2015-06

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NEAPOLIS UNIVERSITY OF PAPHOS (NUP)

WORKING PAPERS SERIES

2015/6

TITLE

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December 2014

Abstract

The Phillips curve represents the link between the business cycle and inflation and constrains the actions of policymakers. In this paper, we study the optimal long-run rate of inflation in the presence of a hybrid Phillips curve, which nests a purely backward-looking Phillips curve and the purely forward-looking New Keynesian Phillips curve as special limiting cases. The monetary authority possesses commitment and its objective function is derived as an approximation to the utility of the representative household.

We show that the commitment solution for the monetary authority leads to steady-state outcomes in which inflation is positive. Rising from zero under the purely forward-looking New Keynesian Phillips curve, the optimal long-run rate of inflation reaches its maximum under the purely backward-looking Phillips curve. The consequences of positive steady-state inflation differ between the limiting case of a purely backward-looking Phillips curve and the case of a hybrid Phillips curve.

JEL classification: E31, E32, E52.

Keywords: Optimal monetary policy, Phillips curve, inflation persistence.

1 Introduction

Delivering the Nobel Prize lecture in 2002, George Akerlof [1] commented *probably the most important macroeconomic relationship is the Phillips curve.* Testament to this importance is the fact that no other macroeconomic relationship has lived through so many fundamental revisions. Having come to life as a statistical regularity by virtue of Phillips [15], the Phillips curve quickly found its place in macroeconomic theory following Samuelson and Solow [17]. Shortly after, Phelps [14] and Friedman [7] highlighted the importance of economic agents' expectations of inflation for the Phillips curve and then Lucas [13] criticized the adaptive formation of expectations, thus starting the rational expectations revolution.

The Phillips curve has never left its prime spot in macroeconomic theory as it represents the link between the business cycle in an economy and inflation. It is the key constraint for policymakers in deciding how they should set interest rates, in the case of central banks, or how they should use fiscal policy, in the case of governments, to stabilize the economic cycle. As such, while the New Classical Phillips curve has been used in many well-known examples of optimal monetary policy, such as those of Kydland and Prescott [11] and Barro and Gordon [3], today the New Keynesian Phillips Curve (NKPC) underpins the analysis of optimal monetary policy within New Keynesian (NK) economics (e.g. Woodford [21], Clarida et al. [6]).

In this paper, we study the deterministic component of the optimal monetary policy commitment in a basic NK model which encompasses different Phillips curves. The analysis is directed toward understanding the nature of the optimal long-run (or steady-state) rate of inflation, OLIR for short, in the presence of a hybrid Phillips curve, which nests both the purely backward-looking Phillips curve and the purely forward-looking NKPC.

In an otherwise basic NK model, characterized by monopolistic competition in product markets and Calvo [5] price staggering, the Phillips curve features such an appealing property as we consider rule-of-thumb behavior à la Steinsson [19], which generalizes

the specification in Galì and Gertler [9]. Galì and Gertler [9] assume that a fraction of firms allowed to re-optimize their prices within the Calvo lottery follow a rule-of-thumb: their prices are equal to the average price set in the previous period plus a correction for expected inflation, which is based on lagged inflation. Steinsson [19] augments this rule-of-thumb by assuming that rule-of-thumb price setters correct the average price set in the previous period not only for lagged inflation but also for expected demand conditions, with the correction being based on lagged output gap.

In the tradition of Woodford [21], we analyze the steady-state inflation rate using a fully microfounded set-up, which is derived as a linear-quadratic approximation around the deterministic steady state with zero inflation and small distortions. Within the same linear-quadratic framework, Woodford [21] shows that zero OLIR is optimal in the basic NK model. This is true despite the inefficiency of the deterministic steady state and despite the existence of a positively sloped long-run Phillips-curve trade-off, as implied by the NKPC.[1] Rule-of-thumb behavior by price setters leads to intrinsic persistence in inflation and alters the central bank's welfare-based objective function. The welfare-based objective function we derive differs from the one in Steinsson [19].[2] In the presence of rule-of-thumb behavior by price setters, we show how the period loss function has one additional term with respect to its counterpart in the purely forward-looking NK model. Interestingly, this extra term can be seen as penalizing variations in the difference between inflation and the change in the price level due to rule-of-thumb price setters.

Our main result is that the commitment solution for the monetary authority leads to steady-state outcomes in which inflation is positive. Starting from zero in the basic NK model, the OLIR rises monotonically with the degree of rule-of-thumb behavior.

Under the purely forward-looking NKPC, if the central bank were to organize some

[1] Moreover, Woodford [21] shows that zero OLIR is also robust to intrinsic inflation persistence due to indexation to lagged inflation by price setters.

[2] We reported the error to the author, who acknowledged it in the Erratum available at http://www.columbia.edu/~js3204/papers/STjme03erratum.pdf

output expansion, the private sector would anticipate higher inflation in the future. As discussed in Woodford [21], the fact that the cost of having higher inflation in the future is discounted at the same rate by the monetary authority and by the private sector means that expanding output is not worth doing for the policymaker. From the standpoint of the discounted welfare-based loss function, the output cost of higher anticipated inflation exactly offsets the stimulative effect of higher current inflation. It follows that there is no welfare gain from a commitment to inflation that can be anticipated in advance. What we show here is that with a backward-looking, either partially or fully, Phillips curve there is a welfare gain from a commitment to a positive steady-state rate of inflation. The monetary authority is prepared, in the short run, to trade off the gain in utility, obtained from having a higher level of output, with the loss in utility caused by having higher inflation. The increase in inflation is actually permanent. However, because of discounting, the policymaker gives a finite value to the loss in utility caused by having permanently higher inflation. As a result, there is a long-run incentive for positive inflation under an optimal commitment. Indeed, with commitment and a backward-looking Phillips curve, either partially or fully, zero OLIR would only obtain if the policymaker were not to discount the future. In this case, permanent inflation would lead to losses with an infinite present value and there would thus be no welfare gain from a commitment to a positive steady-state rate of inflation.

The consequences of positive OLIR then differ between the limiting case of a purely backward-looking Phillips curve and the case of a hybrid Phillips curve. In the latter, given the existence of a long-run Phillips curve trade-off, positive inflation in the steady state implies in turn a long-run output increase. In the former, the positive OLIR reaches it maximum level. However, the purely backward-looking Phillips curve implies that the long-run Phillips curve is vertical. It follows that there is no long-run output increase as a result of inflation being positive at steady state. We thus obtain the well-known inflation bias stressed by Kydland and Prescott [11] and Barro and Gordon [3], namely a positive steady-state rate of inflation without any positive effect on the steady-state level of output. Indeed, expectations play no role in the purely backward-

looking Phillips curve, and so discretionary and commitment solutions are identical, reflecting the fact that optimization under discretion is truly optimal in backward-looking systems.

The inflation target we derive depends on the model's structural parameters. In particular, the OLIR is directly proportional to the gap that represents steady-state distortions originating from monopolistic competition: if one were to assume a subsidy to production aimed at offsetting these distortions, one would fail to obtain our simple result. We calibrate the model to U.S. data and we consider ample ranges for two key structural parameters. The inflation target turns out to be small in magnitude.

