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Thw performance of demand models into the Canadian economy

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THE PERFORMANCE OF DEMAND MODELS INTO THE CANADIAN ECONOMY

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INTRODUCTION

This paper presents four consumer demand models. The Rotterdam, the Linear Expenditure, the Indirect Addilog and the Translogarithmic. In the first part their derivation and theoretical properties are outlined. Afterwards all models are estimated empirically with the help of the same set of data.

The paper aims at providing additional sets of estimates for the important coefficients of the models for the Canadian market and not to question the validity of the Neoclassical demand theory. However, the estimates of the Rotterdam and Translogarithmic model are checked for their compatibility with the theory.

THE ROTTERDAM MODEL

If prices (p), quantities (q) and income y) are related in the form:

$$(1) \quad d(\log q_i) = e_{io}\, d(\log y) + \sum_{j=1}^{n} b_{ij}\, d(\log p_j) \qquad i,j = 1,2 \ldots n$$

then b_{io} and e_{ij} stand respectively for the compensated cross price elasticity and the income elasticity of the ith good.

Multiplying both sides by $p_i q_i / y$ we get the expenditure share weighted equation

$$(2)' \quad w_i\, d(\log q_i) = \mu_i\, d(\log y) + \sum_{j=1}^{n} \Pi_{ij}\, d(\log p_j)$$

where:

$$\mu_i = \frac{\partial(p_i q_i)}{\partial y} \qquad \Pi_{ij} = \frac{p_i p_j}{y} S_{ij} \qquad i,j = 1,2 \ldots n$$

If this function were to be thought as a Neoclassical demand function, it would have to satisfy the Neoclassical properties, namely homogeneity, symmetry, semi-negativity and definiteness of the Slutsky matrix and finally the "adding up" property.

To be specific "adding up" requires $\sum_{i=1}^{n} \mu_i = 1$; Homogeneity requires $\sum_{j=1}^{n} \Pi_{ij} = 0$; The Slutsky condition implies symmetry of the matrix $[\Pi_{ij}]$ since Π's are directly related with the Slutsky terms. Finally second order maximization conditions require that the matrix $[\Pi_{ij}]$ be negative semi-definite.

It is important to observe that (2) is an apriori formulation and not the outcome of a utility maximization problem. It is therefore neccessary to be checked if it really satifies the Neoclassical demand condition every time it is estimated.

LINEAR EXPENDITURE DEMAND MODEL

Given the utility function:

$$(3)\quad U = \prod_{i=1}^{n} (q_i - \gamma_i)^{\beta_i} \qquad i = 1, 2 \ldots n$$

A monotonic transformation of it might be

$$(4)\quad V = \ln U = \sum_{i=1}^{n} \beta_i \ln (q_i - \gamma_i)$$

To maximize (4) under the budget constraint, the Langrangian has to be formed $L = \sum_{i=1}^{n} \beta_i \ln (q_i - \gamma_i) - \lambda \left[\sum_{i=1}^{n} p_i q_i - y \right]$

Deriving the first order conditions and solving the resulting system of N+1 equations we obtain $q_j p_j = \gamma_j p_j + \frac{\beta_j}{\Sigma \beta_i} (y - \Sigma p_i \gamma_i)$

And by application of the normalization rule

$\sum_{i=1}^{n} \beta_i = 1$: $\quad (5)\quad q_j p_j = \gamma_j p_j + \beta_j (y - \Sigma p_i \gamma_i)$

which is known as the Linear Expenditure Demand Model.

The parameters of this model are β's and γ's. β_i is the derivative of expenditure on good i with respect to income. γ_i is the "subsistance" or minimum level of quantity for the ith good. Obviously the expenditure level on ith good is determined by the basic level of consumption plus a certain proportion of the income that is left after all "basic needs" $(\Sigma p_i \gamma_i)$ are satisfied.

