it to a minimum with the best possible yield.

The central problem of portfolio investment is the choice of the optimal portfolio, ie. Definition of assets set with the highest yield at the lowest or a given level of investment risk. The essential point in the modern theory turns out to be consideration of the mutual correlations between the yields of the assets, which

allows for effective portfolio diversification, significantly reduces the risk portfolio compared with the risk of the included assets.

To make a decision on the investment portfolio formation, as well as for the optimal allocation of financial instruments in the portfolio of methods of classical portfolio theory may complement methods of technical analysis. The study showed that the application of the Markowitz theory most correct to use the results of the technical analysis, assuming, for example, the same expected returns for all assets. Comparative analysis formed two investment portfolios showed that the total return from operations is as follows: investments by Markowitz - 67.2% per annum, alternative investments - 49.4% per annum. Thus, it is seen that the idea of combining even the most basic methods of technical analysis with diversification Markowitz yielded significant benefits with respect to alternative investments with equal shares of assets in the portfolio.

There is no doubt that the use of more sophisticated methods of technical analysis, allowing a good predictor not only trends, but also the price targets of assets, will increase the efficiency of the use of similar trading schemes.

Optimizing trading system for each financial asset in the portfolio can be optimized ratio of return / risk. When using trend-following trading systems with a small number of parameters, the ratio of return / risk has a maximum value for each of the shares. The maximum ratio of return / risk depends on the number of transactions generated by the system for a certain period of time, ie the sensitivity of the system. The optimum sensitivity of the system depends mainly on the average slip and from brokerage commissions. The increase in the ratio of return / risk based on historical data can be achieved by increasing the options trading system, so at the expense of reliability.

Carried out in the analysis showed that:

• Using a trend-following trading systems with a small number of parameters, the ratio of the yield / risk ratio has a maximum value for each of the shares.

• The maximum ratio of return / risk depends on the number of transactions generated by the system for a certain period of time, ie the sensitivity of the system.

The optimum sensitivity of the system depends mainly on the average slip and from brokerage commissions.

• Increase the ratio of return / risk based on historical data can be achieved by increasing the options trading system, so at the expense of reliability.

The original formulation of the conditions of the modern theory of portfolio formation directly indicate the retention of the investor throughout the investment horizon of the so-called "long" position in relation to the market portfolio. Therefore, we can assume that the investor at each time interval consistently predicts the value of assets on the market. Meanwhile, according to the initial assumptions of the G.Markowitz theory change in return on assets are distributed as random continuous quantities, so that means the inability to predict their future values.

However, two essential conditions for the functioning of modern portfolio theory, somewhat at odds with each other. A study in the dissertation a case study for their compliance with the actual investment conditions showed that the application of modern portfolio theory, which is based on the condition of market efficiency as constant values, can not be considered rational for a long period of investment, due to the fact that in practice the extent efficiency of the market is changing the value of a dynamic market efficiency confirms the results obtained.

Our analysis will supplement the basic provisions of portfolio theory, and to formalize the functioning of a possible model of active portfolio investment, to introduce new, more realistic assumptions characterizing heterogeneity, information asymmetry and the high volatility of the Russian financial market. In addition, this analysis allowed us to formulate theoretical propositions to minimize market risk, based on the principles of Pareto optimality.

In modern conditions, a serious problem in the decision-making process on the financial investments portfolio formation is the search for effective options portfolio, which consists in determining the composition of the financial assets in the portfolio. Theoretically, all portfolios resulting in is an increase of its economic value to be taken for implementation. In practice, portfolio investment is carried out in a strictly limited ability to attract funds. Based on the above, we have developed an algorithm for deciding to build a financial investments portfolio, which is based on the

mechanism of directed search Pareto optimal variants of a financial investments portfolio. The proposed algorithm is used to determine the effective range of financial investment ratio "return / risk" and the volume of investments (for well correlated portfolios).

The developed algorithm allows finding all the Pareto optimal variants of the portfolio on the basis of available observations on the dynamics of profitability and risk of the financial assets included in portfolio. The main advantage of the algorithm is incorporated in its base directed search mechanism, which allows omitting the bulk of the known variants of inefficient portfolios.

Given the high volatility of the domestic stock market important problem for investor is finding the optimal portfolio in the account of the level of risk in the optimization of the composition of the portfolio. The solution to this problem for investors is to shun risk, possibly based on risk method, cost, maximizing the smallest value of the portfolio at the end of the period of ownership portfolio. Conducted in the thesis comparative analysis technology VaR (Value at Risk) with Sharpe ratio showed that the approach to portfolio optimization based on VaR technology is more efficient compared with the approach based on the optimization criterion Sharpe, as does not require the construction of efficient set of portfolios, as well as the use of the investor's indifference curve. Investor`s attitude to the risk determined by the level of trust, which has a clear interpretation than the tolerance of the investor's risk appetite.

The obtained results can greatly improve the efficiency of investment activity in the Russian financial market and can be recommended for use by professional participants of the stock market.

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