Our work is linked to some recent contributions that have appeared in the literature. Steinsson [19] studies the implications of his proposed rule-of-thumb price setting for the stochastic component of optimal monetary policy. In so doing, he assumes away steady-state distortions due to monopolistic competition and he finds that, following a supply shock, inflation reverts back to its zero target more gradually than in the purely forward-looking NK model. Our paper is complementary to this work as it characterizes the deterministic component of optimal monetary policy. Pontiggia [16] analyzes the deterministic component of the optimal monetary policy commitment in a basic NK model with rule-of-thumb behavior by price setters as in Galì and Gertler [9]. A comparison between his result and previous results in the presence of indexation to lagged inflation in Woodford [21] provides an example of the microeconomic dissonance emphasized in Levin et al. [12]. The two variants of the Calvo model yield hybrid Phillips curves that are first-order equivalent, but they imply different optimal long-run inflation rates: small and positive under rule-of-thumb behavior à la Galì and Gertler [9] and zero under backward-looking price indexation. Under rule-of-thumb behaviour à la Steinsson [19], the Phillips curve is not first-order equivalent to its counterparts under rule-of-thumb behaviour à la Galì and Gertler [9] or under indexation to lagged inflation, but has the theoretical appealing property of nesting both the purely backward-looking Phillips curve and the purely forward-looking NKPC. With this respect, our analysis shares the aim of Kirsanova et al. [10] as we are concerned with studying the implications for

the OLIR of Phillips curves that can be either backward-looking, forward-looking or hybrid. Kirsanova et al. [10] analyze the steady-state rate of inflation under the hybrid Phillips curve put forward by Fuhrer and Moore [8] as well as the NKPC. In particular, they focus on the implications for the OLIR of a monetary authority that is impatient as it discounts the future more heavily than the private sector. Our analysis differs in one important respect as we consider a fully microfounded set-up that nests a number of Phillips curves rather than a generic setting, characterized by an ad-hoc objective function and different Phillips curves. The fully specified general equilibrium nature of our analysis in turns rules out the possibility of the central bank having a different discount rate than the private sector.

The remainder of the paper is organized as follows. Section 2 spells out the model economy. Section 3 contains the main results and Section 4 concludes.

2 The Model

In this section we extend the model of central-bank behavior in Woodford [21] to allow for rule-of-thumb behavior by price setters à la Steinsson [19].The model is derived as a linear-quadratic approximation taken around the deterministic steady state with zero inflation and a mildly inefficient natural level of output.[3]

Woodford [21, ch. 6] demonstrates that the utility flow to the representative agent each period, U_t, can be approximated to second order by

$$\begin{aligned} U_t &= -\Psi\left[(\sigma^{-1}+\varpi)(x_t-x^*)^2+(1+\varpi\theta)\theta var_i \log p_t(i)\right] \\ &\quad +t.i.p+O\left(\left\|\Phi_y,\widetilde{\xi},\varphi\right\|^3\right) \end{aligned} \tag{1}$$

[3]We use conventional terminology. The efficient (first-best) level of output is the level of output that would prevail in the absence of imperfections. The natural level of output is the level of output that would prevail in the absence of nominal rigidities. The output gap is the log distance between the actual level of output and the natural level of output. The welfare-relevant output gap is the log distance between the actual level of output and the efficient level of output.

where $\Psi > 0$, $\theta > 1$ is the elasticity of substitution among alternative differentiated goods, and the term $t.i.p$ collects terms that are independent of monetary policy. In the model, the *divine coincidence* perceived by Blanchard and Galì [4] holds. The gap between the natural level of output, Y_t^n, and the efficient level of output, Y_t^*, is constant and invariant to shocks, namely

$$\log(Y_t^*/Y_t^n) \equiv x^* = \frac{\Phi_y}{\varpi + \sigma^{-1}} + O\left(\|\Phi_y\|^2\right) \tag{2}$$

where $\sigma > 0$ is the intertemporal elasticity of substitution of aggregate expenditure and $\varpi > 0$ is the elasticity of a firm's real marginal cost with respect to its own output level. The parameter Φ_y summarizes the steady-state distortions in the natural level of output originating from monopolistic competition. Technically, the parameter satisfies $\Phi_y = 1 - \mu^{-1}$ where $\mu \equiv \theta/(\theta - 1)$ is the desired markup as a result of firms' market power. Steady-state distortions are assumed to be small and are treated as an expansion parameter in the derivation of a second-order Taylor-series approximation to the period utility of the representative agent. This implies that there are no linear terms in the second-order approximation to the welfare loss function. The resulting quadratic approximation to social welfare has the property that a correct log-linear approximation to optimal policy can be derived by minimizing the welfare-theoretic loss function subject to the constraints implied by a log-linear approximation to the model structural equations. In particular, the third-order residual in (1) clarifies that the quadratic approximation to social welfare provides a valid second-order approximation to the utility of the representative agent when evaluated using log-linearized structural equations as long as: (i) the disturbances buffeting the economy are small enough (small value of $\widetilde{\xi}$), (ii) the deterministic steady state driven by the policy under consideration is close enough to the deterministic steady state around which the approximations are taken (small value of φ), (iii) steady-state distortions are small enough (small value of Φ_y). Allowing for $\Phi_y > 0$ suffices precisely for the characterization of the first-order effects of a mildly inefficient natural rate of output on optimal monetary policy. Specifically, small steady-state distortions matter for the deterministic component of

optimal policy, namely the long-run levels of the endogenous variables, but, in the log-linear approximation to policy, have no effects on the optimal responses to shocks.[4]

From a welfare point of view, it is thus desirable to stabilize (I) the gap between the actual level of output, Y_t, and the efficient level of output, which, given equation (2), is expressed as $\log(Y_t/Y_t^*) = x_t - x^*$ with $x_t \equiv \log(Y_t/Y_t^n)$ being the output gap; (II) the degree of price dispersion, $var_i \log p_t(i)$, which is costly as relative-price distortions cause inefficient dispersion of consumption and output across goods. The details of the price setting in turn relate the degree of price dispersion to variations in the aggregate price level.

Following Calvo [5], we assume that only a fraction $1-\alpha$, with $\alpha \in (0,1)$, of prices are reset in each period. Following Galì and Gertler [9], we assume that only a fraction $1-\omega$, with $\omega \in [0,1)$, of price setters behave optimally (i.e. in a forward-looking manner) when setting the price, the remaining fraction of price setters use a backward-looking rule-of-thumb when revising their prices. The aggregate price level, P_t,hence evolves according to

$$P_t = \left\{(1-\alpha)(p_t^*)^{1-\theta} + \alpha P_{t-1}^{1-\theta}\right\}^{\frac{1}{1-\theta}} \tag{3}$$

where

$$p_t^* = (1-\omega)p_t^f + \omega p_t^b \tag{4}$$

denotes the overall reset price. The forward-looking reset price, p_t^f, is implicitly defined by the profit-maximizing relation

$$E_t \sum_{s=0}^{\infty} (\alpha\beta)^s \Pi_1(p_t^f, p_t^f, P_{t+s}, Y_{t+s,}\widetilde{\xi}_{t+s}) = 0 \tag{5}$$

where $\Pi_1(p_t^f, p_t^f, P_{t+s}, Y_{t+s,}\widetilde{\xi}_{t+s}) = 0$ is what Woodford [21, Ch. 3] labels the *notional Short-Run Aggregate Supply* curve. The rule-of-thumb backward-looking reset price, p_t^b, is specified as in Steinsson [19]

$$p_t^b = p_{t-1}^* \frac{P_{t-1}}{P_{t-2}} \left(\frac{Y_{t-1}}{Y_{t-1}^n}\right)^{\delta} \tag{6}$$

[4]This is because any effects of $\Phi_y \neq 0$ on the optimal responses to shocks would be of second order and can thus be neglected in a first-order characterization to optimal policy.

where $\delta \in [0,1]$. Rule-of-thumb price setters thus set their prices equal to the average price set in the previous period, p^*_{t-1}, plus a correction for both expected inflation, which is based on lagged inflation (i.e. P_{t-1}/P_{t-2}), and expected demand conditions, which is based on lagged output gap (i.e. Y_{t-1}/Y^n_{t-1}). When $\delta = 0$, the rule-of-thumb collapses to the specification in Galì and Gertler [9], whereby rule-of-thumb firms do not take into account past demand conditions when setting their prices.

The basic NK model assumes that there are no costs associated with varying the nominal interest rate (i.e. purely cashless economy). It follows that the intertemporal IS relation, which relates interest rates to the timing of expenditure and it is not affected by rule-of-thumb price setters, does not impose a real constraint on the central bank. The model of central-bank behavior is therefore fully described by the Phillips curve and the welfare-based loss function. Rule-of-thumb behavior à la Steinsson implies two differences relative to the purely forward-looking NK model. Firstly, the Phillips curve becomes hybrid, namely partially backward-looking. Secondly, the welfare-theoretic loss function includes an additional term.