The model fulfills the "adding up" property. It is also homogeneous of degree zero in prices and income. The Slutsky term is of the form $S_{Kj} = \frac{\beta_k \beta_j}{p_k p_j} (y - \Sigma p_i \gamma_i)$ and hence symmetry if present. But the fact that S_{Kj} is always greater than zero imposes the important restriction that all goods have to be substitutes. Of course the own substitution effect in negative

$$S_{KK} = \frac{q_K - \gamma_K}{p_K} (\beta_K - 1)$$

THE INDIRECT ADDILOG DEMAND MODEL

An indirect utility function is obtained through the introduction of the optimal quantities of the individual goods in a classical utility function. It is thus a function of prices and income. Clearly: $q_i^{o} = D(p_1, p_2 \ldots p_n, y)$

i.e. the quantity of ith good that is determined through the optimization process. The indirect utility function will be

$$U^{o} = U(q_1^{o}, q_2^{o} \ldots q_n^{o}) \Rightarrow U^{o} = U[D_1(p_1, p_2 \ldots p_n\, y), D_2(p_1, p_2, \ldots p_n, y), \ldots D_n(p_1, p_2, \ldots p_n, y)] \Rightarrow$$

$$(6) \quad U^{o} = V(p_1, p_2, \ldots p_n, y)$$

Due to Roy's theorem $q_i = -\dfrac{\partial U^{o}/\partial p_i}{\partial U^{o}/\partial y}$

let $U^{o} = V(p_1, p_2 \ldots p_n, y) = \sum_{i=1}^{n} a_i \left(\frac{y}{p_i}\right)^{b_i}$ where a_i is a preference indicator and b_i a reaction coefficient. The derived demand functions are

$$(7) \quad q_i = \frac{a_i b_i y^{b_i} p_i^{-b_i - 1}}{\sum_{j=1}^{n} a_j b_j y^{b_j - 1} p_j^{-b_j}} \qquad i, j = 1, 2 \ldots n$$

Although the derivation of the above equations didn't follow the common pattern they absolutely satisfy the Neoclassical properties. Note that the second order conditions on the Slutsky matrix requires that $b_i > -1$

THE TRANSCENDENTAL LOGARITHMIC UTILITY FUNCTIONS

The translog demand function can be derived from direct or indirect translog utility functions.

If the direct utility function is quadratic logarithmic in the quantities consumed $(8) \quad -\ln U = a_0 + \sum_{i=1}^{n} a_i \ln q_i + \frac{1}{2} \sum_{i}^{n} \sum_{j}^{n} b_{ij} \ln q_i \ln q_j$

and the budget constraint $\sum p_i q_i = m$ the first order conditions for optimization will be

$$\frac{\partial \ln V}{\partial \ln q_i} = \lambda \frac{q_i P_i}{U} \quad i = 1,2 \ldots n \qquad \frac{\lambda}{U} \sum q_i P_i = \sum \frac{\partial \ln U}{\partial \ln q_i}$$

Combining the above expressions.

$$\frac{\partial \ln V}{\partial \ln q_i} = \frac{q_i P_i}{m} \sum \frac{\partial \ln U}{\partial \ln q_i} \Rightarrow \frac{q_i P_i}{m} = w_i = \frac{\frac{\partial \ln V}{\partial \ln q_i}}{\sum \frac{\partial \ln V}{\partial \ln q_i}}$$

Replacing the terms of the ratio with the differentials of the specified form of the function assuming symmetry

$$w_i = \frac{q_i P_i}{m} = \frac{a_i + \sum_j^n b_{ij} \ln q_j}{\sum_k^n a_K + \sum_k^n \sum_j^n b_{Kj} \ln q_j}$$

To simplify the notation we define $(9)\ a_m = \sum_k^n a_K$, $(10)\ b_{mi} = \sum_K^n b_{Ki}$

thus $(11)\ w_i = \dfrac{a_i + \sum b_{ij} \ln q_j}{a_m + \sum_j b_{mj} \ln q_j} \quad i = 1,2 \ldots n$

Very similar is the process and the result when an indirect translog utility function quadratic in the logarithms of the ratios of prices to the total expenditure is the starting point. i.e.

$$(12)\quad \ln U = a_0 + \sum_i^n a_i \ln \frac{P_i}{m} + \frac{1}{2} \sum_i^n \sum_j^n b_{ij} \ln \frac{P_i}{m} \ln \frac{P_j}{m}$$

following the routine way an indirect demand translog function is obtained $(13)\ w_i = \dfrac{a_i + \sum b_{ij} \ln P_j/m}{a_m + \sum b_{mj} \ln P_j/m} \quad i = 1,2 \ldots n$

Equations (11) and (13) are homogeneous of degree zero in the parameters. Thus the normalization rule $a_m = \sum_k^n a_K = -1$ is applied in both.

THE DATA

The source of our data is Statistics Canada (Canadian Statistical Review). The estimation is based on time series quarterly data, seasonaly adjusted, for the period 1966-1976 included. It is on a national level and is referring to expenditures and implicit prices indexes for four consumer categories of goods and services. Durable goods, semi-durable goods, non-durable goods and services.

Total national and personal income and savings is also available in quarterly, seasonaly adjusted data for the same period.

ESTIMATION

ROTTERDAM DIFFERENTIAL MODEL

For purposes of estimation it is preferred to use the discrete version of the model. (14) $w^{o}_{it} Dq_{it} = \mu_i Dq_t + \sum_{j=1}^{n} \Pi_{ij} Dq_{jt} + u_{it}$ where D stands for the log difference. i.e. $Dq_{it} = \log q_{it} - \log q_{it-1}$

also: $w^{o}_{it} = \frac{1}{2}(w_{it-1} + w_{it})$ and $Dq_t = \sum_{i=1}^{n} w^{o}_{it} Dq_{it}$

Given that the random terms u_{it} of the model are uncorrelated across observations but correlated across equations

i.e. $E[u_{is} u_{jt}] = w_{ij}$ $s = t$
$= 0$ $s \neq t$ $i, j = 1, 2 \ldots n$

The contemporaneous variance covariance matrix Ω is singular.

However, the n equations are not independent. In fact if we add the first n-1 equations: $\sum_{i=1}^{n-1} w_{it} Dq_{it} = \sum_{i=1}^{n-1} \mu_i Dq_t + \sum_{j=1}^{n} (\sum_{i=1}^{n-1} \Pi_{ij}) Dp_{jt} + \sum_{i=1}^{n-1} u_{it}$

Employing the previous relations $\sum^{n} \mu_i = 1$, $\sum_{i}^{n} \Pi_{ij} = 0$ $\sum_{i}^{n} u_{it} = 0$

we obtain $Dq_t - w^{o}_{nt} Dq_{it} = (1-\mu_n) Dq_t + \sum_{1}^{n} (-\Pi_{nj}) Dp_t - u_{nt}$ $\Rightarrow$

$$(15) \quad w^{o}_{nt} Dq_{it} = \mu_n Dq_t + \sum_{j=1}^{n} \Pi_{nj} Dp_t + u_{nt}$$

which is exactly the nth equation. This implies that the nth equation can be deleted without loosing any of the system's information.

Let us now turn to our own empirical analysis where four commodity groups are considered. After deletion of the fourth equation the restrictions on Π^{s} will be the following:

For symmetry

a) $\Pi_{12} = \Pi_{21}$ $\Pi_{13} = \Pi_{31}$ $\Pi_{23} = \Pi_{32}$

For homogeneity

b) $\Pi_{11} + \Pi_{12} + \Pi_{13} + \Pi_{14} = 0$

$$\Pi_{21} + \Pi_{22} + \Pi_{23} + \Pi_{24} = 0$$

$$\Pi_{31} + \Pi_{32} + \Pi_{33} + \Pi_{34} = 0$$

For the second order conditions

c) $\Pi_{11} \leq 0 \quad \begin{vmatrix} \Pi_{11} & \Pi_{12} \\ \Pi_{21} & \Pi_{22} \end{vmatrix} \geq 0 \quad \begin{vmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} \\ \Pi_{21} & \Pi_{22} & \Pi_{23} \\ \Pi_{31} & \Pi_{32} & \Pi_{33} \end{vmatrix} \leq 0$

It is easily understood that the restrictions $\Pi_{14} = \Pi_{41}$, $\sum_{i=1}^{4} \mu_i = 1$ and $\sum_{j=1}^{4} \Pi_{4j} = 0$ are no longer effective.