The hybrid Phillips curve, whose derivation is detailed in Appendix A, implies that the inflation rate, π_t, and the output gap in any period satisfy an aggregate-supply relation of the form

$$\pi_t = \chi_f \beta E_t \pi_{t+1} + \chi_b \pi_{t-1} + \kappa_1 x_t + \kappa_2 x_{t-1} \tag{7}$$

where E_t denotes the expectations operator conditional on information available at time t and $\beta \in (0,1)$ is the subjective discount factor. The coefficients on the terms in inflation, , $\chi_f > 0$ and $\chi_b \geq 0$, are given by

$$\chi_f \equiv \frac{\alpha}{\phi}, \quad \chi_b \equiv \frac{\omega}{\phi} \quad \text{with} \quad \phi \equiv \alpha + \omega\left[1 - (1-\beta)\alpha\right] \tag{8}$$

while the coefficients on the terms in output gap, , $\kappa_1 > 0$ and $\kappa_2 \geq 0$, are given by

$$\begin{aligned} \kappa_1 &\equiv \frac{(1-\omega)\alpha\kappa - (1-\alpha)\alpha\beta\omega\delta}{\phi}, \ \kappa_2 \equiv \frac{(1-\alpha)\omega\delta}{\phi} \\ \text{with } \kappa &\equiv \frac{(1-\alpha)(1-\alpha\beta)(\sigma^{-1}+\varpi)}{(1+\varpi\theta)\,\alpha} \end{aligned} \tag{9}$$

Under rule-of-thumb behavior à la Steinsson [19], current inflation thus depends on a combination of expected future inflation and lagged inflation as well as a combination of current and lagged output gap.[5] The hybrid Phillips curve (7) has the theoretical appealing property of nesting, as special limiting cases, both a purely backward-looking Phillips curve and the purely forward-looking NKPC. First, in the limit where all price setters are rational in the face of Calvo-type price staggering (i.e. $\omega = 0$), (7) is easily seen to collapse to the NKPC in Woodford [21, Eq. 2.12 and 2.13, Ch. 3]

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t \tag{10}$$

Second, in the limit where the fraction of rule-thumb price setters goes to one (i.e. $\omega \to 1$), Steinsson [19] shows how the unique bounded solution of (7) has no forward-looking component, taking the form of a purely backward-looking Phillips curve (e.g. Ball [2] and Svensson [20]), namely

$$\pi_t = \pi_{t-1} + (1-\alpha)\delta x_{t-1} \tag{11}$$

It is important to note that the Phillips curves have different implications for the long-run trade-off between output and inflation. As it is well known, the NKPC (10) implies a steep upward-sloping relation of the form

$$\overline{\pi} = \frac{\kappa}{(1-\beta)}\overline{x} \tag{12}$$

between steady-state inflation, $\overline{\pi}$, and the steady-state output gap, $\overline{x}$. The long-run relation between inflation and output gap is in fact due to the fact that the NKPC (10) has a smaller coefficient on the expected future inflation (i.e. $\beta < 1$) term relative to that on current inflation (i.e. 1). With the presence of rule-of-thumb price setters à la Steinsson, the long-run Phillips curve is still positively sloped. Specifically, the hybrid

[5] If rule-of-thumb price setters do not index their prices to lagged output gap (i.e. $\delta = 0$), (7) is easily seen to collapse to the hybrid Phillips curve in Galì and Gertler [9], whereby current inflation does not depend on lagged output gap.

Phillips curve (7) evaluated at steady state yields

$$\overline{\pi} = \left[\frac{\kappa}{(1-\beta)} + \frac{(1-\alpha)(1-\alpha\beta)\omega\delta}{(1-\beta)(1-\omega)\alpha} \right] \overline{x} \tag{13}$$

which collapses to (12) under $\delta = 0$. The slope of the long-run Phillips curve is observed to increase monotonically with the degree of rule-of-thumb behavior and, in the limit where $\omega \to 1$, the long-run trade-off ceases to exist. Indeed, the purely backward-looking Phillips curve (11) implies that the long-run Phillips curve is vertical.

The welfare-theoretic objective function is derived as a second-order Taylor-series expansion to the discounted sum of utility of the representative agent. Appendix B reports a detailed derivation. Under Calvo [5] staggered price setting and rule-of-thumb behavior by price setters à la Steinsson [19], the welfare-based objective function takes the form[6]

$$\begin{aligned} \sum_{t=0}^{\infty} \beta^t U_t &= -\Omega \sum_{t=0}^{\infty} \beta^t \left[\begin{array}{c} \pi_t^2 + \lambda (x_t - x^*)^2 \\ +\lambda_1 \left[\pi_t - (\pi_{t-1} + (1-\alpha)\delta x_{t-1}) \right]^2 \end{array} \right] \\ &\quad + t.i.p + O\left(\left\| \Phi_y, \widetilde{\xi}, \varphi, \Delta_{t-1}^{1/2} \right\|^3 \right) \end{aligned} \tag{14}$$

where $\Omega > 0$ and the weights $\lambda > 0$ and $\lambda_1 \geq 0$ are given by

$$\lambda \equiv \frac{\kappa}{\theta}, \quad \lambda_1 \equiv \frac{\omega}{(1-\omega)\alpha} \tag{15}$$

Absent rule-of-thumb behavior, $\omega = 0$, (14) is easily seen to collapse to the welfare-based loss function in Woodford [21, Eq. 2.21 and 2.22, Ch. 6], which prescribes that the monetary authority should stabilize inflation and output around its efficient level.

It must be noted that the period loss function differs from the one in Steinsson [19] as the additional quadratic terms reported by the author can indeed be combined in a single quadratic term as given in (14).[7] In the presence of rule-of-thumb behavior by

[6] The third-order residual now includes a bound on the initial degree of price dispersion, Δ_{-1}. As in Woodford ([21], Chapter 6), Δ_{-1} is assumed to be of second order so that price dispersion continues to be only of second order in the case of first-order deviations of inflation from zero.

[7] This is because of an incorrect sign for the welfare-based loss function parameter Steinsson denotes with λ_4. We reported the correct sign to the author, who acknowledged it in the Erratum available at http://www.columbia.edu/~js3204/papers/STjme03erratum.pdf

price setters, the period loss function thus has one additional term with respect to its counterpart in the purely forward-looking NK model. Interestingly, this extra term can be seen as penalizing variations in the difference between inflation and the change in the price level due to rule-of-thumb price setters. In the presence of Galì-Gertler's rule-of-thumb behavior, rule-of-thumb price setters index their prices only to lagged inflation, namely $\delta = 0$ in (14), which would be reflected in the welfare-based loss function having a term in change in inflation, $\pi_t - \pi_{t-1}$. In the presence of Steinsson's rule-of-thumb behavior, rule-of-thumb price setters index their prices to both lagged inflation and lagged output gap, which is reflected in the term $[\pi_t - (\pi_{t-1} + (1-\alpha)\delta x_{t-1})]$.

As the fraction of rule-of-thumb price setters increases, the size of the additional term increases relative to the two terms that obtain in the purely forward-looking NK model. This in turn has implications as for the importance of inflation stabilization relative to output stabilization. Specifically, in the limit where the fraction of rule-thumb price setters goes to one (i.e. $\omega \to 1$), Steinsson's rule-of-thumb would still prescribe a concern for output stabilization whereas under Galì-Gertler's rule-of-thumb the relative importance of output stabilization would shrink to zero.

3 Optimal Long-run Inflation

In this section we study the optimal rate of inflation in a purely deterministic setting, certainty equivalence guarantees that the results we obtain hold in the presence of random disturbances. The monetary authority is assumed to be able to act under commitment.