Following every other previous work in the field we imposed homogeneity.

$$(16) \quad w_{it}^{o} D q_{it} = \mu_i D q_t + \sum_{j=1}^{3} \Pi_{ij} (D p_{jt} - D p_{4t}) + u_{it} \qquad i = 1, 2, 3$$

$$E(u_{it}) = 0, \quad E(u_{is} u_{jt}) = 0 \quad s \neq t; \qquad = w \quad s = t$$

and performed the estimation.

The results are presented in TABLE 1. The model is similar to what Zellner calls "seemingly unrelated" regression equations. The variance-covariance matrix of disturbances $\overline{\Omega}$ is not any longer a singular one. Note that the regressors are identical in each equation. Therefore the Generalized Least Squares estimates absolutely coincide with Ordinary Least Squares and they are best and unbiased in statistical sense. Estimates of the model including a constant term also appear.

<u>TABLE 1</u>

<u>Unconstrained Estimates of the Rotterdam Model</u>

	μ_i	π_{ij}				a_i	μ_i	π_{ij}			
Durables	.004 (.0009)	-.09 (.018)	.102 (.05)	-.015 (.004)	.003	-.246 (.0006)	.245 (.656)	.111 (0007)	-.032 (.001)	.010 (.0009)	-.089
Semi-durables	.004 (.0009)	-.136 (.038)	.021 (.009)	.167 (.081)	-.324	-.224 (.0006)	.228 (.0005)	-.033 (.0005)	.105 (.0009)	.011 (.0007)	-.083
Non-durables	.09 (.01)	-.05 (.020)	.193 (.049)	-.034 (.008)	-.107	-.474 (.0006)	.475 (.0006)	-.070 (0006)	-.067 (.001)	.312 (.0009)	-.175
Services	.902	.006	-.316	-.118	.428		.052	-.008	-.006	-.333	.347

The Π_{ij} matrix is not a symmetric one but symmetry cannot be rejected on the basis of a simple observation of the estimates. Theil has developed a rigorous process to test for symmetry which consists of the following.

He redefines the model as a stack regression one which has a non singular variance-covariance matrix of the disturbances Σ. Assuming that this is known the G.L.S. estimator is $\hat{\beta}=(Z'\Sigma^{-1}Z)^{-1}Z'\Sigma^{-1}Y$ where Z is the regressor's matrix.

The quadratic form in the residuals associated with the G.L.S. estimator $(Y-Z\hat{\beta})'\Sigma^{-1}(Y-Z\hat{\beta})$ has a $\chi^2(N-K)$ distribution.

Also the expression $\hat{\beta}'R'[R(Z'\Sigma^{-1}Z)^{-1}R]^{-1}R\hat{\beta}$ (17) is distributed as a $\chi^2(q)$ and it is independent of (17). Therefore their ratio follows an $F(q, N-K)$ distribution.

$$F = \frac{\hat{\beta}'R'[R(Z'\Sigma^{-1}Z)^{-1}R]^{-1}R\hat{\beta}}{(Y-Z\hat{\beta})'\Sigma^{-1}(Y-Z\hat{\beta})} \times \frac{N-K}{q}$$

And this is how symmetry is tested.

Note that R is a known matrix because the symmetry constraints can be written as $R\beta=0$. The degree of accuracy of the test somehow deteriorates due to the fact that Σ is not given and should be approximated.

To avoid this and the computational difficulties involved in the calculation of the above given matrices an alternative test proposed by E. Malinvaud and asymptotically equivalent to that of Theil was actually used in our project. It is known as the "Maximum Likelihood Test" and is based on the ratio

$$\lambda = \frac{\max L(\hat{\Omega})}{\max L(\Omega)}$$

where $\max L(\hat{\Omega})$ is the maximum value of the likelihood function of the constrained model.

It is proved that $-2\ln\lambda$ follows a chi-square distribution with degrees of freedom equal to the number of the restrictions to be tested, thus:

$$(18) \quad \chi^2_{(df)} \sim -2\ln\lambda = n[\ln \max L(\hat{\Omega}) - \ln \max L(\Omega)]$$

The null hypothesis of symmetry is tested and accepted. (The logarithms of the Likelihood functions are 811.089 and 789.296 for the unconstrained and contrained version respectively.) The following table contains the parameter estimates of the system subject to symmetry constraints.

TABLE 2

Symmetry constrained estimates

	μ_i	π_{ij}			
Durables	.007	-.113	.134	-.011	-.010
	(.002)	(.05)	(.005)	(.003)	
Semi-durables	.007		-.280	.264	-.118
	(.002)		(.059)	(.008)	
Non-durables	.014			-.248	-.005
	(.04)			(.018)	
Services	.72				.0268

The diagonal elements of the matrix are negative except one. It also fails to satisfy the second order conditions for utility maximization.

The Durbin-Watson statistic was sufficiently high in almost all regressions as to reject the existence of serial correlation. The Durbin-Watson test was inconclusive only in two cases of the unrestricted model including a constant term.

Linear Expenditure Model

Since the Linear Expenditure Model is not linear in the parameters the traditional way of estimation was based on an iterative process. An initial set of values for the β's was adopted. Then, after a proper transformation a set for the γ's can be estimated which in turn is used to obtain β's. The method continues until a certain degree of convergence is achieved.

Although this method which was suggested by R. Stone is rather of "historical" interest, it is worhtwhile to be demonstrated mathematically.

Starting with initial values of β and transforming the system as:

$$(19) \quad q_j p_j - \beta_j y = \Sigma \gamma_i (\delta_{ji} - \beta_j) p_i + u_j \qquad \delta_{ji} = 1 \quad i=j, \quad = 0 \quad i \neq j$$

or $(20) \quad w_j = X_j \gamma + u_j \qquad j = 1, 2 \ldots n$

where $X_j = [(\delta_{ji} - \beta_j) p_i]$

combining equations (20) into the system $(21) \quad W^{\infty} = X^{\infty} \gamma + u^{\infty}$

where

$$W^{\infty} = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} \qquad X^{\infty} = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix} \qquad U^{\infty} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}$$

This can be estimated through an O.L.S.Q. to obtain γ. Then using the γ estimates a new set for β is obtained through the equations (22) $q_j p_j - \gamma_j p_j = \beta_j (y - \sum_{i=1}^{n} \gamma_i p_i) + u_j \quad j = 1, 2 \ldots n$

Thanks to relatively recent availability of non-linear estimation procedures our model was directly estimated saving time and energy.

TABLE 3

Estimates of the Linear Expenditure Model Parameters through a Non-Linear Technique

β_1	.05	(.03)
β_2	.07	(.02)
β_3	.15	(.03)
β_4	.13	(.08)
γ_1	119.66	(35.63)
γ_2	14.52	(7.25)
γ_3	767.55	(392.54)
γ_4	682.24	(638.37)

The parameters of β's and γ's are all positive. Note that a negative β indicates the existence of an inferior good which is not easily accomodated under the theory underlying the model. Parks also estimates a model incorporating a linear trend but we don't feel that there is adequate reasoning for doing so, besides the fact, of course, that it might bring additional evidence regarding the performance of the model.

The Addilog Demand Functions

The indirect addilog model is of the form (23) $q_i = \frac{a_i b_i y_t^{b_i - 1} p_{it}^{-b_i - 1} e^{\varepsilon_{it}}}{\sum_{j=1}^{n} a_j b_j y_t^{b_j - 1} p_j^{-b_j}}$ and the stochastic factor has the properties

$$E[\varepsilon_{it}] = 0 \qquad E[\varepsilon_{it}\varepsilon_{js}] = 0 \quad t \neq s$$
$$= w \quad t = s$$

For estimation purposes the expenditure shares version has to be considered. $w_i = \frac{q_i p_i}{y} = \frac{a_i b_i (p/y)_t^{b_i} e^{\varepsilon_{it}}}{\sum_{j=1}^{n} a_j b_j (p_j/y)_t^{b_j}}$

$$\frac{w_i}{w_n} = \frac{a_i b_i (p_i/y)_t^{b_i} e^{\varepsilon_{it}}}{a_n b_n (p_n/y)_t^{b_n} e^{\varepsilon_{nt}}}$$

And taking the logarithms:

$$(24) \quad \log \frac{w_i}{w_n} = \log \frac{a_i b_i}{a_n b_n} + b_i \log \left(\frac{p_i}{y}\right)_t - b_n \log \left(\frac{p_n}{y}\right)_t + \varepsilon_t \qquad i = 1, 2 . n \qquad A = \log \frac{a_i b_i}{a_n b_n}$$

The model is linear and O.L.S.Q. estimators are Best Linear and Unbiased. It is implied by the model's formulation that b_n has to be the same across equations.