The analysis of the optimal long-run inflation target under commitment takes the form of a constrained optimization problem. The monetary authority chooses bounded paths for inflation and the output gap, $\{\pi_t, x_t\}_{t=0}^{\infty}$, to minimize the welfare-based objective function (14) subject to the constraint that the sequences satisfy the hybrid

Phillips curve (7) each period.[8] We form the following Lagrangian

$$L = \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \frac{1}{2}\left[\pi_t^2 + \lambda(x_t - x^*)^2 + \lambda_1 \left[\pi_t - (\pi_{t-1} + (1-\alpha)\delta x_{t-1})\right]^2\right] \\ +\vartheta_t \left[\pi_t - \chi_f \beta E_t \pi_{t+1} - \chi_b \pi_{t-1} - \kappa_1 x_t - \kappa_2 x_{t-1}\right] \end{array} \right\} \tag{16}$$

where ϑ_t is the Lagrange multiplier associated with the period t aggregate-supply relation. Differentiation of the Lagrangian with respect to inflation and output gap, yields a pair of first-order conditions

$$\pi_t + \vartheta_t - \chi_f \vartheta_{t-1} - \beta \chi_b \vartheta_{t+1} + \lambda_1 \left\{ \begin{array}{c} \left[\pi_t - (\pi_{t-1} + (1-\alpha)\delta x_{t-1})\right] \\ -\beta \left[\pi_{t+1} - (\pi_t + (1-\alpha)\delta x_t)\right] \end{array} \right\} = 0 \tag{17}$$

$$\lambda(x_t - x^*) - \kappa_1 \vartheta_t - \beta \kappa_2 \vartheta_{t+1} - \beta \lambda_1 (1-\alpha)\delta \left[\pi_{t+1} - (\pi_t + (1-\alpha)\delta x_t)\right] = 0 \tag{18}$$

It is worth noting that the optimal long-run inflation target does not depend on the form of policy commitment. Technically, the difference between fully optimal (or zero-optimal) policy and timeless-perspective policy relates to the time invariance of the optimality conditions. On the one hand, the structure of the optimality conditions associated with timeless perspective is time invariant. On the other hand, under the zero-optimal policy, the inflation optimality condition in the initial period differs from that applying to all later periods (i.e. $\vartheta_{-1} = 0$ in (17) for $t = 0$). The different structure of the optimality conditions does not affect the determination of the steady-state inflation rate, but matters for the optimal transition paths towards the common target. Without loss of generality, we therefore consider timeless-perspective commitment policy, so that the optimality conditions above hold for each $t \geq 0$, and we later characterize the optimal transition paths under both commitment policies.

[8] Given equilibrium paths for inflation and output gap, the intertemporal IS equation then determines the setting for the nominal interest rates, which must always be nonnegative. This is true for all the cases we consider. Indeed, in the presence of rule-of-thumb price setters, the optimality of positive steady-state values for the inflation rate, hence the output gap, implies that the nominal interest rate is also positive at steady state.

Formally, the definition of the OLIR is the same as in Woodford [21, p. 475]: *a constant inflation target $\overline{\pi}$ is optimal from a timeless perspective if the problem of minimizing (14) subject to the constraint that the bounded sequences, $\{\pi_t, x_t\}_{t=0}^{\infty}$, satisfy (7) for each $t \geq 0$, and the additional constraint that $\pi_0 = \overline{\pi}$, has a solution in which $\pi_t = \overline{\pi}$ for all t.*

Condition (17) has a solution with inflation constant over time only if the Lagrange multiplier is also constant over time. The two optimality conditions can be simultaneously satisfied only if

$$\overline{\pi} = \frac{(1-\alpha)(1-\beta)(1-\omega)(\delta\theta-1)\alpha\omega\kappa}{(1-\omega)\alpha\theta\left[(1-\omega)\alpha\kappa+(1-\alpha)^2\beta\omega\delta\right]}\overline{x} + \frac{(1-\alpha)(1-\beta)\omega\kappa}{\theta\left[(1-\omega)\alpha\kappa+(1-\alpha)^2\beta\omega\delta\right]}x^* \tag{19}$$

where (8), (9) and (15) are used to replace the coefficients in the hybrid Phillips curve and the period loss function in terms of structural parameters. The hybrid Phillips curve (7) in turn implies an upward-sloping relation between steady-state inflation and the steady-state output gap as given by (13). Combining (19) and (13) yields the optimal long-run inflation target

$$\overline{\pi} = \frac{(1-\alpha)(1-\beta)\kappa\theta^{-1}\omega\left[(1-\omega)\alpha\kappa+(1-\alpha)(1-\alpha\beta)\omega\delta\right]}{\left\{\begin{array}{c}(1-\omega)(1-\alpha)(\theta^{-1}-\delta)(1-\beta)^2\alpha\omega\kappa+\\ \left[(1-\omega)\alpha\kappa+(1-\alpha)^2\beta\omega\delta\right]\left[(1-\omega)\alpha\kappa+(1-\alpha)(1-\alpha\beta)\omega\delta\right]\end{array}\right\}}x^* \tag{20}$$

Proposition 1 *Consider a cashless economy with flexible wages, Calvo pricing, rule-of-thumb behavior by price setters à la Steinsson, and no real disturbances. Assume that the initial dispersion of prices, Δ_{-1}, is small and real distortions (measured by Φ_y) are small as well, so that a quadratic approximation to the welfare of the representative household of the form (14) is possible, with $x^* > 0$ a small parameter ($x^* = O(\Phi_y)$). Then the policy that is optimal from a timeless perspective involves a constant inflation rate equal to the right-hand side of (20), up to an error of order $O(\left\|\Delta_{-1}^{1/2}, \Phi_y\right\|^2)$.*

3.1 Discussion

The optimal long-run rate of inflation in (20) depends on the model's seven structural parameters: α, β, θ, ϖ, σ^{-1}, δ and ω. Specifically, the OLIR is valid for the degree

of price stickiness entire range, namely $\alpha \in (0, 1)$, and for the degree of rule-of-thumb behavior entire range, namely $\omega \in [0, 1)$. It follows that (20) nests the optimal steady-state rate of inflation both in the presence of the purely forward-looking NKPC and the purely backward-looking Phillips curve. The relationship between the optimal long-run rate of inflation and the Phillips curve can be best summarized by means of a diagram plotting the OLIR against the proportion of price setters who use Steinsson's rule of thumb. To this end, we proceed to calibrate the model where the time period is one quarter. Table 1 summarizes the calibration.

Parameter Definition	Value
Subjective discount factor	$\beta = 0.99$
Output-elasticity of firms' marginal cost	$\varpi = 0.47$
Share of firms keeping prices fixed	$0.25 < \alpha < 0.85$
Share of rule-of-thumb firms	$0 \leq \omega < 0.99$
Degree of indexation to output gap by rule-of-thumb firms	$\delta = 0.052$
Price elasticity of demand	$\theta = 7.88$
Intertemporal elasticity of substitution	$\sigma = 6.25$

Table 1. Calibration (quarterly)

The values for four structural parameters (β, θ, ϖ, and σ^{-1}) are taken from tables 5.1 and 6.1 of Woodford [21]. The efficient level of the output gap, x^*, is accordingly set equal to 0.2, which is the value implied by (2), under the assumption that the elasticity of substitution among alternative differentiated goods is equal to 7.88. In calibrating the degree of indexation to lagged output gap, δ, by rule-of-thumb price setters, we follow Steinsson [19] and set it to 0.052.

The remaining two structural parameters are in fact the key model's primitives: the degree of price stickiness, α, and the degree of rule-of-thumb behavior, ω. In order to characterize the relationship between the OLIR and the Phillips curve, we consider the whole range for ω. As for the degree of price stickiness, the uncertainty surrounding

its estimates and the disconnect between estimates based on macro data or micro data are well-known. Available empirical estimates of the degree of price rigidity vary from less than two quarters to more than five quarters. We thus consider values of α ranging from 0.25 to 0.85.

Figure 1 shows the size of the optimal steady-state inflation rate in three dimensions, while the two-dimensional Figure 2 selects some specific values of α.

[Insert figure 1 here.]

[Insert figure 2 here.]