Most of the studies in the field strongly rejected this hypothesis and our likelihood test followed the tradition. In addition most of the b's of the undonstrained estimates reveal a wrong sign or are below -1.0

The Durbin-Watson statistics indicate a strong serial correlation. To alleviate the problem the Hildreth-Lu iterative technique was employed. This method is much more flexible than just taking the first differences which in fact confines the ρ of the relation $\varepsilon_t = \rho\, \varepsilon_{t-1} + V_t$ to equal the unity.

Clearly the results are more comptatible with the theory. Also the degree of fitness is improved substantially. (R^2 was as low as .17, .14 and .33 for the three unconstrained cases of the model not correlated for auto correlation.)

TABLE 4

Indirect Addilog Model
Ordinary Least Squares Estimates

unconstrained estimates

	A_i	b_i	b_4
i=1	-6.08 (2.87)	1.12 (.69)	-1.89 (.98)
i=2	-3.77 (1.09)	-1.03 (.42)	-1.43 (.58)
i=3	-.75 (.28)	.89 (.20)	-.97 (.23)

with constrained b_4

	A_i	b_i	b_4
i=1	-.96 (.41)	-.07 (.15)	-.08 (.20)
i=2		-.06 (.15)	-.08 (.20)
i=3		-.19 (.15)	-.08 (.20)

Corrected for Autocorrelation

(Hildreth-Lu) technique.

unconstrained estimates

	A_i	b_i	b_4
$i=1$	4.48 (2.12)	-.52 (.21)	-.09 (.05)
$i=2$	-.82 (.40)	-.91 (.33)	-.62 (.18)
$i=3$	2.23 (1.03)	.15 (.04)	.29 (.19)

with constrained b_4

$i=1$	.55 (.16)	-1.09 (.37)	-.44 (.10)
$i=2$		-.86 (.30)	
$i=3$		-.22 (.04)	

The Translog Model

Starting the estimation process for the four commodity groups, we first observe that the fourth equation can be deleted due to $\Sigma w_i = 1$

Thus the system will actually be

$$\frac{q_1 p_1}{m} = \frac{a_1 + b_{11}\ln q_1 + b_{12}\ln q_2 + b_{13}\ln q_3 + b_{14}\ln q_4}{-1 + b_{m1}\ln q_1 + b_{m2}\ln q_2 + b_{m3}\ln q_3 + b_{m4}\ln q_4}$$

$$\frac{q_2 p_2}{m} = \frac{a_2 + b_{21}\ln q_1 + b_{22}\ln q_2 + b_{23}\ln q_3 + b_{24}\ln q_4}{-1 + b_{m1}\ln q_1 + b_{m2}\ln q_2 + b_{m3}\ln q_3 + b_{m4}\ln q_4} \tag{25}$$

$$\frac{q_3 p_3}{m} = \frac{a_3 + b_{31}\ln q_1 + b_{32}\ln q_2 + b_{33}\ln q_3 + b_{34}\ln q_4}{-1 + b_{m1}\ln q_1 + b_{m2}\ln q_2 + b_{m3}\ln q_3 + b_{m4}\ln q_4}$$

We note that demand theory imposes certain restriction on the coefficient of the model.

Equality restrictions. The denominators of the equations contain the same parameters i.e. $\beta_{m1}, \beta_{m2}, \beta_{m3}, \beta_{m4}$.

Symmetry restrictions i.e. $\beta_{ij} = \beta_{ji}$. There are three such restrictions in the system to be estimated. In addition since

$$
\begin{aligned}
\beta_{41} &= \beta_{m1} - \beta_{11} - \beta_{21} - \beta_{31} \\
\beta_{42} &= \beta_{m2} - \beta_{12} - \beta_{22} - \beta_{32} \qquad (26) \\
\beta_{43} &= \beta_{m3} - \beta_{13} - \beta_{23} - \beta_{43}
\end{aligned}
$$

there will be another three restrictions in the whole system.