Figure 2 shows our main results. The outcome in the presence of the purely forward-looking NKPC lies at the extreme left: when $\omega = 0$, the OLIR is zero. The outcome in the presence of the purely backward-looking Phillips curve lies at the extreme right: when $\omega \to 1$, the OLIR is positive. All points in between represent the outcomes for setups in which the Phillips curve is hybrid: when $\omega \in (0, 1)$, the OLIR is positive. In particular, the OLIR is seen to rise monotonically with the degree of rule-of-thumb behavior. Moreover, the deviation from zero long-run inflation is observed to be small so that the policy-driven steady state is found to be close to the zero-inflation steady state around which the model is approximated.[9]

Given the hybrid nature of the Phillips curve in (7), higher inflation in any period increases output in the same period, but decreases output both in the previous period as a result of the anticipation of that higher inflation and in the subsequent period as a result of the realization of that higher inflation. This is shown in the inflation optimality condition (17): while the Lagrange multiplier associated with the period t aggregate-supply relation, namely ϑ_t, enters the inflation optimality condition with a positive sign, the Lagrange multipliers associated with the period $t-1$ and period $t+1$ aggregate-supply relation, respectively ϑ_{t-1} and ϑ_{t+1}, enter the inflation optimality condition with a negative sign. Moreover, the output costs of anticipated inflation and

[9] The inflation targets we derive are based on a hypothetical measure of inflation, which is free of measurement error.

realized inflation depend on the relative weights on lagged and future expected inflation in the hybrid Phillips curve. From the standpoint of the discounted welfare criterion, the stimulative effect of higher current inflation on output is greater or equal than the output cost of higher, both anticipated and realized, inflation if and only if

$$\vartheta_t - \chi_f \vartheta_{t-1} - \beta \chi_b \vartheta_{t+1} \geq 0 \;\Rightarrow\; 1 \geq \frac{\alpha + \beta\omega}{\alpha + \omega\left[1 - \alpha(1-\beta)\right]}$$

The solution is given by

$$\omega(1-\beta)(1-\alpha) \geq 0 \tag{21}$$

The inequality in (21) substantiates the positive and monotonic relationship between the OLIR and the degree of rule-of-thumb behavior, as depicted in Figure 2.

In the absence of rule-of-thumb behavior, $\omega = 0$, the Phillips curve becomes completely forward-looking, taking the NKPC form. In this case, the monetary authority does not find desirable to expand output in the present. If the monetary authority were to organize some output expansion, the private sector would anticipate higher inflation in the future. Specifically, the cost of having higher inflation in the future is discounted at the same rate by the policymaker and by the private sector. This in turn implies that is not worthwhile for the policymaker to engage in any output expansion. From the standpoint of the discounted welfare criterion, the output cost of higher anticipated inflation exactly offsets the stimulative effect of higher current inflation.[10] It follows that there is no welfare gain from a commitment to inflation that can be anticipated in advance. Indeed, it is only under the zero-optimal commitment policy that the monetary authority would arrange initial positive inflation as it would find optimal to exploit the fact that private sector's expectation are given. However, under both zero-optimal and timeless-perspective commitment policy the OLIR is zero, and this is true despite the inefficiency of the deterministic steady state and despite the existence of a long-run Phillips-curve trade-off, as given by (12). Non-zero steady-state inflation rate under

[10] This can be seen by setting $\omega = 0$ in the inflation optimality condition (17), namely $\pi_t + \vartheta_t - \vartheta_{t-1} = 0$.

commitment with the purely forward-looking NKPC can only obtain if the private sector and the policymaker do not share the same discount rate. Kirsanova et al. [10] show that if the monetary authority were to discount the future more heavily than the private sector, negative OLIR would arise as the stimulative effect of higher inflation would be judged to be worth less than the output cost of higher anticipated inflation.

In the limit case where $\omega \to 1$, the Phillips curve becomes completely backward-looking. In this case, the monetary authority finds desirable to expand output in the present. A temporary increase in output, which yields a finite gain, will yield an increase in inflation which is finite, because of the backward-looking nature of inflation. The policymaker is prepared, in the short run, to trade off the gain in utility, obtained from having a higher level of output, with the loss in utility caused by having higher inflation. It is important to note that loss in utility is generated only via the first term in the welfare-based loss function, namely the term in the inflation per se. The third term in the welfare-based loss function, which penalizes variations in the difference between inflation and the change in the price level due to rule-of-thumb price setters, is instead equal to zero under the purely backward-looking Phillips curve, thus implying no additional loss in utility. The increase in inflation is actually permanent. However, because of discounting, the monetary authority gives a finite value to the loss in utility caused by having permanently higher inflation. It follows that there is a welfare gain from a commitment to a positive OLIR. Indeed, with commitment and a fully backward-looking Phillips curve, equation (21) shows how zero OLIR would only obtain if the policymaker were not to discount the future (i.e. $\beta = 1$). In this case, permanent inflation would lead to losses with an infinite present value. It's important to note that the long-run Phillips-curve trade-off ceases to exist when $\omega \to 1$: the purely backward-looking Phillips curve (11) implies that the long-run Phillips curve is vertical. It follows that there is no long-run output increase as a result of inflation being positive at steady state. We thus obtain the well-known inflation bias stressed by Kydland and Prescott [11] and Barro and Gordon [3], namely a positive steady-state rate of inflation without any positive effect on the steady-state level of output. This reflects the fact that

optimization under discretion is truly optimal in backward-looking systems. Indeed, in the limit when $\omega \rightarrow 1$ expectations of inflation play no role in the Phillips curve, resulting in the discretionary and commitment solutions being identical.

In all intermediate cases, $0 < \omega < 1$, the Phillips curve becomes hybrid: expectations of future inflation matter less than in the NKPC and lagged inflation starts to matter. The symmetry that delivers zero OLIR under the purely forward-looking NKPC breaks down. The reduction in the magnitude of the effect of expected future inflation on current inflation implies that the monetary authority now finds desirable to expand output in the present. The policymaker trades off the loss in utility caused by having higher inflation with the gain in utility obtained from having a higher level of output. Under a hybrid Phillips curve, the third term in the welfare-based loss function generates additional losses in utility. This in turns explains the monotonic relationship between the OLIR and the degree of rule-of-thumb behavior. The simple intuition is that the more backward-looking the Phillips curve becomes the smaller will be the loss associated with the third term in the welfare-based loss function and so the more the monetary authority will increase output in the short run, and so the higher will inflation be. The increase in inflation is actually permanent. Because of discounting, the policymaker gives a finite value to the loss in utility caused by having permanently higher inflation. This in turns implies that there is a welfare gain from a commitment to a positive OLIR. Similarly to the purely backward-looking case, with commitment and a hybrid Phillips curve, equation (21) shows how zero OLIR would only obtain if the policymaker were not to discount the future (i.e. $\beta = 1$). In such case, permanent inflation would lead to losses with an infinite present value. Differently from the purely backward-looking case, the hybrid Phillips curve (13) implies an upward-sloping steady-state relation between inflation and output. Given the existence of a long-run Phillips curve trade-off, positive inflation in the steady state implies in turn a long-run output increase. In particular, the linear dependence of the inflation target on the efficient level of the output gap, x^*, conforms to the common belief that larger steady-state distortions justify a higher inflation in a model that incorporates an upward-sloping steady-state relation between

inflation and output.

In the presence of rule-of-thumb behavior there is thus a constant long-run inflation target associated with commitment policy. Two observations are worth making. First, in the absence of indexation to lagged output gap (i.e. $\delta = 0$), that is when rule-of thumb price setters behave as in Galì and Gertler [9], equation (20) collapses to the OLIR under commitment derived in Pontiggia [16]. In particular, the OLIR is observed to be negatively and monotonically related to δ. The degree of indexation to lagged output gap affects welfare via the third-term in welfare-based loss function: the higher the degree of indexation to lagged output gap, the higher is the loss in utility caused by having higher inflation. As a result, the monetary authority will find optimal to increase output by less as the degree of indexation to lagged inflation increases, thus implying a lower steady-state rate of inflation. Second, the form of commitment policy then matters as for the transition to the long-run target. Figure 3 illustrates the time paths of inflation under the zero-optimal commitment policy and under the timeless-perspective commitment policy.[11] Zero-optimal policy leads to the choice of a higher inflation rate in the early periods, reflecting the fact that the central bank exploits the initially given private sector's expectations of inflation, whereas timeless-perspective policy inflation implies that inflation is constant at its steady-state level.

[Insert figure 3 here.]

4 Conclusions

The Phillips curve represents the link between the business cycle and inflation and constrains the actions of policymakers.

The contribution we make in this paper is to analyze the optimal, under commitment, long-run inflation target in the presence of a hybrid Phillips curve, which nests a

[11] The figure is obtained employing the codes in Soderlind [18] under the assumption that the degree of price stickiness is equal to 0.33. In the presence of rule-of-thumb behaviour, we assume that inflation is at steady state in period $t-1$.

purely backward-looking Phillips curve and the purely forward-looking New Keynesian Phillips curve as special limiting cases. The Phillips curve displays such an appealing property as we consider rule-of-thumb price setters à la Steinsson [19]. In the tradition of Woodford [21], we analyze the steady-state inflation rate using a fully microfounded set-up. The welfare-based objective function we derive includes a correction to the one in Steinsson [19]. In the presence of rule-of-thumb behavior by price setters, we show how the period loss function has one additional term with respect to its counterpart in the purely forward-looking NK model. Interestingly, this extra term can be seen as penalizing variations in the difference between inflation and the change in the price level due to rule-of-thumb price setters.

The main finding is that the commitment solution for the monetary authority leads to steady-state outcomes in which inflation is positive. Starting from zero under the purely forward-looking NKPC, the optimal long-run rate of inflation rises monotonically with the degree of rule-of-thumb behavior.

Under the purely forward-looking NKPC, if the central bank were to organize some output expansion, the private sector would anticipate higher inflation in the future. As discussed in Woodford [21], the fact that the cost of having higher inflation in the future is discounted at the same rate by the monetary authority and by the private sector means that expanding output is not worth doing for the policymaker. From the standpoint of the discounted welfare-based loss function, the output cost of higher anticipated inflation exactly offsets the stimulative effect of higher current inflation. It follows that there is no welfare gain from a commitment to inflation that can be anticipated in advance. What we show here is that with a backward-looking, either partially or fully, Phillips curve there is a welfare gain from a commitment to a positive steady-state rate of inflation. The monetary authority is prepared, in the short run, to trade off the gain in utility, obtained from having a higher level of output, with the loss in utility caused by having higher inflation. The increase in inflation is actually permanent. However, because of discounting, the policymaker gives a finite value to the loss in utility caused by having permanently higher inflation. As a result, there is

a long-run incentive for positive inflation under an optimal commitment. Indeed, with commitment and a backward-looking Phillips curve, either partially or fully, zero OLIR would only obtain if the policymaker were not to discount the future. In this case, permanent inflation would lead to losses with an infinite present value and there would thus be no welfare gain from a commitment to a positive steady-state rate of inflation.

The consequences of positive inflation at steady state then differ between the limiting case of a purely backward-looking Phillips curve and the case of a hybrid Phillips curve. Under a hybrid Phillips curve, given the existence of a long-run Phillips curve trade-off, positive inflation in the steady state implies in turn a long-run output increase. In the presence of a purely backward-looking Phillips curve, the positive OLIR reaches its maximum level. However, the purely backward-looking Phillips curve implies that the long-run Phillips curve is vertical. It follows that there is no long-run output increase as a result of inflation being positive at steady state. We thus obtain the well-known inflation bias stressed by Kydland and Prescott [11] and Barro and Gordon [3], namely a positive steady-state rate of inflation without any positive effect on the steady-state level of output. This is because optimization under discretion is truly optimal in backward-looking systems. The inflation target we derive depends on the model's structural parameters. In particular, it is directly proportional to the gap that represents steady-state distortions originating from monopolistic competition: if one were to assume a subsidy to production aimed at offsetting these distortions, one would fail to obtain this result. We calibrate the model to U.S. data and we consider ample ranges for two key structural parameters. The inflation target turns out to be small in magnitude.

Our analysis is naturally sensitive to the basic NK setting, with the assumed rule-of-thumb behavior by price setters à la Steinsson [19] and Calvo [5] price staggering. On the one hand, this allows us to study the nature of OLIR in a single microfounded set-up which encompasses different Phillips curves. On the other hand, this leads us to ignore other factors that may influence the monetary authority when formulating the long-run inflation target. Non-negligible transaction frictions may justify a lower, and

likely negative, steady-state rate of inflation. Other considerations, such as the zero lower bound on nominal interest rates, downward wage rigidity or debt-deflation spiral, may instead justify a higher long-run inflation target.

Acknowledgement *I am particularly indebted to Campbell Leith, Anton Muscatelli, and Patrizio Tirelli for many helpful comments and guidance. I am very grateful to Raffaele Rossi and Giorgio Motta for valuable suggestions and comments. I would also like to thank participants at the 2014 MFS Symposium, Larnaca.*

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Appendix A. The hybrid Phillips curve

The Phillips curve is derived as a log-linear approximation to the dynamics of aggregate inflation in a model of Calvo [5] staggered price setting where a fraction of firms use the rule-of-thumb proposed by Steinsson [19]. The structural equations are log-linearized around the natural steady-state level of output, $\overline{Y}$. If $\widetilde{\xi}_t = 0$ and $Y_t = \overline{Y}$ at all times, (3) has a solution with zero inflation at all times. In the case of small enough fluctuations in $\widetilde{\xi}_t$ and Y_t around 0 and $\overline{Y}$ respectively, the solution to the log-linear approximate model is one in which any variable's log-deviation from its natural steady-state value, for instance, $\widehat{P}_t \equiv \log(P_t/\overline{P})$, remains always close to 0. We begin by log-linearizing (3)

$$\widehat{P}_t = (1-\alpha)\widehat{p}_t^* + \alpha\widehat{P}_{t-1} \tag{A.1}$$

with

$$\widehat{p}_t^* = (1-\omega)\widehat{p}_t^f + \omega\widehat{p}_t^b \tag{A.2}$$

As shown in [21, Ch. 3], a log-linearization to the *notional Short-Run Aggregate Supply* curve is given by

$$\log(p_t^f/P_t) = \zeta x_t \tag{A.3}$$

where ζ is the elasticity of the notional SRAS curve, which, under the assumption of specific labor markets, is given by $\zeta = (\sigma^{-1} + \varpi)(1+\varpi\theta)^{-1} > 0$. Combining (5) with (A.3) and quasi-differencing, we obtain

$$\widehat{p}_t^f = (1-\alpha\beta)\zeta x_t + (1-\alpha\beta)\widehat{P}_t + \alpha\beta E_t\widehat{p}_{t+1}^f \tag{A.4}$$

Log-linearizing (6) delivers

$$\widehat{p}_t^b = \widehat{p}_{t-1}^* + \pi_{t-1} + \delta x_{t-1} \tag{A.5}$$

Combining (A.1) with (A.2), the aggregate inflation rate, π_t, evolves according to

$$\pi_t = \frac{1-\alpha}{\alpha}\left[(1-\omega)(\widehat{p}_t^f - \widehat{P}_t) + \omega(\widehat{p}_t^b - \widehat{P}_t)\right] \tag{A.6}$$

Using (A.1) at time $t-1$, $\widehat{p}_t^b - \widehat{P}_t$ is given by

$$\widehat{p}_t^b - \widehat{P}_t = \frac{1}{1-\alpha}\pi_{t-1} - \pi_t + \delta x_{t-1} \tag{A.7}$$

Rewriting (A.4) in terms of $\widehat{p}_t^f - \widehat{P}_t$ yields

$$\widehat{p}_t^f - \widehat{P}_t = (1-\alpha\beta)\zeta x_t + \alpha\beta E_t(\widehat{p}_{t+1}^f - \widehat{P}_t) \tag{A.8}$$

Combining, at time $t+1$, (A.2) and (A.5) gives

$$\widehat{p}_{t+1}^* - \widehat{P}_t = (1-\omega)(\widehat{p}_{t+1}^f - \widehat{P}_t) + \omega(\widehat{p}_t^* - \widehat{P}_{t-1} + \delta x_t) \tag{A.9}$$

(A.1) can be rewritten as

$$\widehat{p}_t^* - \widehat{P}_{t-1} = \frac{1}{1-\alpha}\pi_{t-1} \tag{A.10}$$

Accordingly substituting in (A.9) and taking the expected value at t, yields

$$E_t(\widehat{p}_{t+1}^f - \widehat{P}_t) = \frac{1}{(1-\alpha)(1-\omega)}E_t(\pi_{t+1} - \omega\pi_t) - \frac{\omega\delta}{(1-\omega)}x_t \tag{A.11}$$

Substituting (A.11) in (A.8), $\widehat{p}_t^f - \widehat{P}_t$ is given by

$$\widehat{p}_t^f - \widehat{P}_t = (1-\alpha\beta)\zeta x_t + \frac{\alpha\beta}{(1-\alpha)(1-\omega)}E_t(\pi_{t+1} - \omega\pi_t) - \frac{\alpha\beta\omega\delta}{(1-\omega)}x_t \tag{A.12}$$

Combining (A.6) with (A.7) and (A.12), we obtain Eq. (7) in the main text, with the parameters defined as in (8) and (9).