These restrictions can be imposed in the estimation because they come straight forward from the utility maximization. However, to highlight the "goodness" of the model and more specifically its performance in the Canadian statistical environment we will estimate it without and under the restrictions. It is true that also other restrictions occur from hypotheses about the functional form of the utility function but testing such hypotheses is much beyond the aim of this work.

The model is estimated non-linearly for the direct and the indirect demand functions. In no case a satisfactory high coefficient of multiple determination is achieved, ranging from .08 to .25. The Durbin-Watson criterion does not indicate any possibility of autocorrelation. A grear part of the parameter estimates are not statistically significant.

TABLE 5

The Direct Translog Demand Function

	Unrestricted estimates	Equality restricted	Symmetry restricted	Equality and symmetry restricted
a_1	-2.7 (6.9)	3.15 (135.)	.86 (1.25)	-.42 (1.57)
b_{11}	-.48 (5.6)	-.53 (14.)	.38 (.52)	-.44 (3.12)
b_{12}	-.55 (9.1)	1.07 (7.1)	-1.08 (.97)	-1.16 (1.63)
b_{13}	1.33 (1.87)	-.66 (5.2)	-1.17 (2.31)	-.68 (.93)
b_{14}	-.36 (3.2)	-.21 (4.8)	-1.62 (3.04)	-1.15 (5.47)
b_{m1}	-.63 (.12)	-7.78 (20.)	.07 (.58)	-.89 (.10)
b_{m2}	1.25 (6.7)	4.7 (3.4)	.08 (2.4)	-1.26 (2.07)
b_{m3}	.88 (1.6)	3.2 (69)	-.39 (.88)	-1.11 (3.42)
b_{m4}	1.03 (12.)	.77 (9.5)	1.06 (1.09)	-.67 (1.35)
a_2	-.69 (15.)	2.40 (4.1)	-2.34 (4.06)	-.77 (1.14)
b_{21}	-2.2 (7.4)	-1.03 (6.5)	-1.08 (.97)	-1.16 (1.63)
b_{22}	.49 (8.2)	1.46 (7.2)	- 2.17 (4.6)	-3.55 (4.18

SEMI DURABLES

	Unrestricted estimates	Equality restricted	Symmetry restricted	Equality and symmetry restricted
b_{23}	-.69 (3.2)	-.48 (.52)	-.91 (1.69)	-1.02 (1.16)
b_{24}	2.11 (9.3)	-.16 (.39)	1.77 (2.12)	-.01 (.17)
b_{m1}	-.79 (11)	-7.78 (20.)	-.56 (.48)	-.89 (10.)
b_{m2}	-.62 (1.5)	4.7 (3.4)	2.12 (8.32)	-1.26 (2.07)
b_{m3}	1.38 (7.15)	3.2 (69)	.04 (.13)	-1.11 (3.42)
b_{m4}	4.3 (9.8)	.77 (9.5)	-.97 (2.1)	-.67 (1.35)
a_3	-1.59 (4.9)	4.14 (36)	6.12 (18.)	.62 (1.33)
b_{31}	7.2 (139)	-2.6 (7.8)	1.17 (2.31)	-.68 (.93)
b_{32}	.89 (10.)	3.0 (8.9)	-91 (1.64)	-1.02 (1.16)
b_{33}	.42 (.55)	-.27 (45.)	.58 (.64)	-.10 (.41)
b_{34}	1.17 (3.6)	-.44 (5.1)	-.24 (.47)	.62 (.57)

b_{m1}	-.22 (4.1)	-7.78 (20)	.69 (.78)	-.89 (10.)
b_{m2}	1.46 (6.2)	4.7 (3.4)	-1.52 (1.74)	-1.26 (2.07)
b_{m3}	6.4 (9.5)	3.2 (6.9)	-1.61 (2.26)	-1.11 (3.42)
b_{m4}	-.07 (.21)	.77 (9.5)	-.48 (.80)	-.67 (1.35)