Appendix B. Welfare-based loss function

Under Calvo staggered price setting and backward-looking rule-of-thumb behavior by price setters, the distribution of prices in any period, $\{p_t(i)\}$, consists of α times the distribution of prices in the previous period, $\{p_{t-1}(i)\}$, an atom of size $(1-\alpha)(1-\omega)$ at the forward-looking reset price, p_t^f, and an atom of size $(1-\alpha)\omega$ at the rule-of-thumb backward-looking reset price, p_t^b

$$\{p_t(i)\} = \alpha \{p_{t-1}(i)\} + (1-\alpha)(1-\omega)p_t^f + (1-\alpha)\omega p_t^b \tag{B.1}$$

Let $\Delta_t \equiv var_i \log p_t(i)$ denote the degree of price dispersion and $\overline{P}_t \equiv E_i \{\log p_t(i)\}$ denote the average price, hence $\overline{P}_t - \overline{P}_{t-1} = E_i \left[\log \{p_t(i)\} - \overline{P}_{t-1}\right]$. Recalling $\log p_t^* = (1-\omega)\log p_t^f + \omega \log p_t^b$ and using (B.1), $\overline{P}_t - \overline{P}_{t-1}$ can be rewritten as

$$\begin{aligned} \overline{P}_t - \overline{P}_{t-1} &= \overbrace{\alpha E_i \left[\{\log p_{t-1}(i)\} - \overline{P}_{t-1}\right]}^{0} + (1-\alpha)(1-\omega)(\log p_t^f - \overline{P}_{t-1}) \\ &\quad +(1-\alpha)\omega(\log p_t^b - \overline{P}_{t-1}) \\ &= (1-\alpha)(\log p_t^* - \overline{P}_{t-1}) \end{aligned} \tag{B.2}$$

Similarly, Δ_t can be rewritten as

$$\begin{aligned} \Delta_t &= var_i \left[\log \{p_t(i)\} - \overline{P}_{t-1}\right] = E_i \left\{ \left[\log \{p_t(i)\} - \overline{P}_{t-1}\right]^2 \right\} \\ &\quad - \left[E_i \log \{p_t(i)\} - \overline{P}_{t-1}\right]^2 \\ &= \begin{bmatrix} \alpha E_i \left\{ \left[\log \{p_{t-1}(i)\} - \overline{P}_{t-1}\right]^2 \right\} \\ +(1-\alpha)(1-\omega)(\log p_t^f - \overline{P}_{t-1})^2 \\ +(1-\alpha)\omega(\log p_t^b - \overline{P}_{t-1})^2 - (\overline{P}_t - \overline{P}_{t-1})^2 \end{bmatrix} \end{aligned} \tag{B.3}$$

$\overline{P}_t$ is related to the Dixit-Stiglitz price index through the log-linear approximation

$$\overline{P}_t = \log P_t + O\left(\left\|\Delta_{-1}^{1/2}, \widetilde{\xi}, \varphi\right\|^2\right) \tag{B.4}$$

the second-order residual follows from the fact that the equilibrium inflation process (as the equilibrium output process) satisfies a bound of second order $O(\left\|\widetilde{\xi}, \varphi\right\|^2)$ together with a second-order bound on the initial (i.e. date -1, policy is implemented at date

0) degree of price dispersion, Δ_{-1}. Note that, as in [21], Δ_{-1} is assumed to be of second order (that is why it enters the second-order residual in (B.4) to the power of 1/2). It then follows that this measure of price dispersion continues to be only of second order in the case of first-order deviations of inflation from zero. Recalling $\log p_t^b = \log p_{t-1}^* + \pi_{t-1} + \delta x_{t-1}$ and using (B.4), $\log p_t^b - \overline{P}_{t-1}$ is given by

$$\begin{aligned} \log p_t^b - \overline{P}_{t-1} &= \log p_{t-1}^* - \overline{P}_{t-2} - (\overline{P}_{t-1} - \overline{P}_{t-2}) + \pi_{t-1} + \delta x_{t-1} \\ &= \log p_{t-1}^* - \overline{P}_{t-2} + \delta x_{t-1} + O\left(\left\|\Delta_{-1}^{1/2}, \widetilde{\xi}, \varphi\right\|^2\right) \end{aligned} \tag{B.5}$$

Recalling $\log p_t^* = (1-\omega)\log p_t^f + \omega \log p_t^b$, $\log p_t^b = \log p_{t-1}^* + \pi_{t-1} + \delta x_{t-1}$, and using (B.4), $\log p_t^f - \overline{P}_{t-1}$ is given by

$$\begin{aligned} \log p_t^f - \overline{P}_{t-1} &= \frac{1}{1-\omega}\log p_t^* - \frac{\omega}{1-\omega}(\log p_{t-1}^* + \pi_{t-1} + \delta x_{t-1}) - \overline{P}_{t-1} \\ &= \left[\begin{array}{c} \frac{1}{1-\omega}(\log p_t^* - \overline{P}_{t-1}) - \frac{\omega}{1-\omega}(\log p_{t-1}^* - \overline{P}_{t-2}) \\ -\frac{\omega\delta}{1-\omega}x_{t-1} + O\left(\left\|\Delta_{-1}^{1/2}, \widetilde{\xi}, \varphi\right\|^2\right) \end{array}\right] \end{aligned} \tag{B.6}$$

Using (B.4), (B.2) can be rewritten as

$$\pi_t = (1-\alpha)(\log p_t^* - \overline{P}_{t-1}) + O\left(\left\|\Delta_{-1}^{1/2}, \widetilde{\xi}, \varphi\right\|^2\right) \tag{B.7}$$

Accordingly, (B.5) and (B.6) become respectively

$$\log p_t^b - \overline{P}_{t-1} = \frac{1}{1-\alpha}\pi_{t-1} + \delta x_{t-1} + O\left(\left\|\Delta_{-1}^{1/2}, \widetilde{\xi}, \varphi\right\|^2\right) \tag{B.8}$$

$$\begin{aligned} \log p_t^f - \overline{P}_{t-1} &= \frac{1}{(1-\omega)(1-\alpha)}(\pi_t - \omega\pi_{t-1}) - \frac{\omega\delta}{(1-\omega)}x_{t-1} \\ &\quad + O\left(\left\|\Delta_{-1}^{1/2}, \widetilde{\xi}, \varrho\right\|^2\right) \end{aligned} \tag{B.9}$$

Substituting (B.4), (B.8), and (B.9) in (B.3), we get that

$$\begin{aligned} \Delta_t &= \alpha\Delta_{t-1} + \frac{\alpha}{(1-\alpha)}\pi_t^2 + \frac{\omega}{(1-\omega)(1-\alpha)}\left[\pi_t - \pi_{t-1} - (1-\alpha)\delta x_{t-1}\right]^2 \\ &\quad + O\left(\left\|\Delta_{-1}^{1/2}, \widetilde{\xi}, \varphi\right\|^2\right) \end{aligned}$$

Integrating forward, starting from any small initial degree of price dispersion, Δ_{-1}, the degree of price dispersion in any period $t \geq 0$ is given by

$$\begin{aligned} \Delta_t &= \sum_{s=0}^{\infty} \alpha^{t-s} \left[\begin{array}{c} \frac{\alpha}{(1-\alpha)} \pi_t^2 + \\ \frac{\omega}{(1-\omega)(1-\alpha)} \left[\pi_t - \pi_{t-1} - (1-\alpha)\delta x_{t-1} \right]^2 \end{array} \right] + \alpha^{t-1} \Delta_{-1} \\ &\quad + O\left(\left\| \Delta_{-1}^{1/2}, \widetilde{\xi}, \varphi \right\|^3 \right) \end{aligned} \tag{B.10}$$

The term $\alpha^{t-1}\Delta_{-1}$ is independent of monetary policy. Taking the discounted value of (B.10) over all periods $t \geq 0$ gives

$$\begin{aligned} \sum_{t=0}^{\infty} \beta^t \Delta_t &= \frac{1}{1-\alpha\beta} \sum_{t=0}^{\infty} \beta^t \left[\begin{array}{c} \frac{\alpha}{(1-\alpha)} \pi_t^2 + \\ \frac{\omega}{(1-\omega)(1-\alpha)} \left[\pi_t - \pi_{t-1} - (1-\alpha)\delta x_{t-1} \right]^2 \end{array} \right] \\ &\quad + t.i.p + O\left(\left\| \Delta_{-1}^{1/2}, \widetilde{\xi}, \varphi \right\|^3 \right) \end{aligned} \tag{B.11}$$

Taking the discounted value of (1) in the main text over all periods $t \geq 0$ delivers

$$\begin{aligned} \sum_{t=0}^{\infty} \beta^t U_t &= -\Psi \left[(\sigma^{-1} + \varpi) \sum_{t=0}^{\infty} \beta^t (x_t - x^*)^2 + (1 + \varpi\theta)\theta \sum_{t=0}^{\infty} \beta^t \Delta_t \right] \\ &\quad + t.i.p + O\left(\left\| \Phi_y, \widetilde{\xi}, \varphi \right\|^3 \right) \end{aligned} \tag{B.12}$$

Combining (B.11) with (B.12) and normalizing on inflation, we obtain Eq. (14) in the main text, with the parameters defined as in (15).

Figures

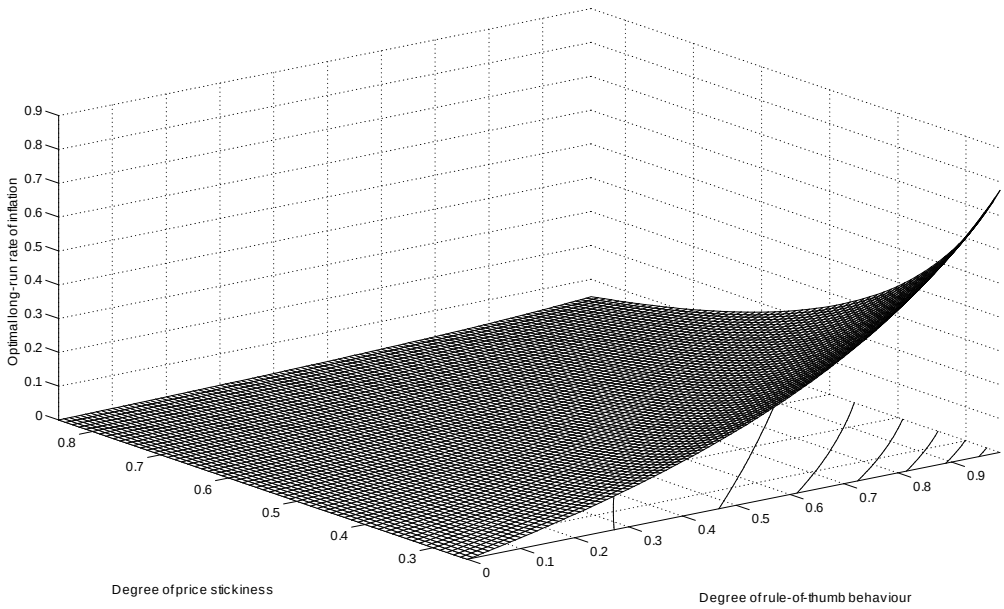

Figure 1: Optimal long-run inflation target in percentage points (z-axis) for varying degrees of price stickiness (x-axis) and rule-of-thumb behaviour by price setters (y-axis).

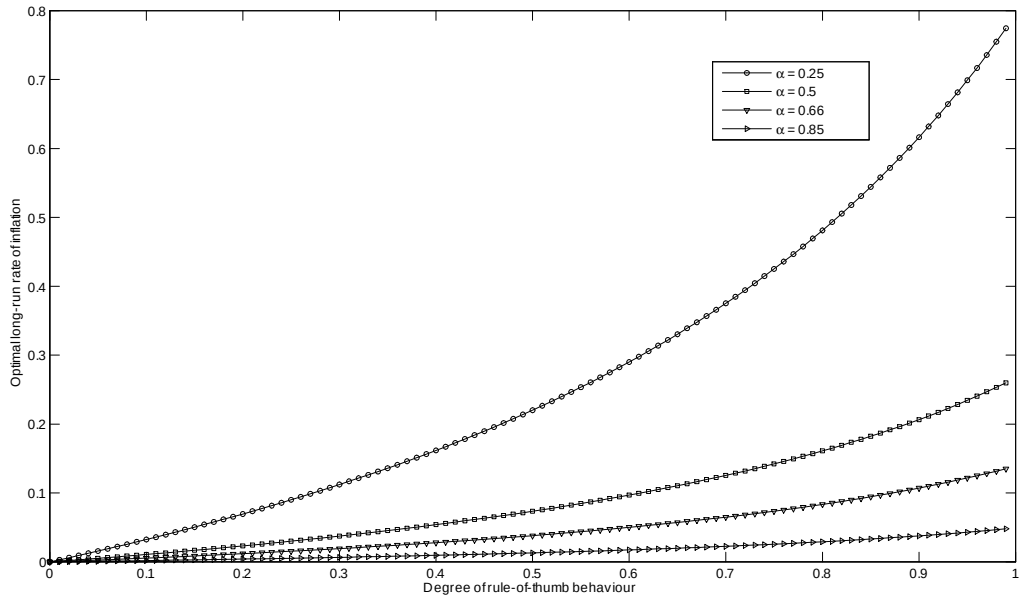

Figure 2: Optimal long-run inflation target in percentage points (y-axis) for varying degrees of rule-of-thumb behaviour by price setters (x-axis) and selected degrees of price stickiness.

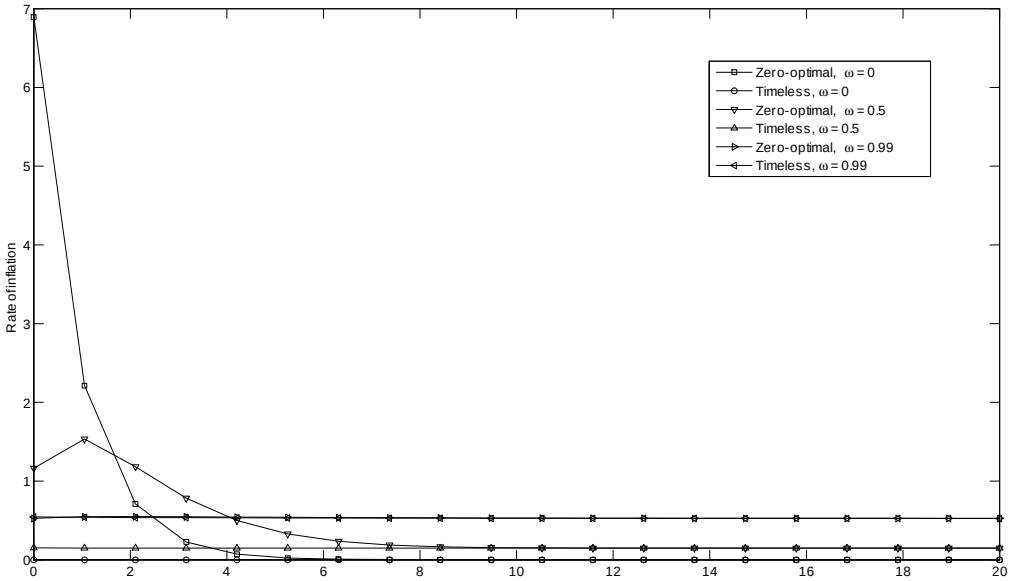

Figure 3: Optimal transition paths for the rate of inflation in percentage points under timeless-perspective commitment policy and zero-optimal commitment policy. Quarters on the horizontal axis.