The Indirect Translog Demand Functions

	Unrestricted estimates	Equality restricted	Symmetry restricted	Equality and symmetry restricted
a_1	-.192 (.05)	-.415 (2.4)	.58 (.17)	-.06 (.009)
b_{11}	-.46 (.28)	-1.46 (.79)	-.04 (.04)	-.18 (.08)
b_{12}	-.91 (55)	-.04 (.05)	.62 (.13)	-.67 (1.14)
b_{13}	1.53 (2.12)	.09 (.07)	-.51 (.18)	.15 (.13)
b_{14}	-.04 (.02)	.52 (.31)	.06 (.10)	.08 (.05)
b_{m1}	.03 (.05)	-.59 (.27)	.07 (.09)	-.37 (.16)
b_{m2}	1.17 (1.06)	-.14 (.09)	-.13 (.03)	.21 (.52)
b_{m3}	-.56 (.41)	-.17 (.09)	-.14 (.06)	-.08 (.07)
b_{m4}	-.06 (.008)	-.22 (.33)	-.09 (.08)	-.19 (.12)
a_2	.45 (.06)	-.07 (.02)	-1.12 (.48)	-1.66 (.85)

b_{21}	-.47	.06	.62	-.67
	(.81)	(.11)	(.13)	(1.14)
b_{22}	-.09	-1.81	1.27	.26
	(.11)	(1.07)	(1.81)	(.12)
b_{23}	-1.84	-.52	-.62	-.12
	(.52)	(.61)	(.27)	(.06)
b_{24}	.62	.34	.07	-.17
	(.37)	(1.13)	(.02)	(.05)
b_{m1}	-.39		1.21	
	(22)		(1.51)	
b_{m2}	-3.4		-1.23	
	(.09)		(.25)	
b_{m3}	-.06		-.32	
	(.01)		(.14)	
b_{m4}	.59		-.26	
	(.18)		(.19)	
a_3	-.03	-.58	-.09	1.52
	(.01)	(.41)	(.14)	(1.04)
b_{31}	-.17	.32	-.51	.15
	(.14)	(.07)	(.18)	(.13)
b_{32}	-.51	-.006	-.62	-.12
	(.48)	(.002)	(.27)	(.06)
b_{33}	.004	-.08	.36	-.36
	(.008)	(.03)	(.22)	(.14)
b_{34}	.002	-1.15	-.05	.009
	(.002)	(.89)	(.02)	(.002)
b_{m1}	.61		-.36	
	(.27)		(.19)	
b_{m2}	1.32		.04	
	(1.21)		(.05)	
b_{m3}	-1.16		.18	
	(.52)		(.16)	
b_{m4}	-.006		-.51	
	(.004)		(.46)	

The likelihood ratio test which was explained previously, accepts the null hypothesis when symmetry or equality is individually tested and also does not fail to accept symmetry and equality simultaneously, for the direct demand functions. Neither equality nor symmetry is likely to exist for indirect demand function coefficients at a statistically significant level. Does it imply that statistical data proves the neo-classical demand theory invalid? Let's now attempt to draw our conclusions.

Conclusions

A serious and concrete attempt to compare the models would involve the "information inaccuracy" criterion, $\left(I = \sum_{i=1}^{n} w_{it} \log \frac{w_{it}}{\hat{w}_{it}} \right.$ where w_{it} and $\hat{w}_{it}$ are the true and the predicted expenditure shares$\left.\right)$ and the inplicit elasticity estimates at mean expenditure shares but such a task was not undertaken because in general the performance of the models was not found satisfactory. In terms of their fitness to the sample data the Linear Expenditures and the unconstrained Rotterdam model seem to be superior with R^2 in the range of .98 to 1. It is also interesting to mention that the two versions of the Indirect Addilog model have R^2 equal to .20 (unconstrained) and .93 (constrained)

However, the important finding of this work is that the Rotterdam and the Translogarithmic model fail to confirm the Neoclassical Demand Theory with the Canadian Statistical information. There is a catch somewhere. Either in the data, or in the model's formulation and estimation or, God bless us, in the Economic Theory.

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